

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Prep-Year Math Program
Math002 Quiz #1

St. ID: _____ St. Name: _____ Sec#: _____ Serial#: _____

Q1: Given the function $f(x) = 1 - 2^{x-2}$

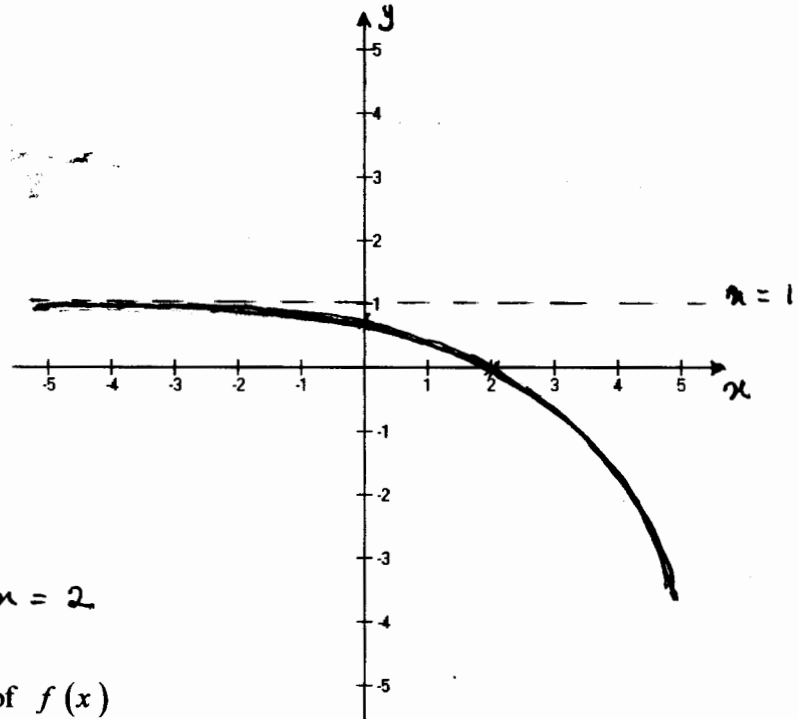
(a) Sketch the graph of $f(x)$

$x=0$:

$$\begin{aligned} y &= 1 - 2^{-2} \\ &= 1 - \frac{1}{4} \\ &= \frac{3}{4} \end{aligned}$$

$y=0$:

$$\begin{aligned} 0 &= 1 - 2^{x-2} \\ \Rightarrow 1 &= 2^{x-2} \\ \log_2 1 &= x-2 \\ 0 &= x-2 \Rightarrow x = 2 \end{aligned}$$



(b) Find the x- and y- intercepts of $f(x)$

x-intercept: $(2, 0)$

y-intercept: $(0, \frac{3}{4})$

(c) Find the domain and the range of $f(x)$

Domain: $(-\infty, \infty)$

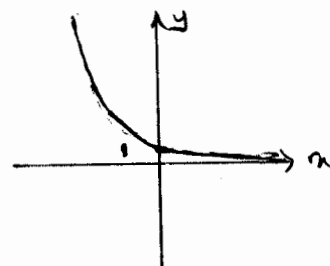
Range: $(-\infty, 1)$

(d) Find the asymptote (if any) of $f(x)$

line $y = 1$ (horizontal asymptote)

(e) Find the range of $n(x) = \frac{e^{-x}}{e^x} = e^{-2x}$

Range: $y > 0$



Q2: Given the function $f(x) = -\log_{\frac{1}{2}}(2-3x)$

(a) Sketch the graph of $f(x)$

$x=0$:

$$y = -\log_{\frac{1}{2}} 2$$

$$-y = \log_{\frac{1}{2}} 2$$

$$\left(\frac{1}{2}\right)^{-y} = 2$$

$$2^y = 2$$

$$\Rightarrow y = 1 //$$

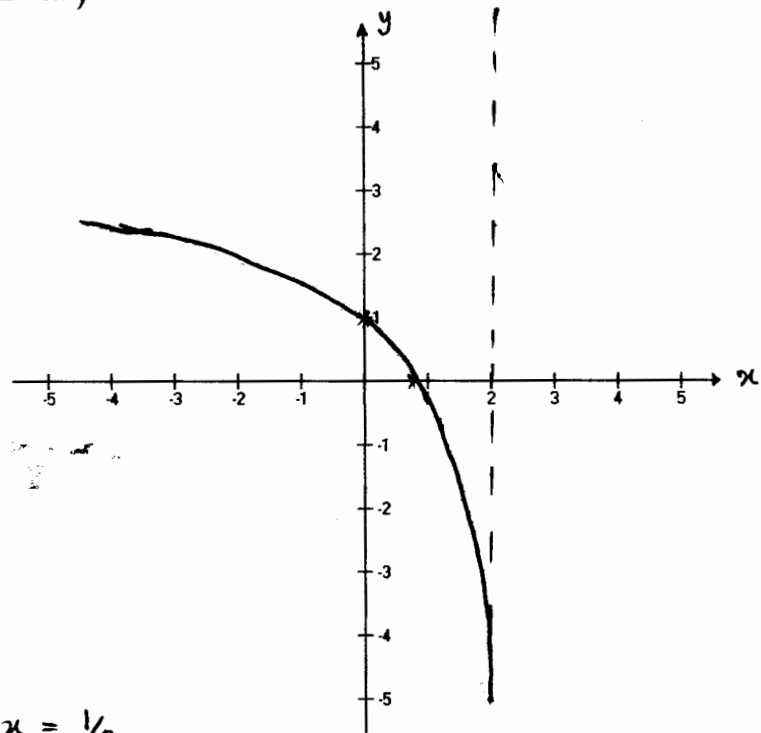
$y=0$:

$$0 = -\log_{\frac{1}{2}}(2-3x)$$

$$0 = \log_{\frac{1}{2}}(2-3x)$$

$$\left(\frac{1}{2}\right)^0 = 2-3x$$

$$\Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$$



(b) Find the x- and y- intercepts of $f(x)$

$$x\text{-intercept: } \left(\frac{1}{3}, 0\right)$$

$$y\text{-intercept: } (0, 1)$$

(c) Find the domain and the range of $f(x)$

$$\text{Domain: } (-\infty, 2)$$

$$\text{Range: } (-\infty, \infty)$$

(d) Find the asymptote (if any) of $f(x)$

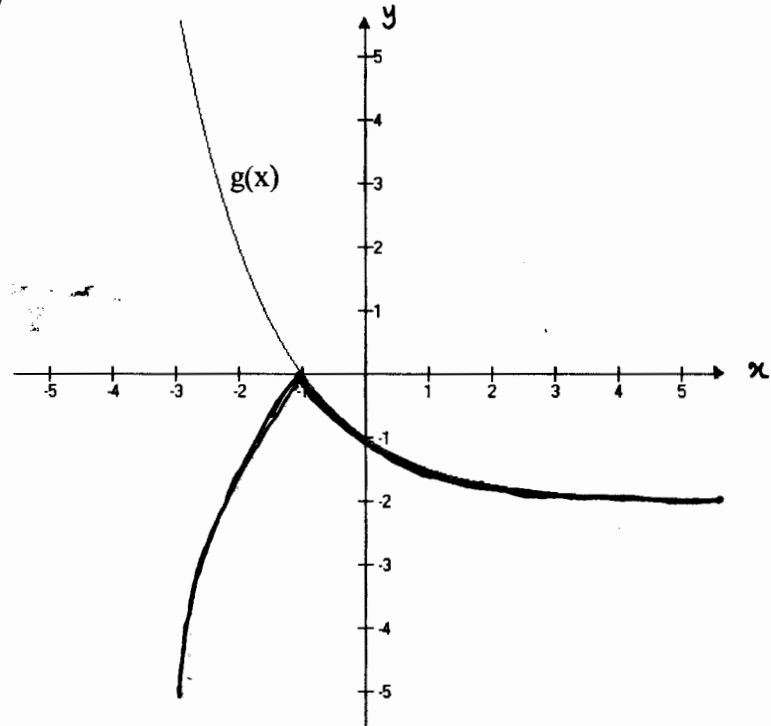
line $x=2$ (vertical asymptote)

(e) Is the function increasing or decreasing?

decreasing

Q3: Given that $f(x) = -\left(\frac{1}{2}\right)^x - 2$

- (a) Use the graph of $g(x) = \left(\frac{1}{2}\right)^x - 2$ in the adjacent figure to sketch the graph of $f(x)$



- (b) Find the x- and y- intercepts of $f(x)$

x-intercept: $(-1, 0)$

y-intercept: $(0, -1)$

- (c) Find the domain and the range of $f(x)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

- (d) The graph of $f(x)$ lies below the x-axis over what interval?

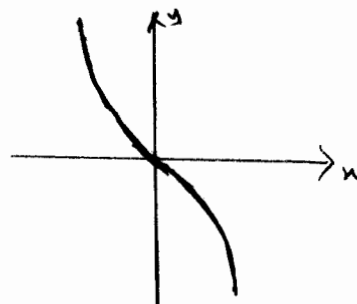
$(-\infty, \infty)$

- (e) Find the range of $h(x) = e^{-x} - e^x$

$x=0$

$e^0 - e^0 = 1 - 1 = 0$

Range: $(-\infty, \infty)$



Q4: Let $f(x)$ be a logarithmic function such that $f\left(\frac{1}{4}\right) = 2$

(a) Find the value of $f\left(\frac{1}{2}\right) = 1$

$$\therefore f\left(\frac{1}{2}\right) = \log_{\frac{1}{2}} \frac{1}{2} = 1 //$$

$$f(x) = \log_b x$$

$$\therefore f\left(\frac{1}{4}\right) = \log_b \frac{1}{4} = 2$$

$$\Rightarrow (b)^2 = \frac{1}{4}$$

$$b^2 = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow b = \frac{1}{2}$$

(b) Find the value of $f^{-1}(-1) = 2$

$$f(x) = \log_{\frac{1}{2}} x$$

$$y = \log_{\frac{1}{2}} x$$

$$\text{inverse: } x = \log_{\frac{1}{2}} y \Rightarrow \left(\frac{1}{2}\right)^x = y = f^{-1}(y)$$

Thus

$$f^{-1}(-1) = \left(\frac{1}{2}\right)^{-1}$$

$$= 2 //$$

(c) Use the symbols "<" and ">" to determine the relationship between $(x^x)^x$ and $x^{(x^x)}$ when $x = 3$.

$$(x^x)^x < x^{(x^x)}$$

$$\left(3^3\right)^3 = 3^9$$

$$3^{(3^3)} = 3^{27}$$

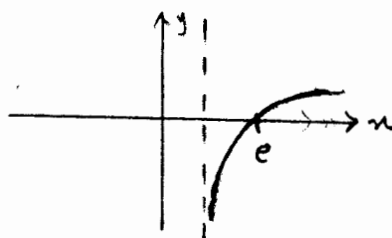
(d) Find the domain of $g(x) = \ln(\ln x)$

$$\ln x > 0$$

$$e^{\ln x} > e^0$$

$$x > 1$$

Domain:



(e) Find the domain of $r(x) = \sqrt{\log x}$

$$\log x \geq 0$$

$$\log x \geq \log 1$$

$$\text{Domain: } x \geq 1$$

