1. **The coordinates of the focus** of the parabola that passes through the origin and the points $(-3,12)$ and $(3,12)$ are

~~(a)~~ $\left(0,\frac{3}{16}\right)$

See example 2 p. 687

See problems 27 and 28 p. 692

(b) $\left(0,\frac{3}{4}\right)$ If horizontal $(12-k)^2 = 4p(3-h) = 4p(-3-h)$

$\Rightarrow 3-h = -3-h \Rightarrow 3=-3$ impos

(c) $\left(0,\frac{5}{16}\right)$ If vertical $(3-h)^2 = 4p(12-k) = (-3-h)^2$

$\Rightarrow \boxed{h=0}$

(d) $\left(0,-\frac{3}{4}\right)$ $(0,0)$ $0^2 = 4p(0-k) \Rightarrow \boxed{k=0}$

$x^2 = 4py$

(e) $\left(0,\frac{1}{16}\right)$ $(3,12) \to 9 = 4p(12) \Rightarrow p = \frac{9}{4\cdot 12} = \boxed{\frac{3}{16}}$

$\Rightarrow F(0, 0+p) = \left(0, \frac{3}{16}\right)$

2. If $\tan\alpha = -\frac{4}{3}, \frac{3\pi}{2} < \alpha < 2\pi,$ then $\sec\frac{\alpha}{2} =$

~~(a)~~ $-\frac{\sqrt{5}}{2}$

See Problems 37 to 48 p. 579

(b) $\frac{2}{\sqrt{3}}$

(c) $\frac{2}{\sqrt{5}}$

(d) $-\frac{\sqrt{3}}{2}$

(e) $-\sqrt{5}$

3. The expression $\dfrac{\sin 2x - \sin x}{2\cos^2 x + \cos x - 1}$ simplifies to

~~(a)~~ $\tan \dfrac{x}{2}$

(b) $\cot \dfrac{x}{2}$

(c) $\cos \dfrac{x}{2}$

(d) $\sin \dfrac{x}{2}$

(e) $\sec \dfrac{x}{2}$

See problems 55, 56, 58 and 89
p. 579– 580

4. The equation of the parabola with focus at $(-8, 1)$ and directrix $x - 4 = 0$ is

~~(a)~~ $(y-1)^2 = -24(x+2)$

(b) $(y-1)^2 = -6(x+2)$

(c) $(x+2)^2 = -24(y-1)$

(d) $(x+2)^2 = 24(y-1)$

(e) $(y+1)^2 = -24(x-2)$

See example. 4 p. 689

See problems 29 to 32 p. 692

Directrix $x = 4$ vertical

$\Rightarrow$ eq is $(y-k)^2 = 4p(x-h)$

dist $(F, (D)) = |-8-4| = 12 \Rightarrow \boxed{p = -6}$

$p < 0$,

$F = (h+p, k) = (-8, 1)$

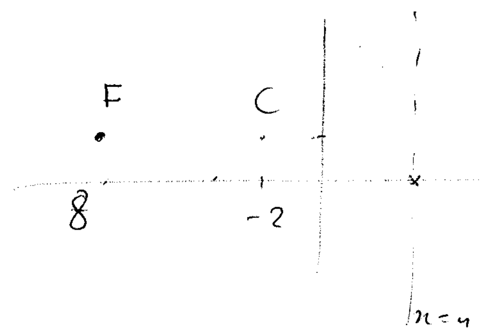
$\boxed{k = 1}$

$h + p = -8$

$h - 6 = -8$

$\boxed{h = -2}$

$(y-1)^2 = -24(x+2)$



5. If the function $y = -3\sin 2x - 3\cos 2x$ is written in the form $y = k\sin(2x+\beta)$, $0 < \beta < 2\pi$, then the values of k and β are

(a) $k = 3\sqrt{2},\ \beta = \frac{5\pi}{4}$

(b) $k = -6,\ \beta = \frac{5\pi}{4}$

(c) $k = 3\sqrt{2},\ \beta = \frac{5\pi}{8}$

(d) $k = -6,\ \beta = \frac{3\pi}{4}$

(e) $k = 3\sqrt{2},\ \beta = \frac{7\pi}{4}$

See example 5 P. 585

See problems 49 to 66 P. 588

6. The exact value of $\sin^{-1}\left(\sin\frac{7\pi}{6}\right) + \tan\left(\cos^{-1} -\frac{1}{2}\right)$ is

(a) $-\frac{\pi}{6} - \sqrt{3}$

(b) $\frac{7\pi}{6} + \frac{\sqrt{3}}{3}$

(c) $\frac{\pi}{6} + \sqrt{3}$

(d) $\frac{5\pi}{6} - 1$

(e) $-\frac{\pi}{6} + \sqrt{3}$

See examples 2 and 3 P. 596

See problems 21 to 52 P. 602

$\sin^{-1}\left(\sin\frac{7\pi}{6}\right) + \tan\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$

III → sin ⊖ $\quad\theta$

$\cos\theta = -\frac{1}{2}$

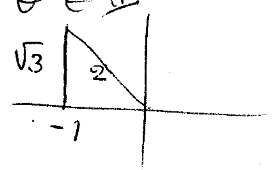
→ $\theta \in$ II

$\sin^{-1}\left(-\sin\left(\frac{\pi}{6}\right)\right) + \frac{\sqrt{3}}{-1}$

$\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right) - \sqrt{3}$

$\boxed{-\frac{\pi}{6} - \sqrt{3}}$

$\in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



7. The eccentricity of the ellipse $9x^2+4y^2+36x-8y+4=0$ is equal to

See example 2 p. 699
and example 4 p. 702
See problems 49 and 50 p. 705

(a) $\frac{\sqrt{5}}{3}$

(b) $\frac{\sqrt{5}}{4}$

(c) $\frac{\sqrt{11}}{3}$

(d) $\frac{\sqrt{13}}{4}$

(e) $\frac{4}{5}$

$9(x^2+4x+4)+4(y^2-2y+1) = -4+36+4$

$9(x+2)^2 + 4(y-1)^2 = 36$

$\frac{(x+2)^2}{4} + \frac{(y-1)^2}{9} = 1 \Rightarrow a^2 = 9, \ b^2 = 4$

$c^2 = 5.$

$\boxed{e = \frac{c}{a} = \frac{\sqrt{5}}{3}}$

8. The equation of the ellipse in the standard form with vertices $(-2,4)$ and $(-2,-2)$, and passing through $(0,1)$ is

(a) $\frac{(x+2)^2}{4}+\frac{(y-1)^2}{9}=1$

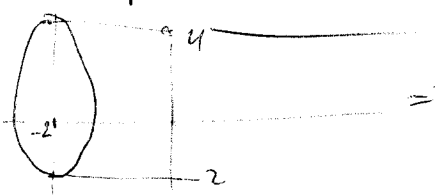
(b) $\frac{(x+2)^2}{4}+\frac{(y-1)^2}{25}=1$

(c) $\frac{(x-2)^2}{4}+\frac{(y-1)^2}{25}=1$

(d) $\frac{(x+2)^2}{3}+\frac{(y-2)^2}{12}=1$

(e) $\frac{(x-2)^2}{4}+\frac{(y+1)^2}{9}=1$

See example 3 p. 701
See problems 35 to 44 p. 705



$\Rightarrow$

$C = \left(-2, \frac{4-2}{2}\right) = (-2, 1)$

$2a = 4-(-2) = 6 \Rightarrow \boxed{a = 3}$

$\frac{(x+2)^2}{b^2} + \frac{(y-1)^2}{9} = 1$

$(0,1)$ pt of ellipse $\Rightarrow \frac{2^2}{b^2} + \frac{(1-1)^2}{9} = 1$

$\frac{4}{b^2} = 1 \Rightarrow b^2 = 4$

$\boxed{\frac{(x+2)^2}{4} + \frac{(y-1)^2}{9} = 1}$

9. If $0 \le x < 2\pi$, then the sum of all solutions of the equation $2\cos^2 x + \cos x - 1 = 0$ is qual to

~~(a)~~ 3π

(b) π

(c) 2π

(d) $\frac{4\pi}{3}$

(e) $\frac{5\pi}{3}$

See example 1 p. 605
and example 3 p. 606
See problems 43 and 44 p. 614

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x = 1$

$\cos x = \frac{1}{2} \; < \; \mathrm{I}, \mathrm{IV}$

$x' = \frac{\pi}{3}$

$x = \frac{\pi}{3} \; , \; \frac{5\pi}{3}$

$\cos x = -1$

$x = \pi$

$\pi + \frac{\pi}{3} + \frac{5\pi}{3} = \boxed{3\pi}$

10. If θ is the smallest positive angle between the vectors $u = 3\mathbf{i} - 4\mathbf{j}$ and $v = -2\mathbf{i} + \mathbf{j}$, then $\tan\theta$ is equal to

~~(a)~~ $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $-\frac{1}{3}$

(e) $-\frac{1}{5}$

See example 9 p. 658
See problems 53 to 60 p. 662

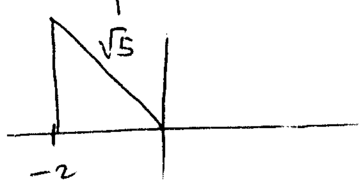
$\cos\alpha = \frac{u \cdot v}{\|u\| \cdot \|v\|}$

$\|u\| = \sqrt{25} = 5, \qquad \|v\| = \sqrt{5}$

$u \cdot v = -6 - 4 = -10$

$\cos\alpha = \frac{-10}{5\sqrt{5}} = \frac{-2}{\sqrt{5}} \; \Rightarrow \; \alpha \in \mathrm{II}$

$0 < \alpha < 180^\circ$



$\Rightarrow x = -2, \quad r = \sqrt{5}$

$y = +\sqrt{5-4} = 1$

$\Rightarrow \tan\alpha = \frac{\cancel{(+1)}}{\cancel{x}} = \frac{y}{x} = \frac{1}{-2} = \boxed{-\frac{1}{2}}$

11. Given the vectors $\mathbf{u} = \langle -4, 10 \rangle$ and $\mathbf{v} = \langle -5, 1 \rangle$. If the vector $\mathbf{w} = \langle a, b \rangle$ is a **unit** vector **in the opposite direction** of $\frac{1}{2}\mathbf{u} - \mathbf{v}$, then $a + b$ is equal to

~~(a)~~ $-\frac{7}{5}$

(b) $-\frac{3}{5}$

(c) $-\frac{4}{5}$

(d) $-\frac{2}{5}$

(e) $-\frac{9}{5}$

See example 3 p. 653

and example 4 p. 654

See problems 7 to 20 p. 661

$$\frac{1}{2}u - v = \frac{1}{2}\langle -4, 10\rangle - \langle -5, 1\rangle = \langle -2, 5\rangle + \langle 5, -1\rangle = \langle 3, 4\rangle$$

$$\left\|\frac{1}{2}u - v\right\| = \sqrt{3^2 + 4^2} = 5$$

$$W = -\left\langle \frac{3}{5}, \frac{4}{5}\right\rangle = \left\langle -\frac{3}{4}, -\frac{4}{5}\right\rangle = \langle a, b\rangle$$

$$\boxed{a + b = -\frac{7}{5}}$$

12. Which one of the following statements is **FALSE** about the graph of $y = \tan^{-1}(x+1) - \frac{\pi}{2}$?

See example 7 p. 599 and problem 81 p. 602

~~(a)~~ the graph has only one x-intercept

(b) the graph has only one y-intercept

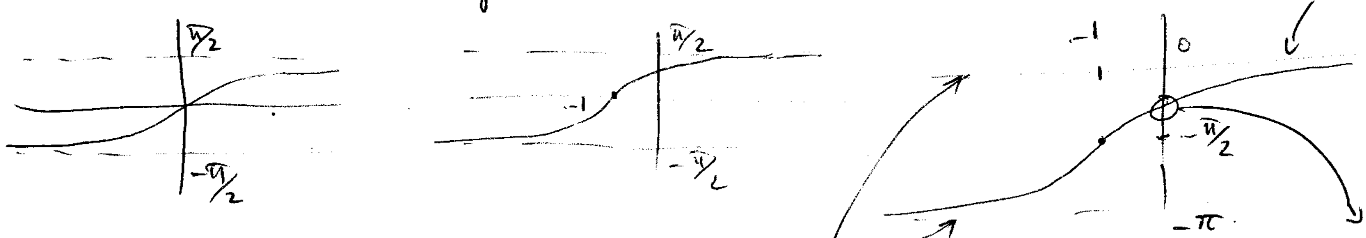
(c) the graph has two asymptotes $y = -\pi$ and $y = 0$

(d) the graph increases for all real numbers x

(e) the graph lies completely under the x-axis

$$y = \tan^{-1}(x) \xrightarrow[\text{1 to left}]{} y = \tan^{-1}(x+1) \xrightarrow[\frac{\pi}{2}\text{ down}]{} y = \tan^{-1}(x+1) - \frac{\pi}{2}$$

No x-int



1 y-int

H. asymp $y = 0$, $y = -\pi$

13. The exact value of the expression

$$\sin\frac{13\pi}{12}\cos\frac{\pi}{12}$$

is equal to

~~(a)~~ $-\frac{1}{4}$

(b) $\frac{1}{4}$

(c) $\frac{1}{8}$

(d) $-\frac{1}{8}$

(e) $\frac{1}{2}$

An application of the identity

$\sin\theta = \frac{1}{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}$.

14. If $\cos^{-1}\frac{x}{2} + \sin^{-1}\left(-\frac{3}{5}\right) = \frac{\pi}{3}$, then $x =$

~~(a)~~ $\frac{4-3\sqrt{3}}{5}$

(b) $\frac{3-4\sqrt{3}}{5}$

(c) $\frac{8-6\sqrt{3}}{5}$

(d) $\frac{4+3\sqrt{3}}{10}$

(e) $\frac{4-3\sqrt{3}}{20}$

See example 5 p.598

See problems 57 & 66 P.602

$\cos^{-1}\frac{x}{2} = \frac{\pi}{3} - \sin^{-1}\left(-\frac{3}{5}\right)$

Apply cos $\frac{x}{2} = \cos\left(\frac{\pi}{3} - \overbrace{\sin^{-1}\left(-3/5\right)}^{\theta}\right)$

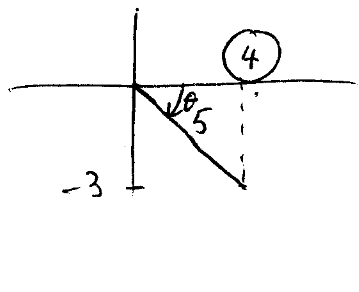
$x = 2\left(\cos\frac{\pi}{3}\cos\theta + \sin\frac{\pi}{3}\sin\theta\right)$

$= 2\left(\frac{1}{2}\frac{4}{5} + \frac{\sqrt{3}}{2}\left(-\frac{3}{5}\right)\right)$

$= \boxed{\frac{4-3\sqrt{3}}{5}}$

$\theta = \sin^{-1}(-3/5)$

$\in Q\,IV$ $\sin\theta = -\frac{3}{5}$



15. If $0 \le x < 2\pi$, then the number of all solutions of the equation $2\sin\left(2x+\dfrac{\pi}{6}\right)-1=0$ is

(a) ~~4~~
(b) 6
(c) 8
(d) 2
(e) 10

See problems 61 to 70 p. 615

$$2\sin\left(2x+\pi/6\right)=1$$

$$\sin\left(2x+\pi/6\right)=1/2$$

$0 \le x < 2\pi$

Graph

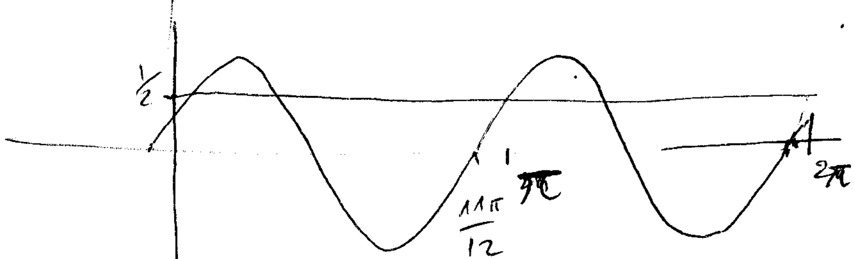
$P = \frac{2\pi}{2} = \pi$, $PS = ?$

$2x + \pi/6 = 0$

$x = -\pi/12$

Cycle ? $PS + P$

$-\pi/12 + \pi = \frac{11\pi}{12}$



4 sol^{ns}