

Recitation 6.5 & 6.6.

Question 1

$$a) \cot^{-1}(-\sqrt{3}) = \theta \quad \Leftrightarrow \quad \left. \begin{array}{l} \cot \theta = -\sqrt{3} \\ 0 < \theta < \pi \end{array} \right\} \Rightarrow \theta \in \text{QII}$$

$$\cot \theta' = \sqrt{3} \Rightarrow \tan \theta' = \frac{1}{\sqrt{3}} \Rightarrow \theta' = \frac{\pi}{6}$$

$$\Rightarrow \theta = \cot^{-1}(-\sqrt{3}) = \pi - \theta' = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$\theta \in \text{QII}$

$$b) \tan^{-1}\left(\tan \frac{4\pi}{3}\right) = \tan^{-1}\left(\tan\left(\frac{4\pi}{3} - \pi\right)\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{3}\right)\right)$$

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$= \boxed{\frac{\pi}{3}}$$

$$c) \sin^{-1}(\sin \pi) = \sin^{-1}(0) = \theta \quad \Leftrightarrow \quad \begin{array}{l} \sin \theta = 0 \\ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \end{array}$$

$$\Rightarrow \theta = 0 \quad \rightarrow \quad \sin^{-1}(\sin \pi) = 0$$

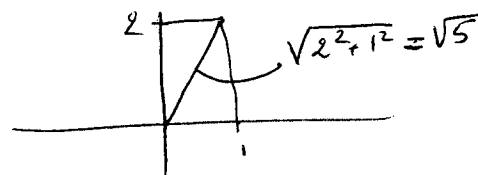
$$d) \cos\left(\cot^{-1}\left(\frac{5}{4}\right)\right) \quad \text{undefined}$$

$[-1, 1] = \text{Dom}(\cos^{-1})$

$$e) \sec(\tan^{-1}(2))$$

$$\text{let } \theta = \tan^{-1}(2) \quad \Leftrightarrow \quad \tan \theta = 2 \Rightarrow \theta \in \text{QI}$$

$\theta \in \text{QI, IV}$ > 0

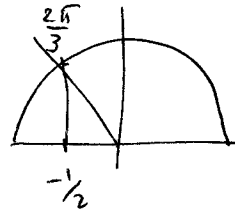


$$\sec(\tan^{-1}(2)) = \sec(\theta) = \frac{r}{x} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

Question 2.

$$\cos^{-1}\left(-\frac{1}{2}\right) + \tan^{-1}\left(\tan \frac{5\pi}{6}\right)$$

$$\begin{aligned} \cos^{-1}\left(-\frac{1}{2}\right) &= \pi - \theta' \\ &= \pi - \frac{\pi}{3} = \frac{2\pi}{3} \end{aligned}$$



$$\tan^{-1}\left(\tan \frac{5\pi}{6}\right) = \tan^{-1}\left(\tan \frac{5\pi}{6} - \pi\right) = \tan^{-1}\left(\tan\left(-\frac{\pi}{6}\right)\right) = -\frac{\pi}{6}$$

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\rightarrow \frac{2\pi}{3} - \frac{\pi}{6} = \frac{4\pi}{6} - \frac{\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$$

Question 3.

$$\tan\left(2 \underbrace{\cos^{-1} x}_{\theta}\right) = \frac{2x\sqrt{1-x^2}}{2x^2-1}$$

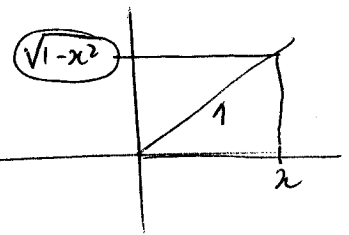
$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\theta = \cos^{-1}(x) \Leftrightarrow \cos \theta = x$$

$\theta \in I, II$

$$\tan 2\theta = \frac{2 \frac{\sqrt{1-x^2}}{x}}{1 - \left(\frac{\sqrt{1-x^2}}{x}\right)^2} = \frac{2\sqrt{1-x^2}}{x}$$

$$\frac{1}{\frac{x^2 - (1-x^2)}{x^2}}$$

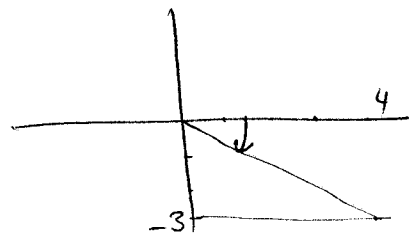


$$= \frac{2x\sqrt{1-x^2}}{2x^2-1} \quad \checkmark$$

Question 4.

$$\cos^{-1}\left(\frac{x}{2}\right) + \sin^{-1}\left(-\frac{3}{5}\right) - \frac{\pi}{3} = 0$$

$$\cos^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{3} - \underbrace{\sin^{-1}\left(-\frac{3}{5}\right)}_{\theta}$$



$$\frac{x}{2} = \cos\left(\frac{\pi}{3} - \theta\right) = \cos\frac{\pi}{3} \cos\theta + \sin\frac{\pi}{3} \sin\theta$$

$$= \frac{1}{2} \cdot \frac{4}{5} + \frac{\sqrt{3}}{2} \left(-\frac{3}{5}\right) = \frac{4 - 3\sqrt{3}}{10}$$

$$x = 2 \left(\frac{4 - 3\sqrt{3}}{10} \right) = \frac{4 - 3\sqrt{3}}{5}$$

Question 5.

$$-\frac{\pi}{2} \leq \sin^{-1}(2x-1) \leq \frac{\pi}{2}$$

$$\frac{3\pi}{2} \geq -3 \sin(2x-1) \geq -\frac{3\pi}{2}$$

Range is $\left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$

Domain $-1 \leq 2x-1 \leq 1$

$$0 \leq 2x \leq 2$$

$$0 \leq x \leq 1$$

Domain = $[0, 1]$

Question 6.

Find the solutions of the equation

$$2 \sin x \cos x - \sqrt{2} \cos x - 2 \sin x = -\sqrt{2}$$

$$2 \sin x \cos x - \sqrt{2} \cos x - 2 \sin x + \sqrt{2} = 0$$

4 terms \Rightarrow Factor by grouping

$$\cos x (2 \sin x - \sqrt{2}) - (2 \sin x - \sqrt{2}) = 0$$

Same term

$$(2 \sin x - \sqrt{2})(\cos x - 1) = 0$$

$$2 \sin x - \sqrt{2} = 0$$

$$\text{or } \cos x = 1$$

$$\sin x = \frac{\sqrt{2}}{2} > 0 \begin{cases} \text{QI} \\ \text{QII} \end{cases}$$

$$\cos x = 1 \quad (1 \text{ sol}^n \text{ per period})$$

$$x = 45^\circ$$

$$x = \frac{\pi}{2} = 90^\circ$$

$$\text{I} \Rightarrow x = 45^\circ$$

$$\text{II} \rightarrow x = 180 - 45^\circ = 135^\circ$$

$$SS = \{45^\circ, 90^\circ, 135^\circ\}$$