An isolated conducting sphere can be considered as one element of a capacitor (the other element being a concentric sphere of infinite radius). (a) If the capacitance of this system is 9.1×10^{-11} F. What is the radius of the sphere? (b) If the potential at the surface of the sphere is 2.8×10^{4} V, what is the corresponding surface charge density.

Given $C = 9.11 \times 10^{-11}$ F

$V = 2.8 \times 10^{4}$ V

$r = ?$

$\sigma = ?$

(a) $C = \frac{q}{V}$ and $V = \frac{Kq}{r}$

$\therefore$

$$C = \frac{q \times r}{Kq}$$

$\therefore \quad r = CK$

$\therefore \quad r = 9.1 \times 10^{-11} \times 9 \times 10^{9}$

$$r = \boxed{0.819 \text{ m}}$$

(b) $\sigma = q/4\pi r^2$ eq 23.13

for this we need to calculate q

$\therefore \quad C = \frac{q}{V}$

OR $\quad q = CV = 9.1 \times 10^{-11} \times 2.8 \times 10^{4}$

$q = 2.55 \times 10^{-6}$ C

$\therefore$

$$\sigma = \frac{q}{4\pi r^2} = \frac{2.55 \times 10^{-6}}{4 \times 3.14 \times (0.819)^2}$$

$$\sigma = \boxed{3.03 \times 10^{-7} \text{ C/m}^2}$$

An air filled capacitor consists of two parallel plates, each with an area of 7.6 cm², separated by a distance of 1.8 mm. If a 20-V potential difference is applied to these plates, calculate
(a) the electric field between the plates
(b) the surface charged density, (c) the capacitance
and (d) the charge on each plate?

Given $A = 7.6\ cm^2 = 7.6 \times 10^{-4}\ m^2$

$d = 1.8\ mm = 1.8 \times 10^{-3}\ m$

$V = 20\ V$, $\epsilon_0 = 8.85 \times 10^{-12}\ F m^{-1}$

(a) $V = Ed$

or $E = \frac{V}{d} = \frac{20}{1.8 \times 10^{-3}} = \boxed{1.1 \times 10^{4}\ V/m}$

(b) $E = \frac{\sigma}{\epsilon_0}$ eq 24.11

$\therefore \sigma = E\epsilon_0 = 1.11 \times 10^{4} \times 8.85 \times 10^{-12} = \boxed{9.83 \times 10^{-8}}$ C/m^2

(c) the capacitance is

$C = \frac{\epsilon_0 A}{d}$ eq. 26.3

$\therefore C = \frac{8.85 \times 10^{-12} \times 7.6 \times 10^{-4}}{1.8 \times 10^{-3}} = \boxed{3.74 \times 10^{-12}}\ F$

(d) for charge on each plate

$V = \frac{Q}{C}$

$\therefore Q = V \times C = 20 \times 3.74 \times 10^{-12} = \boxed{7.5 \times 10^{-11}\ C}$

OR $75 \times 10^{-12} = 75\ pC$

Four Capacitors are connected as shown in fig. (a) Find the equivalent capacitance between points (a) and (b). (b) Calculate the charge on each capacitor if $V_{ab} = 15\,V$

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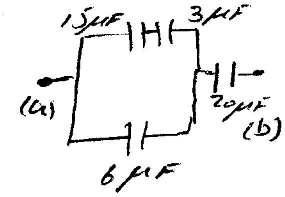
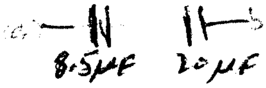


15 μF and 3 μF are in series

$$\therefore \frac{1}{C_{eq}} = \frac{1}{15} + \frac{1}{3} = \frac{3+15}{15 \times 3}$$

$$C_{eq} = \frac{45}{18} = 2.5\ \mu F$$



$\therefore$ the sum 2.5 μF and 6 μF are in parallel

$$\therefore C_{eq} = 2.5 + 6 = 8.5\ \mu F$$

Finally 8.5 μF and 20 μF are in series

$$\therefore C_{eq} = \left(\frac{1}{20} + \frac{1}{8.5}\right)^{-1} = (0.05 + .1176)^{-1}$$

$$\therefore \boxed{C_{eq} = 5.97\ \mu F}$$

(b)

$$Q = CV = 15 \times 5.97 \times 10^{-6} = 89.5\ \mu C$$

on 20 μF

$$\therefore V = Q/C = \frac{89.5}{20} = 4.48\ V$$

Remain voltage is for others which is $15 - 4.48 = 10.52\,V$

$$\therefore Q = VC = 10.52 \times 6 = 63.1\ \mu C \text{ on } 6\ \mu F$$

$$\therefore 89.5 - 63.1 = 26.4\ \mu C \text{ on } 15\ \mu F \text{ and } 3\ \mu F$$

Two Capacitors, $C_1 = 25\mu F$ and $C_2 = 5\mu F$ are connected in parallel and charged with a 100 V power supply. (a) Calculate the total energy stored in the two capacitors. (b) What potential difference would be required across the same two capacitors connected in series in order that the combination store the same energy as in (a)?

the energy eq. is $U = \frac{1}{2} C V^2$

(a) $C_p = C_1 + C_2 = 25 + 5 = 30\mu F = 30\times10^{-6} F$

$\therefore U = \frac{30\times10^{-6}}{2} \times (100)^2 = 15\times10^{-6}\times10^{4} = \boxed{.15\ J}$ $\quad 15\times10^{-6}\times10^{4} = 0.15\ J$

(b) $C_s = \left(\frac{1}{25} + \frac{1}{5}\right)^{-1} = 4.17\mu F = 4.17\times10^{-6} F$

.04

$\therefore U = \frac{1}{2} C V^2$

$\therefore V^2 = \frac{2U}{C} = \frac{2\times.15}{4.17\times10^{-6}}$

$\therefore V = \sqrt{\frac{2\times.15}{4.17\times10^{-6}}} = \boxed{268.54\ V}$

A 16 pF parallel plate capacitor is charged by a 10 V battery. If each plate of the capacitor has an area of 5 cm², what is the energy stored in the capacitor? What is the energy density (energy per unit volume) in the electric field of the capacitor if the plates are separated by air

Given $A = 5\,cm^2 = \left(\frac{5}{10000}\right) = 5\times10^{-4}\,m^2$

$C = 16\,pF = 16\times10^{-12}\,F$

Let's find first $d = ?$

$$\Rightarrow \quad C = \frac{K\epsilon_0 A}{d} \quad OR \quad d = \frac{K\epsilon_0 A}{C}$$

$$\Rightarrow \quad d = \frac{1\times(8.85\times10^{-12})(5\times10^{-4})}{16\times10^{-12}} = \underline{2.76\times10^{-4}\,m}$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\times16\times10^{-12}\times10^2 = \underline{8\times10^{-10}\,J}$$

density

$$U_d = \frac{U}{Ad} = \frac{8\times10^{-10}}{5\times10^{-4}\times2.76\times10^{-4}}$$

$$= 5.79\times10^{-3}\,J/m^3$$

A parallel plate capacitor has a plate area of 0.64 cm^2. When the plates are in vacuum, the capacitance of device is 4.9 PF.
(a) Calculate the value of the capacitance if the space between the plate is filled with nylon. (b) What is the maximum potential difference that can be applied to the plates without causing dielectric breakdown.

Given
$$C_0 = 4.9 \times 10^{-12}\ F$$
$$A = 0.64\ cm^2 = 6.4 \times 10^{-5}\ m^2$$
$$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/N.m^2$$
$$d = ?\ ,\ C_n = ?$$
$$V_{max} = ?$$

(a)

Capacitance in Vacuum is
$$C_0 = \frac{\epsilon_0 A}{d} \quad OR \quad d = \frac{\epsilon_0 A}{C_0} = \frac{8.85\times10^{-12} \times 6.4\times10^{-5}}{4.9\times10^{-12}} = .116\ mm$$

$$\therefore C_n = K C_0 = 3.4 \times 4.9 \times 10^{-12}$$
$$C = 16.7 \times 10^{-12}\ F = 16.7\ PF$$

(b)

E_{max} for nylon is $= 14 \times 10^6\ V/m$
$$V_{max} = E_{max} d = 14 \times 10^6 \times 0.116 \times 10^{-3}$$
$$= 1624\ V$$