King Fahd University of Petroleum and Minerals
College of Sciences Prep-Year Math Program

Master

Math 002 Final Exam
Term 023
Thursday, August 21, 2003
Time Allowed: 2-1/2 hours

Master

Student's Name: KEY SOLUTIONS

ID #: ____________ Section #: ____________

Important Instructions:

1. All types of Calculators, Pagers or Telephone are NOT allowed during the examination.
2. DO NOT any mark on a choice of any answer on the exam paper.
3. Use HB 2.5 pencils only.
4. Use a good eraser. DO NOT use the erasers attached to the pencil.
5. Write your name, ID number and Math Section number on both the examination paper and the OMR sheet.
6. Detach the OMR sheet carefully.
7. When bubbling your ID number and Math Section number, be sure that the bubbles match with the number that your write.
8. Math the Test Code Number already bubbled in your answer sheet with the Test Code Number printed on your question paper.
9. When erasing a bubble, make sure that you do not leave any trace of penciling.
10. Check that the exam paper has 30 questions.

1. Let $f(x)=e^{-x}+1$, then which one of the following statements is FALSE?

a) The range of f is $(0,\infty)$. F

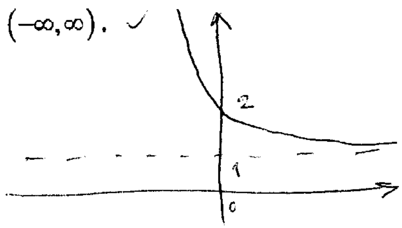
b) The graph of f has no x-intercept. ✓

c) The graph of f has a y-intercept at $(0,2)$. ✓

d) The graph of f decreases on the interval $(-\infty,\infty)$. ✓

e) The domain of f is $(-\infty,\infty)$. ✓

Graph $f(x) = e^{-x}+1$



2. The domain of the logarithmic function $f(x)=1+\log(x^3-x)$ is:

a) $(-1,0)\cup(1,\infty)$

b) $(-1,1)\cup(1,\infty)$

c) $(-1,\infty)$

d) $(-2,1)\cup(1,\infty)$

e) $(-2,0)\cup(2,\infty)$

$x^3 - x > 0$

$x(x^2-1) > 0$

$x(x+1)(x-1) > 0$

x		-1		0		1	
x	$-$		$-$	0	$+$		$+$
$x-1$	$-$		$-$		$-$	0	$+$
$x+1$	$-$	0	$+$		$+$		$+$
	$-$		$+$		$-$		$+$

$D = (-1, 0) \cup (1, \infty)$

3. The number of real solutions of the exponential equation
$\frac{10^x - 10^{-x}}{2} = 20$ is:

a) 1

b) 0

c) 2

d) 3

e) 4

$10^x - 10^{-x} = 40$

$(10^x)^2 - 1 = 40 \cdot 10^x \quad \times 10^x$

put $y = 10^x$

$y^2 - 40y - 1 = 0$

$\Delta = \sqrt{(40)^2 - 4(-1)} = \sqrt{1604} > 40$

$y = \frac{+40 \pm \sqrt{\Delta}}{2}$

$y_1 = \frac{40 - \sqrt{1604}}{2} = 10^x$

$y_1 < 0 \Rightarrow$ No solⁿ.

$y_2 = \frac{40 + \sqrt{1605}}{2} = 10^x$

$x = \log\left(\frac{40 + \sqrt{1604}}{2}\right)$

1 solⁿ

4. Which one of the following statements is TRUE?

a) 40° and 400° are coterminal angles. $400^\circ - 40^\circ = 360^\circ \Rightarrow$ coterminal

b) Angles that have a measure greater than 90° but less than 180° are acute angles. False they are obtuse

c) 90° angles are straight angles. a straight angle $= 180^\circ$

d) π radian $= \pi^\circ$ π radian $= 360^\circ$ not 3.14°

e) π radian is less than π° π radian $< \pi^\circ$

$360^\circ < 3.14^\circ$ False

5. If $W\left(\frac{-17\pi}{6}\right) = P(x,y)$, then $x-y=$

cos even

$x = \cos\left(-\frac{17\pi}{6}\right) = \cos\left(\frac{17\pi}{6}\right) = \cos\left(\frac{12\pi}{6} + \frac{5\pi}{6}\right)$ 2π

$= \cos\left(\frac{5\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

II → cos −

$\theta' = \frac{\pi}{6}$

a) $\frac{1-\sqrt{3}}{2}$

b) $\frac{-\sqrt{3}-1}{2}$

c) 0

d) $\frac{\sqrt{3}-1}{2}$

e) $\frac{\sqrt{3}+1}{2}$

$y = \sin\left(-\frac{17\pi}{6}\right) = -\sin\frac{17\pi}{6} = -\sin\frac{5\pi}{6}$

$= -\sin\frac{\pi}{6} = -\frac{1}{2}$

II → sin ⊕

$x - y = -\frac{\sqrt{3}}{2} - \left(-\frac{1}{2}\right) = \frac{-\sqrt{3}+1}{2}$

$= \boxed{\frac{1-\sqrt{3}}{2}}$

6. Let $\frac{\pi}{2} < t < \pi$. By writing $\tan t$ in terms of $\sin t$, we get:

a) $\tan t = \frac{-\sin t}{\sqrt{1-\sin^2 t}}$

b) $\tan t = \frac{\sin t}{\sqrt{1-\sin^2 t}}$

c) $\tan t = \frac{\sin t}{\sqrt{1+\sin^2 t}}$

d) $\tan t = \frac{-\sin t}{\sqrt{1+\sin^2 t}}$

e) $\tan t = \frac{-\sqrt{1-\sin^2 t}}{\sin t}$

$\tan t = \frac{\sin t}{\cos t} = \frac{\sin t}{\pm\sqrt{1-\sin^2 t}}$

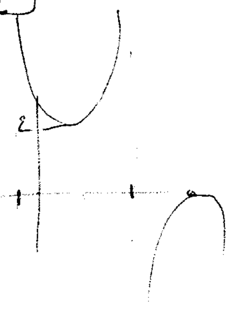
$= \frac{\sin t}{-\sqrt{1-\sin^2 t}} = \boxed{\frac{-\sin t}{\sqrt{1-\sin^2 t}}}$

$t \in$ II → cos ⊖

7. Let $f(x)=1+\csc\left(2x+\frac{\pi}{6}\right)$. Then which one of the following statements is TRUE? $P = \frac{2\pi}{2} = \pi$

a) The graph of f has infintely many x-intercepts. → true

b) The phase shift of f is $\frac{-\pi}{6}$. False $PS = -\frac{\frac{\pi}{6}}{2} = \boxed{-\frac{\pi}{12}}$



c) The period of f is 2π. False $P = \frac{2\pi}{2} = \pi$

d) The graph of f has no x-intercept. False . the graph touches the x-axis

e) One cycle of the graph of f is completed on the interval $\frac{-\pi}{6} \le x \le \frac{\pi}{6}$.

No the length is not a period π.

8. $\frac{\sin 3x}{\sin 2x}+\frac{\cos 3x}{\cos 2x}=$

a) $\frac{2\sin 5x}{\sin 4x}$ $= \frac{\sin 3x \cos 2x + \cos 3x \sin 2x}{\sin 2x \cos 2x}$

b) $\frac{5}{2}$

c) $\frac{\sin 5x}{\sin 4x}$ $= \frac{\sin(5x)}{\frac{1}{2}\sin(4x)} = \boxed{2\frac{\sin 5x}{\sin 2x}}$

d) $\frac{\sin 6x}{\sin 4x}$ → $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$

e) 3

9. $\cos^2 112.5^\circ =$

a) $\frac{2-\sqrt{2}}{4}$

b) $\frac{2+\sqrt{2}}{4}$

c) $\frac{-2-\sqrt{2}}{4}$

d) $\frac{-2+\sqrt{2}}{4}$

e) 1

$$\cos^2 112.5^\circ = \cos^2 \frac{225^\circ}{2} = \left(\pm\sqrt{\frac{1+\cos 225^\circ}{2}}\right)^2$$

$\rightarrow$ III $\rightarrow \cos \ominus$

$\theta' = 45^\circ$

$$= \frac{1+\cos 225^\circ}{2} = \frac{1-\cos 45^\circ}{2}$$

$$= \frac{1-\frac{\sqrt{2}}{2}}{2} = \boxed{\frac{2-\sqrt{2}}{4}}$$

10. The sum of the solutions of the trigonometric equation $\sqrt{3}\sin x + \cos x = 1$, where $\pi < x < 3\pi$, is:

a) $\frac{14\pi}{3}$

b) 4π

c) $\frac{10\pi}{3}$

d) $\frac{8\pi}{3}$

e) $\frac{18\pi}{3}$

$k \sin(x+\alpha)$

$2 \sin\left(x + \pi/6\right) = 1$

$k = \sqrt{3+1} = 2$

$\cos\alpha = \frac{\sqrt{3}}{2}$, $\sin\alpha = \frac{1}{2}$ $\Rightarrow \alpha = \pi/6$

$\sin\left(x + \pi/6\right) = 1/2$

$y = x + \frac{\pi}{6} \Rightarrow \pi + \frac{\pi}{6} < y < 3\pi + \frac{\pi}{6}$

1 period & half.

~~$y = x + \frac{\pi}{6}$~~

$\sin y = 1/2 \rightarrow \oplus$ I, II $\quad \theta' = \pi/6$

$y = \frac{\pi}{6} + 2n\pi$ or $y = \frac{5\pi}{6} + 2n\pi$

$x = y - \pi/6 = 2n\pi$ or $x = \frac{4\pi}{6} + 2n\pi = \frac{2\pi}{3} + 2n\pi$

$n = 0$: $x = 0 \notin [\pi, 3\pi]$ $\quad x = \frac{2\pi}{3}$ ✗

$n = 1$: $x = 2\pi$ ✓ $\quad x = \frac{2\pi}{3} + 2\pi$ ✓

Sum $= 2\pi + \left(\frac{2\pi}{3} + 2\pi\right) = \frac{6\pi}{3} + \frac{2\pi}{3} + \frac{6\pi}{3} = \frac{14\pi}{3}$

11. The solution set of the inverse trigonometric equation

$\sin^{-1}\frac{-3}{5}+\tan^{-1}x=\frac{\pi}{2}$ is:

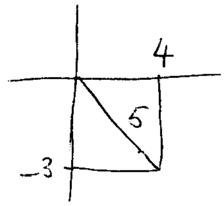
$$\tan^{-1}x = \frac{\pi}{2} - \sin^{-1}(-3/5)$$

$$\tan(\tan^{-1}x) = \tan\left(\frac{\pi}{2} - \sin^{-1}(-3/5)\right)$$

$$x = \cot(\underbrace{\sin^{-1}(-3/5)}_{\theta})$$

$\theta \in Q\,IV$

$\sin\theta = -3/5$



$$x = \frac{4}{-3} = \boxed{-\frac{4}{3}}$$

$\Rightarrow \cot\theta = \frac{4}{-3}$

a) $-\frac{4}{3}$

b) $\frac{4}{3}$

c) $\frac{4}{5}$

d) $-\frac{4}{5}$

e) 1

12. If $u=\langle 3,3\rangle$ and $v=3j$, then a vector of length 3 in the opposite direction of $u+\frac{1}{3}v$ is:

$$w = u+\frac{1}{3}v = \langle 3,3\rangle + \langle 0,1\rangle = \langle 3,4\rangle$$

$$\|w\| = \sqrt{3^2+4^2} = 5$$

The vector is

$$-3\frac{w}{\|w\|} = -3\left\langle \frac{3}{5}, \frac{4}{5}\right\rangle$$

$$= \left\langle -\frac{9}{5}, \frac{-12}{5}\right\rangle$$

a) $\left\langle \frac{-9}{5},\frac{-12}{5}\right\rangle$

b) $\left\langle \frac{9}{5},\frac{-12}{5}\right\rangle$

c) $\left\langle \frac{-9}{5},\frac{12}{5}\right\rangle$

d) $\left\langle \frac{-3}{5},\frac{-12}{5}\right\rangle$

e) $\left\langle \frac{-9}{5},\frac{-4}{5}\right\rangle$

13. Given the vectors $w=\langle y,x\rangle$ and $v=\langle x,y\rangle$, then $proj_w v + proj_v w$ is equal to:

a) $\frac{4xy}{\|w\|}$

b) $\frac{2xy}{\|v\|}$

c) $\frac{2xy}{\|v+w\|}$

d) $\|w\|$

e) $2(x^2+y^2)$

14. The equation in standard form of the parabola that has vertex $(3,-5)$, has its axis of symmetry parallel to the x-axis and passes through the point $(4,3)$ is:

a) $(y+5)^2=64(x-3)$

b) $(y+5)^2=-16(x-3)$

c) $(y+5)^2=16(x-3)$

d) $(x-3)^2=64(y+5)$

e) $(x-3)^2=16(y+5)$

axis of symmetry // to x-axis

$\Rightarrow$ horizontal

$\Rightarrow (y-k)^2 = 4p(x-h)$

$(y+5)^2 = 4p(x-3)$

$(3+5)^2 = 4p(4-3)$

$64 = 4p$

$\rightarrow \boxed{p = 16}$

$$\boxed{(y+5)^2 = 64(x-3)}$$

15. The coordinates of one of the foci of the ellipse that has eccentricity $\frac{2}{3}$, minor axis of length $2\sqrt{20}$ on the x-axis and center at $(0,0)$ is:

$2b = 2\sqrt{20} \Rightarrow b = \sqrt{20}$

Minor on x-axis & Center (0,0) $\Rightarrow \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

a) (0,4)

b) (−4,0)

c) (0,−8)

d) (0,−6)

e) (2,0)

$e = \frac{c}{a} = \frac{2}{3} \Rightarrow c = \frac{2}{3}a$

$a^2 = b^2 + c^2$

$a^2 = 20 + \frac{4}{9}a^2$

$\frac{5}{9}a^2 = 20$

$a^2 = \frac{4}{\cancel{5}}\cancel{20} \cdot 9 = 36$

$\frac{x^2}{20} + \frac{y^2}{36} = 1$

$c = \frac{2}{3} 6 = 4$

$\boxed{F = (0, \pm 4)}$

16. One equation of the asymptotes of the hyperbola $4x^2 - 25y^2 + 16x + 50y - 109 = 0$ is:

a) $y = \frac{2}{5}x + \frac{9}{5}$

b) $y = \frac{2}{5}x + 1$

c) $y = \frac{-2}{5}x - \frac{9}{5}$

d) $y = \frac{2}{5}x + 9$

e) $y = \frac{-2}{5}x + 1$

$(4x^2 + 16x) - (25y^2 - 50y) = 109$

$4(x^2 + 4x + 4) - 25(y^2 - 2y + 1) = 109 + 16 - 25$

$4(x+2)^2 - 25(y-1)^2 = 100$

$\frac{(x+2)^2}{25} - \frac{(y-1)^2}{4} = 1$

$y - 1 = \pm \frac{2}{5}(x+2)$

Try $+ \frac{5}{2}$

$y = \frac{2}{5}x + \frac{2}{5} + 1$

$y = \frac{2}{5}x + \frac{4}{5} + \frac{5}{5}$

$\boxed{y = \frac{2}{5}x + \frac{9}{5}}$

17. The equation in standard form of the hyperbola that has foci $(0,3)$ and $(0,-3)$ and passes through the point $\left(\frac{5}{2},3\right)$ is:

a) $\frac{y^2}{4}-\frac{x^2}{5}=1$

b) $\frac{y^2}{8}-\frac{x^2}{5}=1$

c) $\frac{y^2}{4}-\frac{x^2}{10}=1$

d) $\frac{x^2}{16}-\frac{y^2}{25}=1$

e) $\frac{x^2}{4}-\frac{y^2}{5}=1$

$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

$c=3$

$2a = dist(F_1,P) + dist(F_2,P)$

$= \sqrt{(\frac{5}{2}-0)^2+(3-3)^2} + \sqrt{(\frac{5}{2}-0)^2+(3+3)^2}$

$= \frac{5}{2} + \sqrt{\frac{25}{4}+36} = \frac{5}{2} + \sqrt{\frac{25+144}{4}}$

$= \frac{5}{2} + \frac{13}{2} = \frac{18}{2} = 9 \Rightarrow a$

18. If (x, y) is the solution of the system of equations $\begin{cases}2x-5\pi y=3\\3x+4\pi y=2\end{cases}$, then $x+\pi y=$

a) $\frac{17}{23}$

b) $\frac{15}{23}$

c) 1

d) $\frac{13}{23}$

e) $\frac{19}{23}$

(1) × −3
(2) × 2

$$\begin{array}{r} -6x+15\pi y=-9 \\ 6x+8\pi y=4 \\ \hline 23\pi y=-5 \end{array}$$

$y=\frac{-5}{23\pi}$

$2x = 3+5\pi y$

$= 3 + 5\cancel{\pi}\left(\frac{-5}{23\cancel{\pi}}\right) = 3-\frac{25}{23} = +\frac{44-25}{23}$

$= \frac{44}{23}$

$\boxed{x=\frac{22}{23}}$

$x+\pi y = \frac{22}{23} - \frac{5}{23}$

$=\frac{17}{23}$

19. If the graphs of the parabola $y = x^2 - 4x + 3$ and the line $y - 2x = k$ intersect at only one point, then the value of k is equal to:

a) -6

b) 6

c) 3

d) -3

e) -1

$y = 2x + k$

$2x + k = x^2 - 4x + 3$

$x^2 - 6x + 3 - k = 0$

has one solution only $\Rightarrow$ $\Delta = 0$

$36 - 4(3-k) = 0$

$36 - 12 + 4k = 0$

$4k = -24$

$\Rightarrow \boxed{k = -6}$

20. If (a,b) and (c,d) are the solutions of the system $\begin{cases}(x-1)^2 + (y+1)^2 = 5 \\ (x+1)^2 + (y-1)^2 = 1\end{cases}$, then $a+b+c+d$ is equal to:

a) 0

x^2

b) -2

c) 2

d) -1

e) 1

21. The echelon form of the system $\begin{cases} 4x-5y-z=2 \\ 3x-4y+z=-2 \\ x-2y-z=3 \end{cases}$ is:

Find echelon form.

a) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 1 & 2 & -\frac{11}{2} \\ 0 & 0 & 1 & -\frac{13}{6} \end{bmatrix}$

b) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 1 & 2 & -11 \\ 0 & 0 & 1 & -\frac{13}{6} \end{bmatrix}$

c) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 1 & 2 & -\frac{11}{2} \\ 0 & 0 & 0 & -\frac{13}{6} \end{bmatrix}$

d) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 1 & 2 & -\frac{11}{2} \\ 0 & 0 & 1 & -\frac{5}{6} \end{bmatrix}$

e) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 1 & 2 & -\frac{11}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$

22. The system of equations $\begin{cases} x+2y-2z=3 \\ 5x+8y-6z=14 \\ 3x+4y-2z=8 \end{cases}$

a) is dependent

b) is independent

c) is inconsistent

d) has the unique solution $\left\{\left(2,\frac{1}{2},0\right)\right\}$

e) has the unique soluion $\left\{\left(0,\frac{5}{2},1\right)\right\}$

$$\begin{bmatrix} 1 & 2 & -2 & 3 \\ 5 & 8 & -6 & 14 \\ 3 & 4 & -2 & 8 \end{bmatrix} \xrightarrow[-3R_1+R_3]{-5R_1+R_2} \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & -2 & 4 & -1 \\ 0 & -2 & 4 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & -2 & \frac{1}{2} \\ 0 & -2 & 4 & -1 \end{bmatrix}$$

$$\xrightarrow{2R_2+R_3} \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & -2 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2 eqⁿ in echelon form
& 3 variable

$\boxed{\text{Dependent}}$

23. The system $\begin{cases} x+y=1 \\ y+z=1 \\ x+kz=1 \end{cases}$ has no solution if k is equal to:

a) -1

b) 1

c) 0

d) -2

e) 2

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & k & 1 \end{bmatrix} \longrightarrow$$

$$\xrightarrow{-R_1+R_3} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & k & 0 \end{bmatrix}$$

$$\xrightarrow{R_2+R_3} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & k+1 & 1 \end{bmatrix} \rightarrow \begin{cases} x+y=1 \\ y+z=1 \\ (k+1)z=1 \end{cases}$$

$(k+1) = 0$, $1 \neq 0$

the system has no solution if

$k+1=0$

$\boxed{k=-1}$

24. If $A=\begin{bmatrix}1 & x\\2 & 3\end{bmatrix}$ and $A^2-4A=I$, then x is equal to:

a) 2

$$A^2=\begin{pmatrix}1 & x\\2 & 3\end{pmatrix}\begin{pmatrix}1 & x\\2 & 3\end{pmatrix}=\begin{pmatrix}1+2x & x+3x\\ * & *\end{pmatrix}$$

b) 1

$$-4A=\begin{pmatrix}-4 & -4x\\ & \end{pmatrix}$$

c) 0

d) −1

$$A^2-4A=I_n$$

e) −2

$$\begin{pmatrix}1+2x-4 & *\\ * & *\end{pmatrix}=\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$$

$\Rightarrow 1+2x-4=1$

$\Rightarrow 2x=4$

$\boxed{x=2}$

25. The matrix $\begin{bmatrix}1 & 0 & 2\\0 & 3 & 1\\0 & k & 2\end{bmatrix}$ is a **singular matrix** if k is equal to:

a) 6

b) −6

c) −2

A singular if $|A|=0$

d) −3

e) 3

$$|A|=1\begin{vmatrix}3 & 1\\k & 2\end{vmatrix}=6-k=0$$

$\boxed{k=6}$

26. The system of equations $\begin{cases} 3x-5y=-18 \\ 2x-3y=-9 \end{cases}$, has the solution in the form:

a) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -18 \\ -9 \end{bmatrix}$

b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} -18 \\ -9 \end{bmatrix}$

c) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -18 \\ -9 \end{bmatrix}$

d) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -18 \\ -9 \end{bmatrix} \begin{bmatrix} -3 & 5 \\ -2 & 3 \end{bmatrix}$

e) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -18 \\ -9 \end{bmatrix}$

$A = \begin{pmatrix} 3 & -5 \\ 2 & -3 \end{pmatrix}$ $X = \begin{pmatrix} x \\ y \end{pmatrix}$, $C = \begin{pmatrix} -18 \\ -9 \end{pmatrix}$

$X = A^{-1} C$.

$A^{-1} = \frac{1}{-9+10} \begin{pmatrix} -3 & 5 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} -3 & 5 \\ -2 & 3 \end{pmatrix}$

So

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & 5 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -18 \\ -9 \end{pmatrix}$

27. Let $A^{-1} = \begin{bmatrix} 1 & 0 & 7 \\ 2 & 1 & -1 \\ 7 & 3 & 1 \end{bmatrix}$, then the sum of the elements in the 2nd row of the matrix A is:

a) 14

b) 10

c) 9

d) 16

e) 2

Sum $= -5 + 16 + 3$
$= \boxed{14}$

28. If $A=\begin{bmatrix}5 & -2 & -3\\ 2 & 4 & -1\\ 4 & -5 & 6\end{bmatrix}$, then $M_{21}+C_{23}=$

a) -10

b) -18

c) -8

d) -9

e) 0

$M_{21} = \begin{vmatrix}5 & -2 & -3\\ 2 & 4 & -1\\ 4 & -5 & 6\end{vmatrix} = \begin{vmatrix}-2 & -3\\ -5 & 6\end{vmatrix} = -12-15$

$= -27$

$C_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix}5 & -2 & -3\\ 2 & 4 & -1\\ 4 & -5 & 6\end{vmatrix} = -\begin{vmatrix}5 & -2\\ 4 & -5\end{vmatrix}$

$= -(-25+8) = 17$

$M_{21} + C_{23} = \boxed{-10}$

29. Let A, B be two invertible matrices such that $|A|=2$ and $|B|=4$, then $|2A|+|A^{-1}B|=$

$\underbrace{\text{matrices}}_{3\times 3}$

a) 18

b) 10

c) 20

d) 22

e) 16

$|2A| + |A^{-1}B|$

$= 2^3|A| + |A^{-1}|\cdot|B|$

$= 2^3\cdot 2 + \frac{1}{|A|}\cdot|B|$

$= 2^4 + \frac{1}{2}\cdot 4 = 16+2 = \boxed{18}$

30. If $\begin{vmatrix} 2 & 2 & 2 \\ x-1 & y-2 & z-3 \\ 1 & 2 & 3 \end{vmatrix} = 3$, then $\begin{vmatrix} 1 & 2 & 3 \\ x & y & z \\ 4 & 4 & 4 \end{vmatrix} =$

a) −6

b) −8

c) 6

d) −12

e) 0

$$\begin{vmatrix} 1 & 2 & 3 \\ x & y & z \\ 4 & 4 & 4 \end{vmatrix} \overset{-R_1+R_2}{=} \begin{vmatrix} 1 & 2 & 3 \\ x-1 & y-2 & z-3 \\ 4 & 4 & 4 \end{vmatrix}$$

$$\overset{R_1 \leftrightarrow R_3}{=} - \begin{vmatrix} 4 & 4 & 4 \\ x-1 & y-2 & z-3 \\ 1 & 2 & 3 \end{vmatrix} \overset{\frac{R}{2}}{=} -2 \begin{vmatrix} 2 & 2 & 2 \\ x-1 & y-2 & z-3 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= -2(3) = \boxed{-6}$$