King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Prep-Year Math Program
Math002 Quiz #2

St. ID:______________ St. Name:____________________ Sec#:__________ Serial#:_________

Q1: Find the value of each of the following

(a) $\ln \ln e^{e^{x+3}} - e^{-\ln(1/x)}$

$$\ln\left(\ln e^{e^{x+3}}\right) - e^{\ln(1/x)^{-1}}$$

$$\ln e^{x+3} - e^{\ln x}$$

$$x + 3 - x = 3 //$$

(b) $(\log_5 9)(\log_9 35 - \log_9 7)$

$$= \log_5 9 \cdot \left(\log_9 \frac{35}{7}\right) = \log_5 9 \cdot \log_9 5$$

$$= \frac{\log \cancel{9}}{\cancel{\log 5}} \cdot \frac{\cancel{\log 5}}{\log \cancel{9}} = \underline{1}$$

(c) $\log_a\left(\frac{1}{9}\right)$, if $\log_3 a = \frac{1}{3}$

$$\log_a\left(\tfrac{1}{9}\right) = \frac{\log_3 \frac{1}{9}}{\log_3 a} = \frac{\log_3 \frac{1}{9}}{1/3} = 3\log_3 \frac{1}{9} = 3\log_3 3^{-2}$$

$$= \log_3 (3^{-2})^3 = \log_3 3^{-6} = \underline{\underline{-6}}$$

(d) y, if $y^{\frac{1}{3}} = \log_{\frac{1}{10}} 100$

$$y^{1/3} = \log_{\frac{1}{10}} 100 = \log_{10} \frac{1}{100} = \log 10^{-2} = -2$$

i.e. $y^{1/3} = -2 \quad \therefore \left(y^{1/3}\right)^3 = (-2)^3$

$$y = -8 //$$

Q2: Find the solution set of each of the following

(a) $\log(x+4)<0$

$\Rightarrow \log(x+4) < \log 1 \qquad (\text{Since } \log 1 = 0)$

$\Rightarrow (x+4) < 1$ or $x < -3$.

but the domain of $\log(x+4)$ is

$x+4>0 \Rightarrow x>-4$.

$\therefore$ S.S. $(-4,-3)$

(b) $\log_3\left(\log_{\frac{1}{2}} x\right)=1$

$\Rightarrow \log_3\left(\log_{1/2} x\right) = \log_3 3 \qquad (\text{Since } \log_3 3 = 1)$

$\Rightarrow \log_{1/2} x = 3$ or $x = \left(\tfrac{1}{2}\right)^3$

$\therefore x = \frac{1}{8}$ //

(c) $\left(\frac{2}{3}\right)^{|k-5|}=\left(\frac{81}{16}\right)^{-|k|}$

$\Rightarrow \left(\frac{2}{3}\right)^{|k-5|} = \left(\frac{16}{81}\right)^{|k|} \Rightarrow \left(\frac{2}{3}\right)^{|k-5|} = \left(\frac{2}{3}\right)^{4|k|}$

$\Rightarrow |k-5| = 4|k|$ or $(k-5)^2 = (4k)^2$

$\left\{-\frac{5}{3}, 1\right\}$

$\Rightarrow k^2 - 10k + 25 = 16k^2$ or

$15k^2 + 10k - 25 = 0 \Rightarrow \{k$

$3k^2 + 2k - 5 = 0$

$(3k+5)(5k-5) = 0 \Rightarrow k = -5/3$ or $k=1$

(d) $\log_3 x + 2\log_9 x > 2$

$\log_3 x + \log_{3^2} x^2 > 2$

$\log_3 x + \log_3 (x^2)^{1/2} > 2$

$\log_3 x + \log_3 x > 2$

$2\log_3 x > 2$ or $\log_3 x^2 > \log_3 3^2 \Rightarrow x^2 > 3^2$ or $x^2 - 9 > 0$

$(x-3)(x+3) > 0$ i.e. $(-\infty, -3) \cup (3, \infty)$

but the domain of $\log_3 x^2$ is $x^2>0 \Rightarrow x>0$

$\therefore$ S.S. $(3, \infty)$ //

Q3:

(a) Find the radian measure of the angle that is complementary to the angle $79^o 30'$

The complement angle is

$$\begin{array}{r} 89^\circ\ 60' \\ -\ 79^\circ\ 30' \\ \hline 10^\circ\ 30' \end{array} = 10^\circ + 30'\left(\frac{1^\circ}{60'}\right) = \left(\frac{21}{2}\right)^\circ = 10.5^\circ$$

Thus in radian, we have

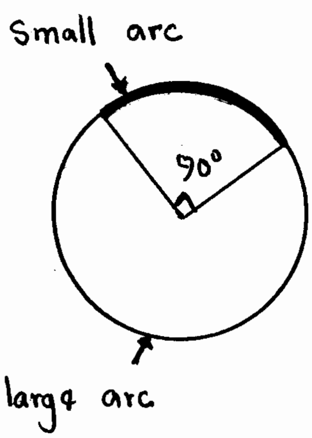
$$\left(\frac{21}{2}\right)^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{7\pi}{120} \quad \text{or} \quad \frac{1.05\pi}{18}$$

(b) Find the smallest positive angle coterminal with $\theta = -761^o$

$$-761^\circ + (3)\,360^\circ = -761^\circ + 1080$$

$$= 319^\circ \quad //$$

(c) Find the length of the smaller arc of a circle of diameter 2 meter whose angle is 90^o



diameter $= 2\,m \Rightarrow$ radius $= \frac{2}{2}\,m = 1\,m$.

Also, $\theta = 90^\circ = 90^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{2}$ radian.

Thus, the length of the smaller arc is

$$S = r\theta = 1\,m\,(\pi/2) = \pi/2\ m \quad ///.$$