1. The expression $\dfrac{\sin^2 x + \tan^2 x + \cos^2 x}{\csc^2 x - 1}$ is equal to

A) $\tan^2 x \sec^2 x$
B) $\cot^2 x \sec^2 x$
C) $\cot^2 x \csc^2 x$
D) $\tan^2 x \csc^2 x$
E) $\csc^2 x$

$$= \frac{\sin^2 x + \cos^2 x + \tan^2 x}{\csc^2 x - 1} = \frac{1 + \tan^2 x}{1 + \cot^2 x - 1}$$

$$= \frac{\sec^2 x}{\cot^2 x} =$$

$$= (\sec^2 x) \cdot \tan^2 x$$

$\boxed{\text{See exercise 21 Page 560}}$

2. The expression $\dfrac{1}{1-\sin x} - \dfrac{\sin x}{1+\sin x}$ is equal to

A) $2\tan^2 x + 1$
B) $2\tan^2 x - 1$
C) $2\sec^2 + 1$
D) $1 - 2\sec^2 x$
E) $2\csc^2 x - 1$

$$\frac{(1+\sin x)}{(1-\sin x)(1+\sin x)} - \frac{\sin x(1-\sin x)}{(1+\sin x)(1-\sin x)}$$

$$= \frac{1 + \sin x - \sin x + \sin^2 x}{1 - \sin^2 x}$$

$$= \frac{1 + 1 - \cos^2 x}{\cos^2 x} =$$

$$= \frac{2 - \cos^2 x}{\cos^2 x} = 2\sec^2 x - 1$$

$$= 2(\tan^2 x + 1) - 1$$

$$= 2\tan^2 x + 1$$

(

See exercise 47 - 52 Page 560
example 4 Page 558

3. The value of the expression $\dfrac{1-\tan 29^o \cot 59^o}{\tan(29^o)+\cot 59^o}$ is

A) $\dfrac{1}{\sqrt{3}}$

B) $\sqrt{3}$

C) 1

D) $-\dfrac{1}{\sqrt{3}}$

E) $-2\tan 29^o$

$$= \frac{1 - \tan 29 \tan(90-59)}{\tan 29 + \tan(90-59)}$$

$$= \frac{1 - \tan(29)\tan(31^\circ)}{\tan 31^\circ + \tan 29^\circ} = \frac{1}{\tan(29+31^\circ)} = \frac{1}{\tan 60^\circ}$$

$$= \frac{1}{\sqrt{3}}$$

See exercise 17 − 18 Page 570
" 57 Page 571

example 36, Page 567

4. If $\tan\alpha = \dfrac{4}{3}$ and $\cos\beta = -\dfrac{12}{13}$ such that α and β are both in quadrant III, then the value of $\cos\left(\dfrac{\pi}{2}+(-\alpha+\beta)\right)$ is equal to

A) $\dfrac{33}{65}$

B) $\dfrac{63}{65}$

C) $\dfrac{16}{65}$

D) $\dfrac{56}{65}$

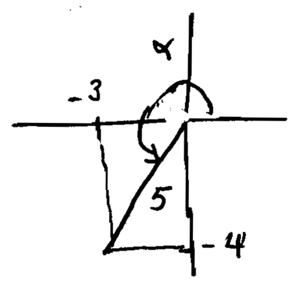
E) $\dfrac{23}{65}$

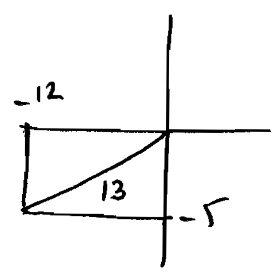
$$\cos\left(\frac{\pi}{2} - (\alpha-\beta)\right) = \sin(\alpha-\beta)$$

$$= \sin\alpha \cos\beta - \cos\alpha \sin\beta$$

$$= \frac{-4}{5}\cdot\frac{-12}{13} - \frac{-3}{5}\cdot\frac{-5}{13}$$

$$= \frac{48-15}{65} = \frac{33}{65}$$





See exercises 69 − 70 Page 571

5. The range of the function $f(x)=1-4\sin 2x\cos 2x\sin 4x$ is

A) $[-1,1]$
B) $[-1,3]$
C) $[-3,5]$
D) $[-2,2]$
E) $[-4,4]$

$$= 1 - 2(2\sin 2x\cos 2x)\sin 4x$$
$$= 1 - 2(\sin 4x)\sin 4x$$
$$= 1 - 2\sin^2 4x$$
$$= \cos 8x$$

The range is $[-1,1]$

$\boxed{\text{See exercise 49, Page 579}}$

6. The expression $\cos 110^\circ$ is equal to

A) $-\sqrt{\dfrac{1-\cos 40^\circ}{2}}$
B) $\sqrt{\dfrac{1-\cos 40^\circ}{2}}$
C) $-\sqrt{\dfrac{1-\cos 220^\circ}{2}}$
D) $\sqrt{\dfrac{1+\sin 50^\circ}{2}}$
E) $-\sqrt{\dfrac{1+\sin 220^\circ}{2}}$

$$\cos 110^\circ = -\sqrt{\frac{1+\cos 220^\circ}{2}} \quad \because \cos 110^\circ < 0$$
$$= -\sqrt{\frac{1-\cos 40^\circ}{2}} \quad \because 220^\circ \in \text{III} \text{ with ref ang } 40^\circ$$

$\boxed{\text{See exercises 10, 13, Page 578}}$

ex 49 - 58 Pg 588

7. The expression $-\sqrt{2}\sin\frac{\pi}{5}+\sqrt{2}\cos\frac{\pi}{5}$ can be written as

A) $2\sin\frac{\pi}{20}$

B) $2\sin\frac{9\pi}{20}$

C) $-2\sin\frac{9\pi}{20}$

D) $-2\cos\frac{9\pi}{20}$

E) $2\sin\frac{7\pi}{20}$

$= k\sin\left(\frac{\pi}{5}+\alpha\right)$,

$k = 2$ $\quad \sin\alpha = \frac{\sqrt{2}}{2}$, $\cos\alpha = -\frac{\sqrt{2}}{2}$

$\Rightarrow \alpha' = \frac{\pi}{4}$ & $\alpha \in Q\,\text{II}$ $\Rightarrow \alpha = \frac{3\pi}{4}$

$2\sin\left(\frac{\pi}{5}+\frac{3\pi}{4}\right) = 2\sin\left(\frac{4\pi+15\pi}{20}\right)$

$= 2\sin\left(\frac{19\pi}{20}\right) = 2\sin\frac{\pi}{20}$

$\downarrow$

$Q\,\text{II} \rightarrow \sin > 0$

$\boxed{\text{See exercises 49 - 58, Page 588}}$

8. The value of $\cos\left(2\tan^{-1}\left(-\frac{4}{5}\right)\right)$

A) $\frac{9}{41}$

B) $-\frac{9}{41}$

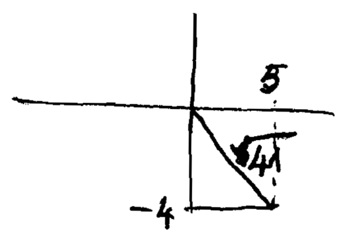
C) $\frac{5}{\sqrt{41}}$

D) $-\frac{40}{41}$

E) $-\frac{4}{\sqrt{41}}$

$\theta = \tan^{-1}\left(-\frac{4}{5}\right) \in Q\,\text{IV}$

& $\tan\theta = -\frac{4}{5}$



$\cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2\left(\frac{-4}{\sqrt{41}}\right)^2 = 1 - \frac{32}{41} = \frac{41-32}{41}$

$= \frac{9}{41}$

$\boxed{\text{See exercises 49 - 52, Page 602}}$

9. The <u>range</u> of the function $y = -\cos^{-1}(2-x) - \frac{\pi}{2}$

A) $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$

B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

C) $\left[-\frac{3\pi}{2}, +\frac{\pi}{2}\right]$

D) $\left[-1-\frac{\pi}{2}, 1-\frac{\pi}{2}\right]$

E) $[1,3]$

function	Range
$y = \cos^{-1} x$	$[0, \pi]$
$y = \cos^{-1}(-x)$	$[0, \pi]$
$y = \cos^{-1}(2-x)$	$[0, \pi]$
$y = -\cos^{-1}(2-x)$	$[-\pi, 0]$
$y = -\cos^{-1}(2-x) - \pi/2$	$\left[-\frac{3\pi}{2}, -\pi/2\right]$

See exercises 75-82, Page 602
recitation 6.5, Question 5

10. The solution set of the equation $\sin^{-1} x + \cos^{-1}\frac{4}{5} = \frac{\pi}{4}$ is equal to

A) $\left\{\frac{\sqrt{2}}{10}\right\}$

B) $\left\{\frac{3}{5}\right\}$

C) $\left\{\frac{7\sqrt{2}}{10}\right\}$

D) $\left\{\frac{3\sqrt{3}-4}{10}\right\}$

E) $\left\{-\frac{\sqrt{2}}{10}\right\}$

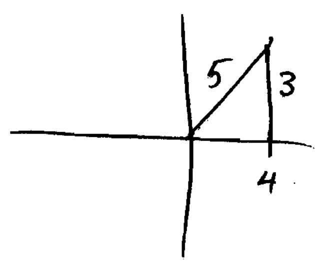
$$\sin^{-1}(x) = \frac{\pi}{4} - \cos^{-1}\left(\frac{4}{5}\right)$$

$$x = \sin\left(\frac{\pi}{4} - \underbrace{\cos^{-1}(4/5)}_{\theta}\right)$$

$$= \sin\frac{\pi}{4}\cos\theta - \cos\frac{\pi}{4}\sin\theta$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{4}{5} - \frac{\sqrt{2}}{2}\,\frac{3}{5}$$

$$= \frac{\sqrt{2}}{10}$$



See exercises 63-66, Page 602
recitation 6.5, Question 4

11. The sum of the solutions of

$2\sin x\cos x+\sqrt{3}\cos x+2\sin x+\sqrt{3}=0$ in the interval $\left[0,\frac{3\pi}{2}\right)$ is

A) $\frac{7\pi}{3}$

B) 3π

C) 4π

D) $\frac{11\pi}{6}$

E) $\frac{7\pi}{6}$

$$\left(2\sin x\cos x+\sqrt{3}\cos x\right)+\left(2\sin x+\sqrt{3}\right)=0$$

$$\cos x\left(2\sin+\sqrt{3}\right)+\left(2\sin x+\sqrt{3}\right)=0$$

$$\left(2\sin x+\sqrt{3}\right)\left(\cos x+1\right)=0$$

$\sin x=-\frac{\sqrt{3}}{2}$ or $\cos x=-1$

$x'=\frac{\pi}{3}$, $x\in \text{III}$ or IV $\qquad x=\pi$ ✓

$x=\pi+\pi/3=\frac{4\pi}{3}$ ✓

or $x=2\pi-\pi/3=\frac{5\pi}{3}\notin\left[0,\frac{3\pi}{2}\right)$

$$\boxed{\pi+\frac{4\pi}{3}=\frac{7\pi}{3}}$$

See exercise 49-50, Page 614

example 5, Page 605

12. The number of solutions of the equation $\cos\left(2x+\frac{2\pi}{3}\right)-\frac{\sqrt{3}}{2}=0$ in $[0,2\pi]$

A) 4

B) 3

C) 2

D) 1

E) 5

$$\cos\left(2x+\frac{2\pi}{3}\right)=\frac{\sqrt{3}}{2}$$

$2x+\frac{2\pi}{3}=\frac{\pi}{6}+2k\pi$ or $2x+\frac{2\pi}{3}=\frac{11\pi}{6}+2k\pi$

$2x=-\frac{4\pi}{6}+\frac{\pi}{6}+2k\pi$ $\qquad 2x=-\frac{4\pi}{6}+\frac{11\pi}{6}+2k\pi$

$=-\frac{\pi}{2}+2k\pi$ $\qquad 2x=\frac{7\pi}{6}+2k\pi$

$x=-\frac{\pi}{4}+k\pi$ $\qquad x=\frac{7\pi}{12}+k\pi$

Sol^{ns} in $[0,2\pi]$ $\qquad$ Sol^{n} in $[0,2\pi]$

$\frac{3\pi}{4}$, $\frac{7\pi}{4}$ $\qquad \frac{7\pi}{12}$, $\frac{19\pi}{12}$

4 sol^{ns}.

See exercises 67-68, Page 615

13. The smallest positive angle between the vectors
$V = -2i - 2\sqrt{3}j$ and $W = -5i + 5\sqrt{3}j$ is

A) $\frac{2\pi}{3}$

B) $\frac{\pi}{3}$

C) $\frac{\pi}{6}$

D) $\frac{5\pi}{6}$

E) $\frac{4\pi}{3}$

$$\cos\alpha = \frac{V \cdot W}{\|V\| \cdot \|W\|} = \frac{10 - 30}{4 \cdot 10} = \frac{-20}{40} = -\frac{1}{2}$$

$$\Rightarrow \alpha' = \frac{\pi}{3}, \quad \alpha \in \mathrm{II} \quad \because (0 < \alpha < \pi)$$

$$\Rightarrow \boxed{\alpha = \frac{2\pi}{3}}$$

See exercises 53 - 60 Page 662
example 9, Page 658

14. Given that a vector V with $\|V\| = 5$ and direction angle $\theta = 30°$, the vector W of magnitude 2 and in the opposite direction of V is

A) $\langle -\sqrt{3}, -1 \rangle$

B) $\langle \sqrt{3}, 1 \rangle$

C) $\langle -2\sqrt{3}, -2 \rangle$

D) $\langle -\frac{1}{2}, \sqrt{3} \rangle$

E) $\langle -1, -\sqrt{3} \rangle$

$$W = -\|W\| \langle \cos 30°, \sin 30° \rangle$$
$$= -2 \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$$
$$= \langle -\sqrt{3}, -1 \rangle$$

See exercises 33 - 36 Page 661
recitation 7.3, Question 1 b

15. A parabola has equation $3x^2+2mx+8y=-24$, its vertex is $(3,k)$. Then the value of k is

A) $\frac{3}{8}$

B) 3

C) -9

D) 24

E) 1

$$3\left(x^2+\frac{2m}{3}x+\left(\frac{m}{3}\right)^2\right)=-8y-24+\frac{m^2}{3}$$

$$3\left(x+\frac{m}{3}\right)^2=-8y-24+\frac{m^2}{3}=-8\left(y+3-\frac{m^2}{24}\right)$$

$$3=-\frac{m}{3} \rightarrow m=-9$$

$$k=-3+\frac{m^2}{24}=-3+\frac{81}{24}=\frac{-72+81}{24}$$

$$=\frac{9}{24}=\boxed{\frac{3}{8}}$$

See exercises 19 – 26 Page 692
recitation 8.1, Question 4

16. A parabola has its focus at $(2,5)$. Its directrix is vertical and passes through $(-4,3)$. Its equation is

A) $y^2-10y-12x+13=0$

B) $(x-2)^2=4(y-4)$

C) $y^2-10y+12x+37=0$

D) $x^2-4x+4y-12=0$

E) $y^2-12y-10x-37=0$

directrix is $x=-4$

focus is $(2,5)$

$\Rightarrow$

$|2p|=6$ & $p>0$

$\Rightarrow p=3$,

Center is $(-1,5)$

The eq^n is $(y-5)^2=4(3)(x+1)$

$y^2-10y+25-12x-12=0$

$y^2-10y-12x+13=0$

See exercises 31 – 32, Page 692

17. The equation of the ellipse with vertices at $(7,3)$ and $(-3,3)$ and one focus at $(-1,3)$ is

A) $\dfrac{(x-2)^2}{25}+\dfrac{(y-3)^2}{16}=1$

B) $\dfrac{(x+2)^2}{25}+\dfrac{(y+3)^2}{64}=1$

C) $\dfrac{(x-2)^2}{25}+\dfrac{(y-3)^2}{64}=1$

D) $\dfrac{(x+2)^2}{64}+\dfrac{(y-3)^2}{25}=1$

E) $\dfrac{(x+2)^2}{25}+\dfrac{(y-3)^2}{21}=1$

Center is $(2,3)$

$a = 5$

$c = |2-(-1)| = 3$

$b^2 = a^2 - c^2 = 25 - 9 = 16$

Equation is

$\dfrac{(x-2)^2}{25} + \dfrac{(y-3)^2}{16} = 1$

See exercises 43-44, Page 705
recitation 8.2, Question 2.

18. The set of all values of k for which the equation $9x^2-18x+4y^2+16y+k=0$ is an ellipse is

A) $(-\infty,25)$

B) $(25,\infty)$

C) $\{-11\}$

D) $\{-1\}$

E) $\{24\}$

$9(x^2-2x+1) + 4(y^2+4y+4) = -k+9+16$

$9(x-1)^2 + 4(y+2)^2 = 25-k$

is an ellipse is $25-k>0$

$k < 25$

See exercises 23-32, Pg 705

19. The equations of the asymptotes of the hyperbola $16y^2-9x^2-64y-18x-89=0$ are

A) $4y-3x-11=0$ and $4y+3x-5=0$
B) $4y-3x+11=0$ and $4y+3x+5=0$
C) $4y-3x-5=0$ and $4y+3x+11=0$
D) $4y-3x+1=0$ and $4y+3x-1=0$
E) $x-y=0$ and $x+y=0$

$$16(y^2-4y+4)-9(x^2+2x+1) = 89+64-9$$

$$16(y-2)^2-9(x+1)^2 = 144$$

$$\frac{(y-2)^2}{9}-\frac{(x+1)^2}{16}=1$$

$a=3$, $b=4$

Asymptotes are $(y-2)=\pm\frac{a}{b}(x+1)$

$$(y-2)=\pm\frac{3}{4}(x+1)$$

$4y-8-3x-3=0$ or $4y-8+3x+3=0$

$\boxed{4y-3x-11=0}$ or $\boxed{4y+3x-5=0}$

See exercises 23-26, Page 719
recitation 8.3, Question 3

20. A hyperbola with center $(2,7)$ is passing through the point $(4,5)$ and has one asymptote with slope 2 and its tranverse axis is horizontal. Its equation is

A) $4x^2-16x-y^2+14y-45=0$
B) $4x^2+16x-y^2+14y+81=0$
C) $4x^2-8x-y^2+14y-54=0$
D) $4x^2-4x-y^2+6y-144=0$
E) $4y^2+8y-x^2+14x-54=0$

$$\frac{(x-2)^2}{a^2}-\frac{(y-7)^2}{b^2}=1$$

$m=\frac{b}{a}=2 \Rightarrow b=2a \Rightarrow b^2=4a^2$

$$\frac{(4-2)^2}{a^2}-\frac{(5-7)^2}{4a^2}=1$$

$\frac{4}{a^2}-\frac{1}{a^2}=1 \Rightarrow a^2=3$

$\Rightarrow b^2=12$

$$\frac{(x-2)^2}{3}-\frac{(y-7)^2}{12}=1$$

$$4(x-2)^2-(y-7)^2=12$$

$$4x^2-16x+16-y^2+14y-49-12=0$$

$$4x^2-16x-y^2+14y-45=0$$

See exercises 45-46, Page 720

21. If the system of equations $\begin{cases} 2x+5y+A &= 0 \\ 3x-By &= 2 \end{cases}$ has an infinite number of solutions, then $A+B$ is equal to

A) $-\frac{53}{6}$

B) $-\frac{17}{4}$

C) $-\frac{19}{3}$

D) -12

E) -25

$$\begin{cases} 2x+5y = -A & \times 3 \\ 3x - By = 2 & \times -2 \end{cases}$$

$$\begin{cases} 6x + 15y = -3A \\ -6x + 2By = -4 \end{cases}$$

$$(15+2B)y = -3A - 4$$

For independent system $15+2B=0$ & $-3A-4=0$

$B = -\frac{15}{2}$, $A = -4/3$

$A+B = -\frac{45}{6} - \frac{8}{6} = -\frac{53}{6}$

Similar to axercise 77, Page 780, racitation 9.1, Question 6

22. Given that the lines with equations
$3x-2y=12$ and $2x-3y=13$ and $5x+ky=19$ intersect at the same point, the number k satisfies

A) $k=-3$

B) $k \neq -\frac{15}{2}$

C) $k=2$

D) $k=-\frac{15}{2}$

E) $k \neq -\frac{15}{2}$ and $k \neq -2$

$$\begin{cases} 3x-2y=12 & \times 2 \\ 2x-3y=13 & \times -3 \end{cases}$$

$$6x - 4y = 24$$
$$-6x + 9y = -39$$
$$5y = -15$$

$\Rightarrow \boxed{y=-3} \Rightarrow x = \frac{12+2y}{3} = \frac{12-6}{3} = \boxed{2 = x}$

Replace in $5x+ky=19$

$5(2)$ $3k = 19$

$-3k = 19 - =$

$k = \frac{9}{-3} = -3$

See axarcises 52, 54 → Page 779