King Fahd University of Petroleum and Minerals
Prep-Year Math Program

Exam II
Prep-Year Math II. Term(052)
April 27, 2006
Time Allowed: 75 Minutes

NAME:______________________ ID# KEY ______ SEC#______

IMPORTANT INSTRUCTIONS:

SHOW ALL YOUR WORK AND WRITE CLEAR STEPS

1) ALL TYPES OF CALCULATORS, PAGERS OR MOBILES ARE NOT ALLOWED DURING THE EXAMINATION.
2) WRITE YOUR NAME, ID NUMBER AND SECTION NUMBER.
3) USE ONLY PENCIL TO ANSWER THE QUESTIONS.
4) USE A GOOD ERASER, DON'T USE THE ERASER ATTACHED TO THE PENCIL.
5) CHECK THAT THE EXAM PAPER HAS 12 QUESTIONS.

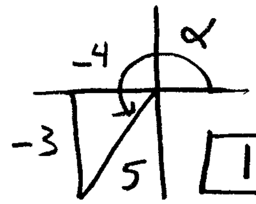
1	2	3	4	5	6	7	8	9	10	11	12
3 pts	4 pts	3 pts	3 pts	4 pts	4 pts	5 pts	4 pts	3 pts	2 pts	6 pts	4 pts

TOTAL ________/ 45

Q1. (3 points) Find the exact value of $\cos\frac{\alpha}{2}$ if $\csc\alpha = \frac{-5}{3}$, $\pi < \alpha < \frac{3\pi}{2}$.

$$\pi < \alpha < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{\alpha}{2} < \frac{3\pi}{2} \quad \boxed{0.5}$$

$$\Rightarrow \alpha \in QII \quad \boxed{0.5}$$



$\boxed{1}$

$$\Rightarrow \cos\frac{\alpha}{2} = -\sqrt{\frac{1+\cos\alpha}{2}} \quad \boxed{0.5}$$

$$= -\sqrt{\frac{1-\frac{4}{5}}{2}} = \frac{-1}{\sqrt{10}} \quad \boxed{0.5}$$

See problems #37 to 48 p. 579

Q2. Given the vectors $\boldsymbol{u} = \left\langle \frac{2\sqrt{3}}{3}, \frac{2}{3} \right\rangle$ and $\boldsymbol{v} = \left\langle -\frac{1}{5}, \frac{\sqrt{3}}{5} \right\rangle$

a) (2 points) Find a unit vector $\boldsymbol{e}$ in the direction of $\boldsymbol{u}$

$$\|u\| = \sqrt{\left(\frac{2\sqrt{3}}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = \sqrt{\frac{4}{3} + \frac{4}{9}} = \sqrt{\frac{16}{9}} = \frac{4}{3} \quad \boxed{1}$$

$$\Rightarrow e = \frac{1}{\|u\|} u = \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle \quad \boxed{1}$$

See example 3 P. 653 Problems 7 to 14 P661

b) (1 points) Find the "dot product" $\boldsymbol{u} . \boldsymbol{v}$

$$u \cdot v = \left(\frac{2\sqrt{3}}{3}\right)\left(-\frac{1}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{\sqrt{3}}{5}\right) \quad \boxed{1}$$

$$= -\frac{2\sqrt{3}}{15} + \frac{2\sqrt{3}}{15} = 0$$

See examples 8 p. 656 Problems 45 to 52 P. 662

c) (1 points) Find the angle θ between $\boldsymbol{u}$ and $\boldsymbol{v}$.

$$\cos\theta = \frac{u \cdot v}{\|u\| \, \|v\|} = 0 \quad \boxed{0.5}$$

$$\Rightarrow \theta = 90^0 \quad \boxed{0.5}$$

See example 9 P. 659 Problems 53 to 60 P. 662

Q3. (3 points) Use a half-angle identity to show that $\tan 165^0 = \sqrt{3} - 2$

$$\tan 165^0 \overset{\boxed{0.5}}{=} \tan\frac{330^0}{2} \overset{\boxed{1}}{=} \frac{1-\cos 330^0}{\sin 330^0}$$

$$\overset{\boxed{0.5}}{=} \frac{1-\cos 30^0}{-\sin 30^0} \overset{\boxed{0.5}}{=} \frac{1-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$$

$$\overset{\boxed{0.5}}{=} \sqrt{3} - 2$$

it is problem 12 P. 578 See also #9 to 24 P. 578

Q4. (3-points) Find the equation in standard form of the parabola that has vertex (-4, 1), has its axis of symmetry parallel to the x-axis, and passes through the point (4, 3).

The equation is of the form

$(y-k)^2 = 4p(x-h)$ $\boxed{0.5}$

$\Rightarrow (y-1)^2 = 4p(x+4)$ $\boxed{1}$

The parabola passes through $(4,3) \Rightarrow$

$(3-1)^2 = 4p(4+4)$ $\boxed{0.5}$

$\Rightarrow 4 = 32p \Rightarrow p = \frac{1}{8}$ $\boxed{0.5}$

$\Rightarrow$ the required equation is

$(y-1)^2 = \frac{1}{2}(x+4)$ $\boxed{0.5}$

see problems 33 and 34 p.692

Q5. (4 points) Find the exact solutions, in radians, of the trigonometric equation $3\sin x - 2\cos^2 x = 0$

$\Rightarrow 3\sin x - 2(1-\sin^2 x) = 0$ $\boxed{0.5}$

$\Rightarrow 2\sin^2 x + 3\sin x - 2 = 0$ $\boxed{0.5}$

$\Rightarrow (2\sin x - 1)(\sin x + 2) = 0$ $\boxed{0.5}$

$\Rightarrow \sin x = -2$ (reject) $\boxed{0.5}$

or $\sin x = 1/2$ $\boxed{0.5}$

$\Rightarrow x = \frac{\pi}{6} + 2\pi k$ $\boxed{0.5}$

or $x = \frac{5\pi}{6} + 2\pi k$ $\boxed{0.5}$

where k is an integer $\boxed{0.5}$

See examples 1 to 5 p 605-8 and problems 41, 42, 44, 59 p 614

Q6. (4 points) Graph $f(x) = \sqrt{3}\cos x - \sin x$ over the interval $\left[-\frac{2\pi}{3}, \frac{4\pi}{3}\right]$.

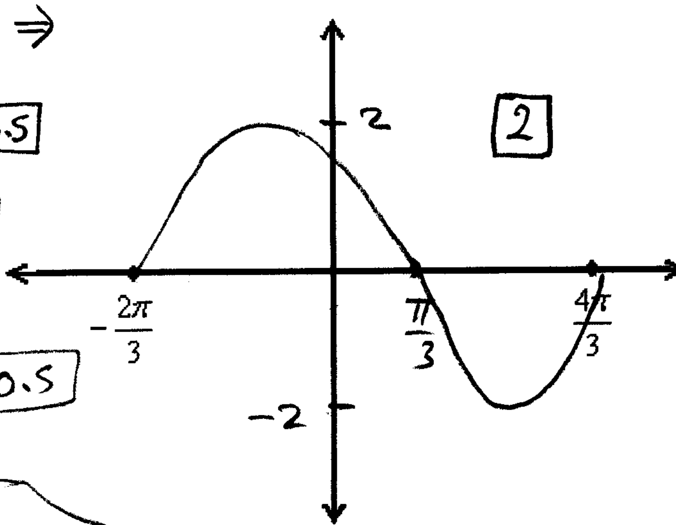
[Hint: Write $f(x)$ in the form $k\sin(x+\alpha)$]

$k = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4} = 2$ $\boxed{0.5}$ $\Rightarrow$

$\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\alpha = -\frac{1}{2}$ $\boxed{0.5}$

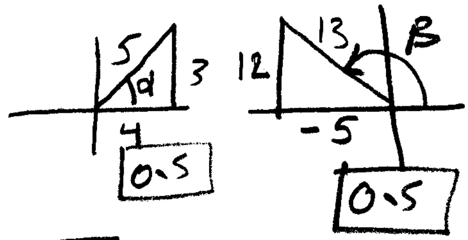
$\Rightarrow \alpha \in Q\,II$, $\alpha = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ $\boxed{0.5}$

$\Rightarrow f(x) = 2\sin\left(x + \frac{2\pi}{3}\right)$ $\boxed{0.5}$



See example 4 p. 585
Problems #55 to 76 p. 588

Q7. Find the exact values of each of the following:



a) (3 points) $\cos\left(\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{-5}{13}\right)\right)$

Let $\alpha = \sin^{-1}\frac{3}{5}$, $\beta = \cos^{-1}\left(\frac{-5}{13}\right)$ $\boxed{0.5}$

Thus $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\boxed{0.5}$

$= \left(\frac{4}{5}\right)\left(\frac{-5}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$ $\boxed{0.5}$

$= -\frac{20}{65} - \frac{36}{65} = -\frac{56}{65}$ $\boxed{0.5}$

See example 4
p. 597
Problems 53, 54
p. 602

b) (2 points) $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)$ $= \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{6}\right)\right)$ $\boxed{0.5}$

$= \tan^{-1}\left(\tan -\frac{\pi}{6}\right)$ $\boxed{0.5}$

$= -\frac{\pi}{6}$ $\boxed{0.5}$

because $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$ $\boxed{0.5}$

See problems
21 to 48
p. 601–602

Q8. (4 points) Find the exact solutions of the trigonometric equation

$\sin\frac{x}{2} + \cos x = 1,\ 0^0 \le x < 360^0$

$\Rightarrow \sin\frac{x}{2} + (\cancel{1} - 2\sin^2\frac{x}{2}) = \cancel{1}$ $\boxed{1}$

$\Rightarrow \sin\frac{x}{2}(1 - 2\sin\frac{x}{2}) = 0$ $\boxed{0.5}$

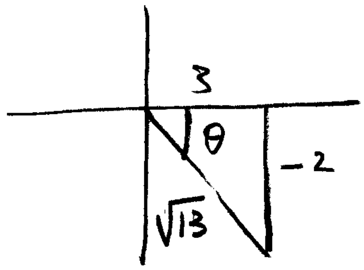
$\Rightarrow \sin\frac{x}{2} = 0$ or $\sin\frac{x}{2} = \frac{1}{2}$ $\boxed{0.5}$

$\Rightarrow \frac{x}{2} = 0^0, 180^0$ or $\frac{x}{2} = 30^0, 150^0,$ $\boxed{1}$

$\Rightarrow x = 0^0, 60^0, 300^0$ $\boxed{1}$

See examples 1 to 5 P. 605–608
See problems 69, 70 p. 615

Q9. (3 points) Find the horizontal and vertical components of the vector $\boldsymbol{v}$ of magnitude $\sqrt{13}$ meters with direction angle $\theta = \tan^{-1}\left(-\frac{2}{3}\right)$. [Write your answer in simplest form].



The horizontal component

$= \|v\| \cos\theta$

$= \sqrt{13}\left(\frac{3}{\sqrt{13}}\right) = 3$ $\boxed{1}$

$\boxed{1}$

The vertical component

$= \|v\| \sin\theta = \sqrt{13}\left(\frac{-2}{\sqrt{13}}\right) = -2$ $\boxed{1}$

See example 5 P.655 and problems 33 to 36 P. 661

Q10. (2 points) Find the equation of the **directrix** of the parabola given by the equation

$2x^2 - 8x - 9y + 4 = 0 \Rightarrow 2x^2 - 8x = 9y - 4$

$\Rightarrow 2(x^2 - 4x) = 9y - 4 \Rightarrow 2(x-2)^2 - 8 = 9y - 4$

$\Rightarrow 2(x-2)^2 = 9y + 4 \Rightarrow (x-2)^2 = \frac{9}{2}(y + \frac{4}{9})$ $\boxed{1}$

$\Rightarrow$ Vertex is at $(2, -\frac{4}{9})$, $P = \frac{9}{8}$ $\boxed{0.5}$

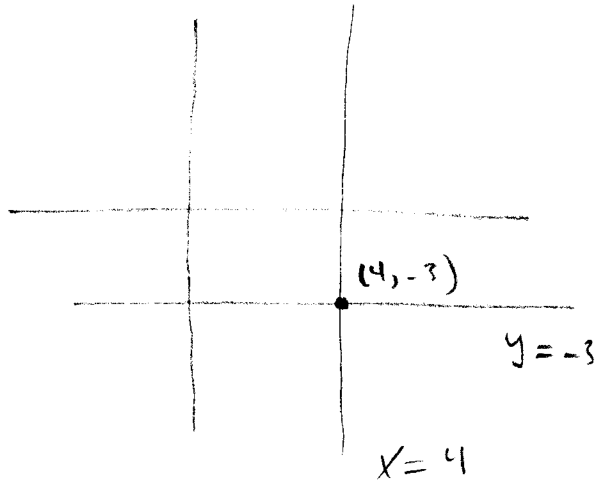
$\Rightarrow$ The directrix $y = -\frac{4}{9} - \frac{9}{8} = -\frac{113}{72}$ $\boxed{0.5}$

See example 3 P. 688

Q11. (6 points) Given an ellipse with foci on the line $x = 4$, minor axis on the line $y = -3$, length of the major axis equals 8, and length of the minor axis equals 4. Find:

a) the coordinates of the center of the ellipse

Center $(4, -3)$ $\boxed{1}$



b) the coordinates of the foci of the ellipse

$a = 4$, $b = 2$ $\boxed{0.5}$

$\Rightarrow c^2 = \sqrt{16-4} = \sqrt{12} = 2\sqrt{3}$ $\boxed{0.5}$

$\Rightarrow$ foci $(4, 2\sqrt{3} - 3), (4, -2\sqrt{3} - 3)$

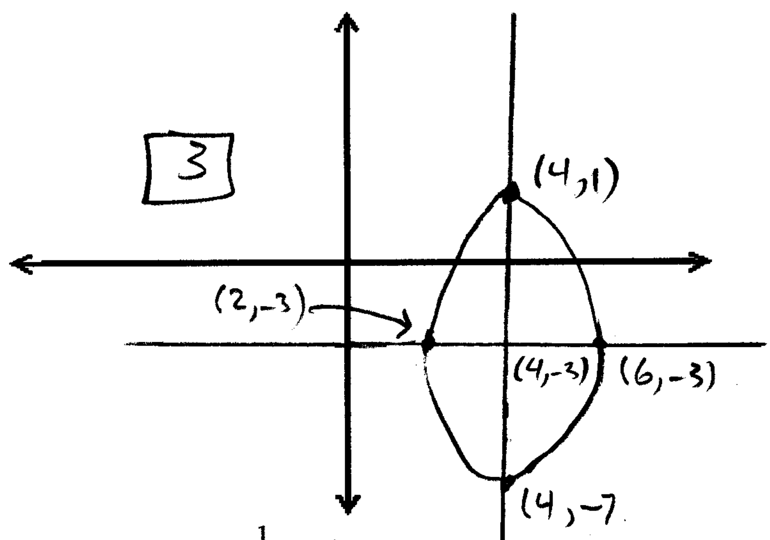
c) the equation of the ellipse in standard form

$$\frac{(x-4)^2}{4} + \frac{(y+3)^2}{16} = 1 \quad \boxed{1}$$

d) sketch the graph of the ellipse (show the coordinates of the vertices of the ellipse)

See problems
1 to 44 p. 704-705

$\boxed{3}$



Q12. (4 points) Find the domain and range of the function $f(x) = \frac{1}{2}\sin^{-1}(3x-2)$

Domain: $-1 \leq 3x - 2 \leq 1 \overset{\boxed{1}}{\Longrightarrow} 1 \leq 3x \leq 3$

$\Rightarrow \frac{1}{3} \leq x \leq 1$ $\boxed{1}$

Range: $-\frac{\pi}{2} \leq \sin^{-1}(3x-2) \leq \frac{\pi}{2} \overset{\boxed{1}}{\Longrightarrow} -\frac{\pi}{4} \leq \frac{1}{2}\sin^{-1}(3x-2) \leq \frac{\pi}{4}$

See the definition p. 592 $\boxed{1}$