

Extra Solved Problems for 4.2.

①

1)  $f(x) = -3^{-x-1} + 2$

Domain, range, x-int, y-int, graph, H.A.

Sol<sup>n</sup>

Dom =  $(-\infty, \infty)$

Range:  $(-\infty, 2)$

x-int:  $y = 0 = -3^{-x-1} + 2 \Rightarrow 3^{-x-1} = 2$

$\log_3 2 = -x - 1 \Rightarrow x = -1 - \log_3 2$

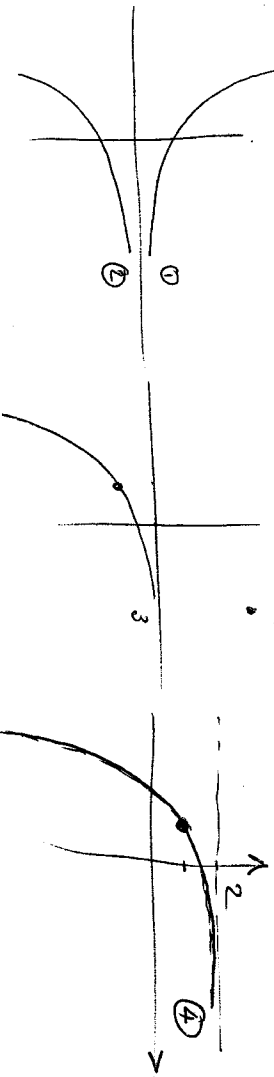
y-int:  $x = 0 \Rightarrow y = -3^{-0-1} + 2 = -\frac{1}{3} + 2 = \frac{5}{3}$

H.A.  $y = 2$

Graph Rewrite:

$y = -3^{-x-1} + 2 = -\left(\frac{1}{3}\right)^{x+1} + 2$

$y = \left(\frac{1}{3}\right)^x$  shift  $\frac{1}{x}$  to  $\frac{1}{x-0}$   $\rightarrow y = \left(\frac{1}{3}\right)^x$  shift  $\frac{1}{x}$  to  $\frac{1}{x-0}$   $\rightarrow y = -\left(\frac{1}{3}\right)^{x+1} + 2$



2)  $y = |3^x - 2|$ , Graph, domain, range, x-int, y-int, H.A.

②

Sol<sup>n</sup> Dom:  $(-\infty, \infty)$

x-int,  $y = 0 = |3^x - 2| \Rightarrow 3^x - 2 = 0$

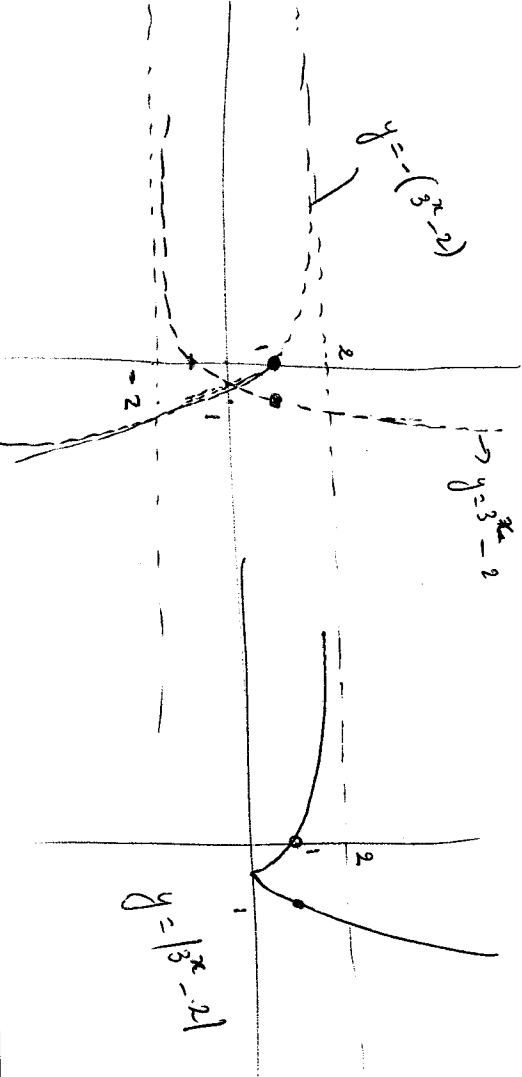
$3^x = 2 \Leftrightarrow x = \log_3 2 \Rightarrow (\log_3 2, 0)$

y-int,  $x = 0 \Rightarrow y = |3^0 - 2| = |-1| = 1 \Rightarrow (0, 1)$

$x \rightarrow -\infty, 3^{-x} \rightarrow 0 \Rightarrow y = |3^x - 2| \rightarrow |0 - 2| = 2 \Rightarrow y = 2$  H.A.

$y = |3^x - 2| = \begin{cases} 3^x - 2 & \text{if } 3^x - 2 \geq 0 \rightarrow \text{above } x\text{-axis} \\ -(3^x - 2) & \text{if } 3^x - 2 < 0 \rightarrow \text{below } x\text{-axis} \end{cases}$

Reflection of  $y = 3^x - 2$  about x-axis



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3) Find the domain of

$$f(x) = \frac{e^{2x} - e^x}{1 - e^x}, \quad g(x) = \sqrt{2^x - 4}$$

for  $f(x)$ ,  $1 + e^x \neq 0$

$$1 \neq e^x \quad (\Rightarrow) \quad x \neq 0$$

Dom:  $(-\infty, 0) \cup (0, \infty)$ .

For  $g(x)$ ,  $2^x - 4 \geq 0$

$$2^x \geq 4 \Rightarrow 2^x \geq 2^2$$

$$\Rightarrow x \geq 2$$

$$\text{Dom} = [2, \infty)$$

4) Find the inverse of  $f(x) = -2^{x+3} - 4$

$$y = -2^{x+3} - 4$$

Interchange  $x$  &  $y$ ,

$$x = -2^{y+3} - 4$$

Solve for  $y$

$$x + 4 = -2^{y+3}$$

Isolate exp

$$-x - 4 = 2^{y+3}$$

Change to log form

$$\log_2(-x-4) = -y+3$$

$$-3 + \log_2(-x-4) = -y$$

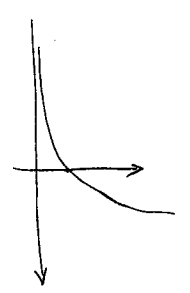
$$f^{-1}(x) = y = 3 - \log_2(-x-4)$$

4

Small Remarks

- Graph of  $y = |e^x|$

$$e^x > 0 \Rightarrow y = |e^x| = e^x$$



- Dom of  $f(x) = \frac{1}{2^x + 3^x}$

$$2^x > 0, \quad 3^x > 0 \Rightarrow 2^x + 3^x > 0 \text{ never zero}$$

$$\text{Dom} = (-\infty, \infty)$$