# - Extra material on 4.4 -

$\log_{\frac{1}{b}} x = -\log_b x$ $\quad \because \log_{\frac{1}{b}} x = \dfrac{\log_b x}{\log_b \frac{1}{b}} = -\log_b x$

$\log_b a = \dfrac{1}{\log_a b}$ $\quad \because \log_b a = \dfrac{\log_a(a)}{\log_a(b)} = \dfrac{1}{\log_a b}$

**Ex1.** If $\log_c 2 = 2/3$ Find $\log_8 c$

**Ex2.** Simplify

$$\log_5 20 \cdot \log_{20}(60) \cdot \log_{60} 100 \cdot \log_{100}(125)$$

**Ex3.** Write as a simple logarithm

$$\log_2(x+2) + \log_4(y-1) - \log_8(4)$$

**Ex4** If $\log x = 2$, $\log y = 3$, $\log z = 5$
Find the value of $\log \dfrac{x^3 y}{\sqrt{z}} - \log_{x^2} z$

**Ex 5:** Graph $y = \log x^2$ using $y = \log x$.

**Ex 6:** True or False

1) $\log x^2 = 2 \log |x|$ for all $x \in (-\infty, \infty)$
2) $\log x^2 = 2 \log x$ " " " "
3) $\dfrac{\log x}{\log y} = \log \dfrac{x}{y}$
4) $(\log x)\cdot(\log y) = \log(x+y)$
5) $(\log x)^r = r \log x$

Ex 1.

$\log_c 2 = 2/3 \implies \log_8 c = \dfrac{\log_c(c)}{\log_c 8} = \dfrac{1}{\log_c 2^3} = \dfrac{1}{3\log_c(2)}$

$= \dfrac{1}{3 \cdot (2/3)} = \boxed{\dfrac{1}{2}}$

Ex 2.

$\log_5 20 \cdot \log_{20}(60) \cdot \log_{60}(100) \cdot \log_{100}(125)$

$= \log_5(20) \cdot \dfrac{\log_5 60}{\log_5(20)} \cdot \dfrac{\log_5(100)}{\log_5(60)} \cdot \dfrac{\log_5(125)}{\log_5(100)} = \log_5(125) =$

$= \log_5(5^3) = 3$

Ex 3

$\log_2(x+2) + \log_4(y-1) - \log_8(4)$

$\log_2(x+2) + \dfrac{\log_2(y-1)}{\log_2(4)} - \dfrac{\log_2 4}{\log_2 8}$

$= \log_2(x+2) + \frac{1}{2}\log_2(y-1) - \frac{1}{3}\log_2 4$

$= \boxed{\log_2 \dfrac{(x+2)(y-1)^{1/2}}{4^{1/3}}}$

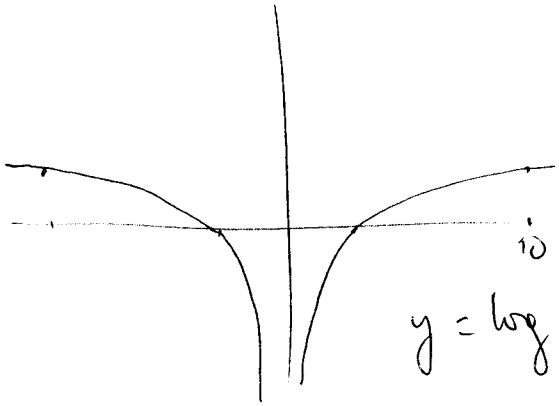
Eg 4

$\log \dfrac{x^3 y}{\sqrt{z}} - \log_x z = 3\log x + \log y - \frac{1}{2}\log z - \dfrac{\log z}{\log x}$

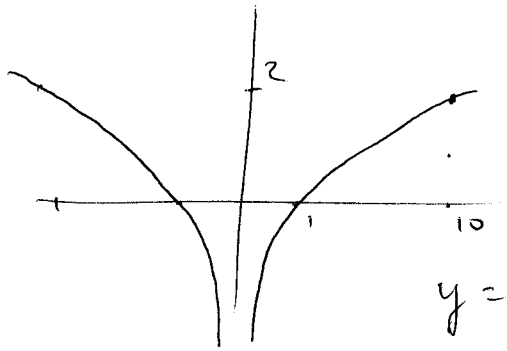
$= 3(2) + 3 - \frac{1}{2}(5) - \dfrac{5}{2} = 6 + 3 - 5 = \boxed{4}$

5) $y = \log x^2 = \log |x|^2 = 2\log|x|$

$y = \log|x| \xrightarrow{\text{V. Streteching by 2.}} y = 2\log|x|$




$y = \log|x|$




$y = \log x^2$

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6)

1) true for

$\log x^2 = \log|x|^2 = 2\log|x|$

2) $\log x^2 = 2\log x$ false

~~because~~ it is undefined for $x < 0$

3) $\frac{\log x}{\log y} = \log \frac{x}{y}$ false

$x = 10^2$, $y = 10$, $\frac{\log 10^2}{\log 10} = 2$ $\log \frac{10^2}{10} = \log 10 = 1$

4) $(\log x)(\log y) = \log(x+y)$ False

$(\log 10)(\log 10)$ $\log(10+10) = \log 2 + \log 10$

$1$ $= (\log 2) + 1$

$\neq 1$

5) $(\log x)^r = r \log x$ False.

$(\log 10)^2 = 2\log 10$

$1^2 = 2$

$1 = 2$ False.