KFUPM
Math002
7 May 2007

NAME:
ID:
Serial: Sec:

QUIZ # 4 (D)

1) Solve $2\sin^2(4x)+3\sin(4x)+2=0$ in $[0,\pi)$

$2\sin^2 4x + 3\sin 4x + 2 = 0$

$(2\sin(4x) + 1)(\sin 4x + 2) = 0$

$\sin(4x) = -1/2$ $\quad 0 \le 4x < 4\pi$ (2 periods)

$4x = 7\pi/6$ or $11\pi/6$

$4x = \frac{7\pi}{6} + 2\pi$, $\frac{11\pi}{6} + 2\pi$ (2^{nd} period)

$= \frac{19\pi}{6}$, $\frac{23\pi}{6}$

$x = 4x/4$

$x = \frac{7\pi}{24}$, $x = \frac{11\pi}{24}$

$x = \frac{19\pi}{24}$, $x = \frac{23\pi}{24}$

$\sin 4x = -2$ No solutions.

2) Let $U = 2\sqrt{3}\,i - 2j$ and $V = 3\sqrt{3}\,i + 3j$

a) Find the direction angle of V

$\|V\| = 3\sqrt{3+1} = 6$

$\cos\theta = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$, $\sin\theta = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$, $\theta \in QI$

$\Rightarrow \theta = \pi/6 = 30^\circ$

b) Find a vector W of magnitude 3 in the same direction of U+V

$U+V = 5\sqrt{3}\,i + j = \langle 5\sqrt{3}, 1\rangle$

$\|U+V\| = \sqrt{25\cdot 3 + 1} = \sqrt{76} = 2\sqrt{19}$

$W = 3\left\langle \frac{5\sqrt{3}}{2\sqrt{19}}, \frac{1}{2\sqrt{19}} \right\rangle$

76 | 2
38 | 2
19 | 19

3) Find the vertex, focus and directrix of the parabola with equation $2x^2+4x+8y-5=0$

$2x^2 + 4x = -8y + 5$

$2(x^2+2x+1) = -8y + 5 + 2 = -8(y - 7/8)$

$(x+1)^2 = -4(y - 7/8)$

$p = -1$, $V = (h,k) = (-1, 7/8)$

Focus: $(-1, k+p) = (-1, \frac{7}{8} - \frac{1}{8})$

Directrix: $y = k - p = \frac{7}{8} + 1 = \frac{15}{8}$

4) Find the equation of the ellipse with vertices at (-7,-1) and (5,-1) and one focus at (3,-1)

$C = \left(\frac{-7+5}{2}, \frac{-1-1}{2}\right) = (-1,-1) = (h,k)$ $\quad (h+a,k)$ $\quad (h+c,k)$

$h + a = 5$ $\quad h + c = 3$

$-1 + a = 5$ $\quad -1 + c = 3$

$a = 6$ $\quad c = 4$

$b^2 = a^2 - c^2 = 36 - 16 = 20$

Major Axis $y = -1$

$\frac{(x+1)^2}{36} + \frac{(y+1)^2}{20} = 1$

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QUIZ # 4 (C)

1) Solve $2\sin^2(4x)+3\sin(4x)+2=0$ in $[0,\pi)$

$2\sin^2(4x)+3\sin(4x)-2=0$

$(2\sin(4x)-1)(\sin(4x)+2)=0$

$\sin(4x)=1/2$ $\qquad$ $\sin(4x)=-2$ No sol.

$\theta=4x$, $\sin\theta=1/2$

$0\le x<\pi$

$0\le 4x<4\pi$

$0\le\theta<4\pi$ → 2 periods

$\theta'=\frac{\pi}{6}$, θ in I or II

$\theta=\boxed{\frac{\pi}{6}}$ or $\boxed{\frac{5\pi}{6}}$ (in 1st period)

$\theta=\frac{\pi}{6}+2\pi=\boxed{\frac{13\pi}{6}}$

$\theta=\frac{5\pi}{6}+2\pi=\boxed{\frac{17\pi}{6}}$

Then $x=\frac{\theta}{4}$.

$=\frac{\pi}{24}$, $\frac{5\pi}{24}$, $\frac{13\pi}{24}$, $\frac{17\pi}{24}$

2) Let $U=2\sqrt{3}\,i-2j$ and $V=3\sqrt{3}\,i+3j$

a) Find the direction angle of U

$\|U\|=2\|\sqrt{3}\,i-j\|=2\sqrt{3+1}=4$

$\cos\theta=\frac{2\sqrt{3}}{4}=\frac{\sqrt{3}}{2}$, $\sin\theta=-1/2$

$\theta'=\pi/6$, $\theta\in QIV$

$\theta=\frac{11\pi}{6}$

b) Find a vector W of magnitude 4 in the opposite direction of U+V

$U+V=5\sqrt{3}i+j \Rightarrow \|U+V\|=\sqrt{25\cdot3+1}=\sqrt{76}=\sqrt{2^2\cdot19}=2\sqrt{19}$

$W=-4\left\langle\frac{5\sqrt{3}}{2\sqrt{19}},\frac{1}{2\sqrt{19}}\right\rangle=\left\langle\frac{-10\sqrt{3}}{\sqrt{19}},\frac{-2}{\sqrt{19}}\right\rangle$

3) Find the vertex, focus and directrix of the parabola with equation $4x^2-12x+12y+7=0$

$4x^2-12x=-12y+7$

$4(x^2-3x+(\frac{3}{2})^2)=-12y-7+9$

$4(x-3/2)^2=-12(y+\frac{1}{6})$

$(x-3/2)^2=-3(y+1/6)$

$C=(3/2,\ 1/6)$

$p=-3/4$

$F=(3/2,\ 1/6-3/4)$

$D: y=1/6+3/4$

4) Find the equation of the ellipse with vertice at (5,6) and (5,-4) and one focus at (5,4)

(h, k+a) $\qquad$ k−a $\qquad$ (h, k+c)

$h=5$

$k+a=6$

$k-a=-4$

$2k=2 \rightarrow k=1$

$a=6-k=5$

$c=4-k=3$ $\Rightarrow b^2=a^2-c^2=25-9=16$

Maj axis is verti $x=5$

$\Rightarrow \boxed{\frac{(y-1)^2}{25}+\frac{(x-5)^2}{16}=1}$