King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Prep-Year Math Program
Math002 Quiz #4

St. ID:_____ St. Name:_____ Sec#:_____ Serial#:_____

Q1 Given the function $f(x) = \frac{3}{2}\tan\left(2x - \frac{\pi}{2}\right)$

(a) Find the equation of all vertical asymptote of $f(x)$ over the interval $[-\pi, \pi]$ **(3 points)**

Note that: $\frac{3}{2}\tan\left(2x - \frac{\pi}{2}\right) = \frac{3}{2}\sin\left(2x - \pi/2\right) \Big/ \frac{3}{2}\cos\left(2x - \pi/2\right)$

Thus, the VA of tangent are the zeros of Cosine

$\therefore \quad 2x - \pi/2 = \pi/2 + k\pi$

$\Rightarrow \quad 2x = (k+1)\pi$

$x = (k+1)\pi/2$

for $k = -3, -2, -1, 0, 1$

when $k = -3$, $x = -\pi$
$k = -2$, $x = -\pi/2$
$k = -1$, $x = 0$
$k = 0$, $x = \pi/2$
$k = 1$, $x = \pi$

(b) Find the $x-$ intercepts of $f(x)$ over the interval $[-\pi, \pi]$ **(2 points)**

The x-intercepts of f(x) are the zeros of Sine.

i.e. $2x - \pi/2 = k\pi$

$2x = (k + 1/2)\pi$

$x = (k + 1/2)\pi/2$

for $k = -2, -1, 0, 1$.

when $k = -3$, $x = -5\pi/4 \notin [-\pi, \pi]$
$k = -2$, $x = -3\pi/4$ ✓
$k = -1$, $x = -\pi/4$ ✓
$k = 0$, $x = \pi/4$ ✓
$k = 1$, $x = 3\pi/4$ ✓
$k = 2$, $x = 5\pi/4 \notin [-\pi, \pi]$

(c) Find the exact value of $\sin 13^\circ \sin 73^\circ + \sin 77^\circ \sin 17^\circ$ **(2 points)**

$= \sin 13^\circ \cos(90^\circ - 73^\circ) + \cos(90^\circ - 77^\circ)\sin 17^\circ$

$= \sin 13^\circ \cos 17^\circ + \cos 13^\circ \sin 17^\circ$

$= \sin(13^\circ + 17^\circ)$

$= \sin 30^\circ = 1/2$

Q2:

(a) If the point $\left(-1, \frac{3}{4}\right)$ lies on the terminal side of an angle θ in standard position, then find the exact value of $\cot\left(\frac{\pi}{2}-\theta\right)$. **(3 points)**

$$(\cos\theta, \sin\theta) = (-1, 3/4)$$

$$\Rightarrow \sin\theta = 3/4 \quad \text{and} \quad \cos\theta = -1$$

but

$$\cot\left(\tfrac{\pi}{2}-\theta\right) = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/4}{-1} = -\frac{3}{4} \;//$$

(b) On the unit circle, if the arc length $\frac{\pi}{12}$ terminates at the point (a, b), then find the value of ab **(3 points)**

$$W\left(\tfrac{\pi}{12}\right) = P(a,b) = \left\{\cos\tfrac{\pi}{12}, \sin\tfrac{\pi}{12}\right\}.$$

$$\Rightarrow a = \cos\tfrac{\pi}{12} = \cos 15^\circ$$

also

$$b = \sin\tfrac{\pi}{12} = \sin 15^\circ$$

$$\therefore ab = \sin 15^\circ \cos 15^\circ = \frac{1}{2}\sin 2(15^\circ) = \frac{1}{2}\sin 30^\circ$$

$$= \tfrac{1}{2}\left(\tfrac{1}{2}\right) = \frac{1}{4}$$

(c) Find the distance between the points $(\cos 85^\circ, \sin 85^\circ)$ and $(\cos 25^\circ, \sin 25^\circ)$ **(2 points)**

$$d = \sqrt{(\cos 25^\circ - \cos 85^\circ)^2 + (\sin 25^\circ - \sin 85^\circ)^2}$$

$$= \sqrt{\cos^2 25^\circ - 2\cos 25^\circ\cos 85^\circ + \cos^2 85^\circ + \sin^2 25^\circ - 2\sin 25^\circ \sin 85^\circ + \sin^2 85^\circ}$$

$$= \sqrt{(\cos^2 25^\circ + \sin^2 25^\circ) + (\cos^2 85^\circ + \sin^2 85^\circ) - 2(\cos 25^\circ\cos 85^\circ + \sin 25^\circ \sin 85^\circ)}$$

$$= \sqrt{1 + 1 - 2\cos(25^\circ - 85^\circ)}$$

$$= \sqrt{2 - 2\cos 60^\circ} = \sqrt{2 - 2\left(\tfrac{1}{2}\right)} = \sqrt{2-1} = \sqrt{1} = 1$$