FARADAY'S LAW OF INDUCTION.

A coil formed by wrapping 50 turns of wire in the shape of a square, is positioned in a magnetic field so that the normal to the plane of the coil makes an angle of 30° with the direction of the field. It is observed that if the magnitude of the magnetic field is increased uniformly from 200 μT to 600 μT in 0.4 s, an emf of 80 mV is induced in the coil. What is the length of the wire?

$$B_2 - B_1 = \Delta B = 600 - 200 = 400\,\mu T$$

$$\mathcal{E} = 80\,mV = 80 \times 10^{-3}\,V$$

$$N = 50,\quad \Delta t = 0.4\,s \text{ i.e } t_2 - t_1 = 0.4 - 0.0$$

$$\therefore \quad \mathcal{E} = -N\frac{d\phi}{dt} = -N\frac{d}{dt}BA\cos\theta$$

$$\therefore \quad |\mathcal{E}| = +NA\cos\theta\frac{dB}{dt}$$

$$\therefore \quad |\mathcal{E}| = NA\cos\theta\left(\frac{\Delta B}{\Delta t}\right)$$

$\frac{\Delta B}{\Delta t}$ is the rate of increment in B about t

Area $=\therefore\ A = \dfrac{|\mathcal{E}|}{N\cos\theta\left(\frac{600-200}{0.4}\right)\times 10^{-6}} = \dfrac{80\times 10^{-3}}{50\times\cos 30\times\frac{400\times 10^{-6}}{0.4}}$

$$= 1.85\,m^2$$

the edge length of the coil, $d = \sqrt{A}$, and the total length of the coil is $l = N(4d)$

$$\therefore \quad L = 50\times 4\times\sqrt{1.85} = \boxed{272\,m}$$

MAGNETIC FLUX

A toroid is constructed from N rectangular turns of the wire. Each turn has height h. The toroid has an inner radius a and outer radius b. (a) If the toroid carries a current I, show that the total magnetic flux through the turns of the toroid is proportional to $\ln\left(\frac{b}{a}\right)$. (b) Evaluate this flux if $N = 200$ turns $h = 1.5$ cm, $a = 2$ cm, $b = 5$ cm and $I = 2A$

(a) We have the general expression for magnetic flux

$$\Phi_m = \int B \cdot dA = \int_a^b B h \, dr$$

$$\Phi_m = \frac{\mu_0 N I h}{2\pi} \int_a^b \frac{dr}{r}$$

$$\Phi_m = \boxed{\frac{\mu_0 N I h}{2\pi} \ln\left(\frac{b}{a}\right)}$$

(b)

$N = 200$ turns, $h = 1.5\text{cm} = 0.015\text{ m}$

$b = 5\text{cm} = .05\text{ m}$, $a = .02\text{ m}$, $I = 2A$

$\mu_0 = 1.26 \times 10^{-6}\ \text{T.m/A}$

$$\Phi_m = \frac{1.26 \times 10^{-6} \times 200 \times 2 \times 0.015}{2 \times 3.14} \ln\left(\frac{.05}{.02}\right)$$

$$\Phi_m = \boxed{1.1 \times 10^{-6}\ \text{Wb}}$$

MOTIONAL emf and LENZ'S LAW.

In the arrangement shown in fig. 31.28 a conducting bar moves to the right along parallel, frictionless conducting rails connected on one end by a 6 Ω resistor. A 2.5 T magnetic field is directed into the paper (perpendicular to l). let $l = 1.2$ m and neglect the mass of the bar. (a) Calculate the applied force required to move the bar to the right at constant speed of 2 m/s (b) At what rate is energy dissipated in the resistor?

Given $l = 1.2$ m, $B = 2.5$ T, $R = 6\,\Omega$

$V = 2$ m/s

Now since the magnetic field is perpendicular to the l $\therefore$

$F = IlB$, Now $I = \frac{\mathcal{E}}{R}$ and $\mathcal{E} = Blv$

(a) $$F = \frac{\mathcal{E} l B}{R} = \frac{B^2 l^2 v}{R} = \frac{(2.5)^2(1.2)^2(2)}{6}$$

$\therefore$ $F = \boxed{3\,N}$

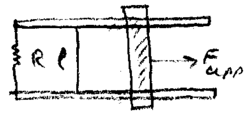
(b) $$P = I^2 R = \frac{B^2 l^2 v^2}{R} = \frac{(2.5)^2(1.2)^2(2)^2}{6}$$

$= \boxed{6\,W}$

OR $P = FV = 3 \times 2 = \boxed{6\,W}$

Note when the bar is moved right side the current is counterclockwise by definition see page 882

A conducting rod of length l moves on two horizontal frictionless rails as shown in fig 31.28. If a constant force of 1N moves the bar at 2 m/s through a magnetic field B which is into the paper, (a) what is the current through an 8-Ω Resistor R? (b) What is the rate of energy dissipation in the resistor? (c) What is the mechanical power delivered by the force F?



Sol.

$$F = 1\ N$$
$$V = 2\ m/s$$
$$R = 8\ \Omega$$

Now $F = IlB$ and $\mathcal{E} = BlV$

$$I = \frac{\mathcal{E}}{R} = \frac{BlV}{R}$$

OR $B = IR/lV$

(a) ⇒ $F = \frac{I^2 \cancel{l} R}{\cancel{l} V}$ OR $I = \sqrt{\frac{FV}{R}} = \sqrt{\frac{1\times 2}{8}} = \boxed{0.5A}$

(b), $I^2R = (0.5)^2(8) = \boxed{2.00\ W}$

(c) for constant force $P = F.V = 1\times 2 = \boxed{2.0\ W}$

INDUCED emfs AND ELECTRIC FIELDS

A Solenoide has a radius of 2 cm and has 1000 turns /m. the current varies with time according to the expression $I = 3e^{0.2t}$, where I is in A and t is in s. Calculate the electric field at a distance of 5 cm from the axis of the solenoide at $t = 10$ s

Given

$$I = 3e^{0.2t}$$

$$t = 10\text{ s}, \quad R = .02\text{ m} = R$$

$$\frac{N}{L} = 1000\text{ turn/m} = N/L$$

$$r = .05\text{ m} = r$$

$$\mathcal{E} = -\pi R^2 \frac{dB}{dt}$$

$$|\mathcal{E}| = \pi R^2 \frac{dB}{dt}$$

$$\therefore \quad |\mathcal{E}| = +\frac{d\phi}{dt} = +\pi R^2 \frac{dB}{dt} = \oint E.dl = \qquad \text{eq 31.14}$$

$$\mathcal{E} = E(2\pi r)$$

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$$\therefore \quad E(2\pi r) = \pi R^2 \frac{dB}{dt} \quad \text{for } r > R$$

$$E = \frac{\mathcal{E}}{2\pi r}$$

Now $B = N\frac{\mu_0 I}{L}$ ← 30.20, page 848

$$\therefore \quad E = \frac{\pi R^2}{2\pi r}\left(\frac{\mu_0 N}{L}\right)\frac{dI}{dt}$$

$$E = \frac{|\mathcal{E}|}{2\pi r} = \frac{\pi R^2}{2\pi r}\frac{dB}{dt}$$

Similarly $I = 3e^{0.2t}$

$$\therefore \quad \frac{dI}{dt} = 0.2 \times 3e^{0.2t} = 0.6 \times e^{0.2t} = 4.43\text{ A/s}$$

$$\therefore \quad E = \frac{(0.02)^2(4\pi \times 10^{-7})(1000) \times 4.43}{2(0.05)}$$

$$E = \boxed{2.23 \times 10^{-5}\text{ N/C}}$$