KFUPM
Math002
7 May 2007

NAME:
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Serial: Sec:

QUIZ # 4 (A)

1) Solve $2\sin^2(2x)-5\sin(2x)-3=0$ in $[0,2\pi)$

Let $\theta = 2x$

$0 \le x < 2\pi \Rightarrow 0 \le \theta = 2x \le 4\pi$ (2 periods of $\sin\theta$)

$2\sin^2\theta - 5\sin\theta - 3 = 0$

$(2\sin\theta + 1)(\sin\theta - 3) = 0$

$\sin\theta = -\frac{1}{2}$ $\quad \sin\theta = 3$ No solution

$\theta' = \frac{\pi}{6}$, θ in III or IV

$\Rightarrow \theta = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ in 1st period

in second period

$\theta = \frac{7\pi}{6} + 2\pi = \frac{19\pi}{6}$

$\theta = \frac{11\pi}{6} + 2\pi = \frac{23\pi}{6}$

$\Rightarrow x = \frac{\theta}{2} = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

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2) Let $U = 2\sqrt{3}\,i - 2j$ and $V = 3\sqrt{3}\,i + 3j$

a) Find the direction angle of U

$\|U\| = 2\|\sqrt{3}\,i - j\| = 2\sqrt{3+1} = 4$

$\cos\theta = \frac{a}{\|U\|} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$, $\sin\theta = \frac{-2}{4} = -\frac{1}{2}$

$\theta' = \frac{\pi}{6}$, θ in IV

$\theta = \frac{11\pi}{6}$

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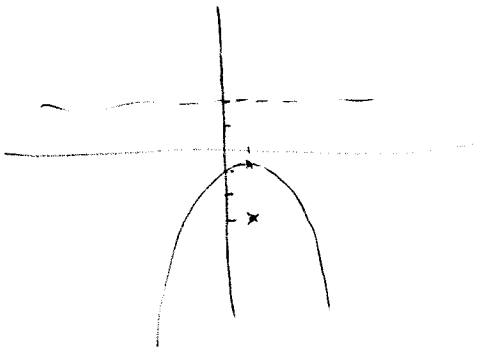
b) The smallest positive angle between U and V

$\|U\| = 4$, $\|V\| = 6$

$\cos\alpha = \frac{U\cdot V}{\|U\|\,\|V\|} = \frac{6\cdot 3 - 6}{4\cdot 6} = \frac{12}{4\cdot 6} = \frac{1}{2} \Rightarrow \alpha = 60^\circ = \frac{\pi}{3}$

3) Find the equation of the parabola that has focus at (1,-3) and directrix at $y = 2$



$V = (1, -\frac{1}{2})$ $\quad p = 2.5 = -\frac{5}{2}$

$(x-1)^2 = 4\left(-\frac{5}{2}\right)\left(y + \frac{1}{2}\right) = -10\left(y + \frac{1}{2}\right)$

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4) Find the center, vertices and foci of the ellipse with equation

$16x^2 + 9y^2 + 36y - 108 = 0$

$16x^2 + 36y + 9y^2 = 108$

$16x^2 + 9(y^2 + 4y + 4) = 108 + 36 = 144$

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