Physics-305
Homework Set (5)

This homework set is due on Thursday, 29^{th} Muharram, 1437 (Nov. 12^{th}, 2015) at 10.00 p.m.

Question #1:
This question is intended to walk you through the method of Multipole Expansion. This method is most useful when you like to find the potential far away from a localized set of charges.
Develop a systematic expansion for the potential of an arbitrary localized charge distribution, in powers by $1/|\mathbf{r}|$, where $\mathbf{r}$ is the position vector from a chosen origin.

a- Define the relevant variables through a sketch.
b- Starting from the general integral relation between potential and electrostatic charge density, show (*) that the potential can be expressed as a sum in $r^{-(n+1)}$ containing integrals over Legendre polynomials.
c- Explicitly express the so-called Monopole term. Argue how that for a source that is merely a point charge at the origin, the Monopole term is the exact potential.
d- Explicitly express the so-called Dipole term, and identify the so-called Dipole Moment.
e- Argue when it is that the Dipole term is the *dominant* term in the expansion for the potential of a charge distribution.
f- Explain when this term is not dependent on your choice of origin of coordinates.
g- Draw the electric field for a perfect (point) dipole and compare it with that of a physical (real) dipole.

(*) You are free to solve from your textbook; however you need to explain your steps, oftentimes *more* than Griffiths does.

Question #2:
A point charge "q" is situated a distance "a" from the center of a hallow grounded conducting sphere of radius "R", such a < R. Find the potential inside the sphere.

Question #3:
In class, we separated the radial from the angular dependence solution for the azimuthally independent electrostatic potential satisfying Laplace's equation.The angular solution satisfies Chapter-3's equation 60 of your textbook. They turn out to be the Legendre polynomials $\{P_l(\cos\theta)$.

a- Use the Rodrigues formula to express $P_3(x)$ and $P_4(x)$
b- Use a computer package to plot $P_3(x)$, $P_4(x)$, and $P_3(x)\ P_4(x)$ over the interval [-1,1], on the same graph.
c- Compute through direct analytical integration that $P_3(x)$ is orthogonal to $P_4(x)$ over [-1,1].

Question #4:
An infinitely long rectangular metal pipe (sides “a” and “b”; a = 4b) is grounded, but one end, at x = 0, is maintained at a specific potential $V_o(y,z)$.

a- Sketch the problem.
b- Find the potential inside the pipe.
c- For x > 5 a, describe how you can evaluate a good approximation for the potential by summing just a *few* terms.

Question #5:
Textbook chapter-3, problem-19. Furthermore, use a computer package to plot the angular distribution of the charge density as a *spherical plot* (where the surface element distance from the origin, at a particular angle, represents the magnitude of the charge density at that angular position on the sphere).

Work hard on this homework set for it is heavily weighted