Question from Exercises booklet

$\arcsin = \sin^{-1}$, $\arccos = \cos^{-1}$

$\arctan = \tan^{-1}$

1. Evaluate.

$$\sec(\arccos x) + \sin(\arccos x)$$

$$= \frac{1}{\cos(\cos^{-1} x)} + \sqrt{1-x^2} = \frac{1}{x} + \sqrt{1-x^2}$$

2. Domain & Range of

$$y = \arcsin(2x) + \frac{\pi}{2}$$

$$2x \in [-1, 1]$$

$$x \in \left[-\tfrac{1}{2}, \tfrac{1}{2}\right]$$

Dom : $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Ran $\left(\arcsin(2x)\right) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Ran $\left(\arcsin(2x) + \frac{\pi}{2}\right) = \left[-\frac{\pi}{2} + \frac{\pi}{2}, \frac{\pi}{2} + \frac{\pi}{2}\right]$

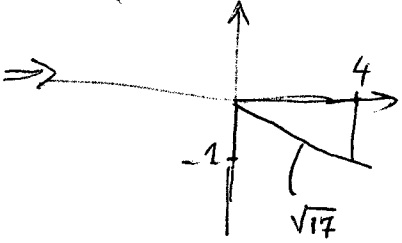
$= [0, \pi]$

3. Evaluate

$$\cos\left(\arctan\left(-\tfrac{1}{4}\right)\right) = y$$

$\theta = \arctan\left(-\frac{1}{4}\right) \Rightarrow \begin{cases} \tan\theta = -\frac{1}{4} \\ -\frac{\pi}{2} < \theta < \frac{\pi}{2} \end{cases}$

$\Rightarrow \theta$ in QIV



$$y = \cos\theta = \frac{x}{r} = \boxed{\frac{4}{\sqrt{17}}}$$

4. Find the domain & Range of

$$y = \arcsin(3x+1) - \pi$$

$$-1 \le 3x+1 \le 1 \Rightarrow -2 \le 3x \le 0$$

$$\boxed{-\frac{2}{3} \le x \le 0} \text{ — Dom}$$

$$\text{Ran} = \left[-\frac{\pi}{2} - \pi, +\frac{\pi}{2} - \pi\right]$$

Rule

$$\text{Ran}\left(\arcsin(ax+b) + d\right) = \left[-\frac{\pi}{2} + d, \frac{\pi}{2} + d\right]$$

Dom : $-1 \le ax + b \le 1$ $\quad a > 0$

Solve for x.

$$-\frac{1}{a} - \frac{b}{a} \le x \le \frac{1}{a} + \frac{b}{a}.$$

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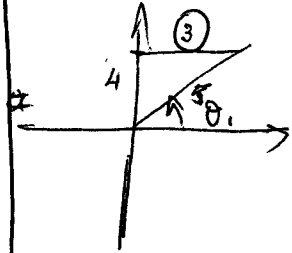
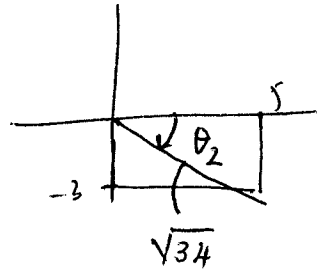
6. Evaluate

$$\sin\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(-\tfrac{3}{5}\right)\right)$$

$$= \cos\left(\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(-\tfrac{3}{5}\right)\right)$$

$$= \cos\underbrace{\left(\sin^{-1}\left(\tfrac{4}{5}\right)\right)}_{\theta_1}\cos\underbrace{\left(\tan^{-1}\left(-\tfrac{3}{5}\right)\right)}_{\theta_2} + \sin\left(\sin^{-1}\left(\tfrac{4}{5}\right)\right)\sin\left(\tan^{-1}\left(-\tfrac{3}{5}\right)\right)$$





$$= \frac{3}{5}\cdot\frac{5}{\sqrt{34}} + \frac{4}{5}\cdot\left(\frac{-3}{\sqrt{34}}\right)$$

$$= \frac{15-12}{5\sqrt{34}} = \frac{3}{5\sqrt{34}} = \boxed{\frac{3\sqrt{34}}{170}}$$

7) Which is true

(a) $2\arccos(x) = \arccos(2x)$

(b) $y = \arcsin(x+2) \Rightarrow x = -2 + \sin y$

(c) $\arctan x = \dfrac{\arcsin x}{\arccos x}$

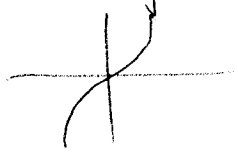
(d) $y = \arcsin x$ is even f^n

e) $\operatorname{arccot} x = \dfrac{1}{\arctan x}$

In general, the properties of sin, cos, tan are not true for $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

So (c) & e) are not true.

arcsin is not even from its graph



(b) is true because

$y = \sin^{-1}(x+2) \Rightarrow x + 2 = \sin y$

$\boxed{x = -2 + \sin y}$

9) Evaluate

$$\operatorname{arccsc}\left(\frac{1}{\sin\left(\frac{3\pi}{4}\right)}\right)$$

$\frac{3\pi}{4}$ in QII, $\sigma' = \frac{\pi}{4}$

$$= \operatorname{arccsc}\left(\csc\left(\frac{3\pi}{4}\right)\right)$$

$\csc$ is defined on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

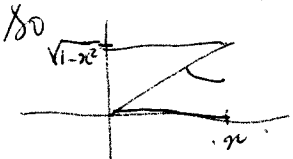
$$= \operatorname{arccsc}\left(\csc\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{4}$$

11) Write in terms of x.

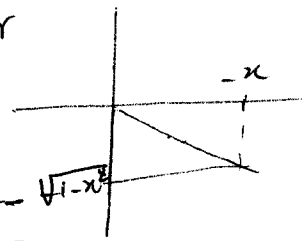
$$\cos\left(\underbrace{\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)}_{\theta}\right)$$

$$\tan\theta = \frac{\sqrt{1-x^2}}{x}$$

So



or



hypotenuse: $\sqrt{x^2 + (1-x^2)} = 1$

$\Rightarrow \boxed{\cos\theta = |x|}$

8) Dom & Range of

$y = \cos(\arcsin 2x)$

is defi for $2x \in [-1, 1]$

Dom = $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Range is $[0, 1]$

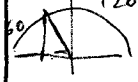
$\because \theta = \arcsin 2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \cos(\theta) \in [0, 1]$

13) Evaluate.

$\cos^{-1}$ is defined on $[0, \pi]$

$$\cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\sin\frac{7\pi}{6}\right)$$



$$= \frac{2\pi}{3} + \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$$

$$= \frac{2\pi}{3} + \sin^{-1}\left(+\sin\left(-\frac{\pi}{6}\right)\right) = \frac{2\pi}{3} - \frac{\pi}{6}$$

$$= \boxed{\frac{\pi}{2}}$$