

Following are Summaries from Two Chemistry Education Web Sites Concerning Significant Figure Rules

From <http://dbhs.wvusd.k12.ca.us/SigFigs/SigFigRules.html>

There are three rules on determining how many significant figures are in a number:

1. Non-zero digits are always significant.
2. Any zeros between two significant digits are significant.
3. A final zero or trailing zeros in the decimal portion ONLY are significant.

Focus on these rules and learn them well. They will be used extensively throughout the remainder of this course. You would be well advised to do as many problems as needed to nail the concept of significant figures down tight and then do some more, just to be sure.

Please remember that, in science, all numbers are based upon measurements (except for a very few that are defined). Since all measurements are uncertain, we must only use those numbers that are meaningful. A common ruler cannot measure something to be 22.4072643 cm long. Not all of the digits have meaning (significance) and, therefore, should not be written down. In science, only the numbers that have significance (derived from measurement) are written.

If you're not convinced significant figures are important, you may want to read the Significant Figure Fable that follows.

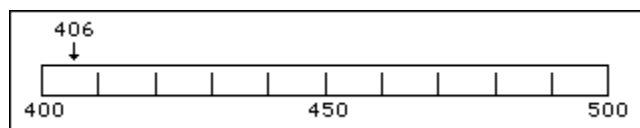
Rule 1: Non-zero digits are always significant.

Hopefully, this rule seems rather obvious. If you measure something and the device you use (ruler, thermometer, triple-beam balance, etc.) returns a number to you, then you have made a measurement decision and that ACT of measuring gives significance to that particular numeral (or digit) in the overall value you obtain.

Hence a number like 26.38 would have four significant figures and 7.94 would have three. The problem comes with numbers like 0.00980 or 28.09.

Rule 2: Any zeros between two significant digits are significant.

Suppose you had a number like 406. By the first rule, the 4 and the 6 are significant. However, to make a measurement decision on the 4 (in the hundred's place) and the 6 (in the unit's place), you HAD to have made a decision on the ten's place. The measurement scale for this number would have hundreds and tens marked with an estimation made in the unit's place. Like this:



Rule 3: A final zero or trailing zeros in the decimal portion ONLY are significant.

This rule causes the most difficulty with ChemTeam students. Here are two examples of this rule with the zeros this rule affects in boldface:

0.005**00**
0.030**40**

Here are two more examples where the significant zeros are in boldface:

2.**30** $\times 10^{-5}$
4.**500** $\times 10^{12}$

What Zeros are Not Discussed Above

Zero Type #1: Space holding zeros on numbers less than one.

Here are the first two numbers from just above with the digits that are NOT significant in boldface:

0.**00**500
0.**03**040

These zeros serve only as space holders. They are there to put the decimal point in its correct location. They DO NOT involve measurement decisions. Upon writing the numbers in scientific notation (5.00×10^{-3} and 3.040×10^{-2}), the non-significant zeros disappear.

Zero Type #2: the zero to the left of the decimal point on numbers less than one.

When a number like 0.00500 is written, the very first zero (to the left of the decimal point) is put there by convention. Its sole function is to communicate unambiguously that the decimal point is a decimal point. If the number were written like this, .00500, there is a possibility that the decimal point might be mistaken for a period. Many students omit that zero. They should not.

Zero Type #3: trailing zeros in a whole number.

200 is considered to have only ONE significant figure while 25,000 has two.

This is based on the way each number is written. When whole number are written as above, the zeros, BY DEFINITION, did not require a measurement decision, thus they are not significant.

However, it is entirely possible that 200 really does have two or three significant figures. If it does, it will be written in a different manner than 200.

Typically, scientific notation is used for this purpose. If 200 has two significant figures, then 2.0×10^2 is used. If it has three, then 2.00×10^2 is used. If it had four, then 200.0 is sufficient. See rule #2 above.

How will you know how many significant figures are in a number like 200? In a problem like below, divorced of all scientific context, you will be told. If you were doing an experiment, the context of the experiment and its measuring devices would tell you how many significant figures to report to people who read the report of your work.

Zero Type #4: leading zeros in a whole number.

00250 has two significant figures. 0.0500×10^{-4} has three.

Exact Numbers

Exact numbers, such as the number of people in a room, have an infinite number of significant figures. Exact numbers are counting up how many of something are present, they are not measurements made with instruments. Another example of this are defined numbers, such as 1 foot = 12 inches. There are exactly 12 inches in one foot. Therefore, if a number is exact, it DOES NOT affect the accuracy of a calculation nor the precision of the expression. Some more examples:

There are 100 years in a century.

2 molecules of hydrogen react with 1 molecule of oxygen to form 2 molecules of water.

There are 500 sheets of paper in one ream.

Interestingly, the speed of light is now a defined quantity. By definition, the value is 299,792,458 meters per second.

Are Significant Figures Important? A Fable

A student once needed a cube of metal that had to have a mass of 83 grams. He knew the density of this metal was 8.67 g/mL, which told him the cube's volume. Believing significant figures were invented just to make life difficult for chemistry students and had no practical use in the real world, he calculated the volume of the cube as 9.573 mL. He thus determined that the edge of the cube had to be 2.097 cm. He took his plans to the machine shop where his friend had the same type of work done the previous year. The shop foreman said, "Yes, we can make this according to your specifications - but it will be expensive."

"That's OK," replied the student. "It's important." He knew his friend has paid \$35, and he had been given \$50 out of the school's research budget to get the job done.

He returned the next day, expecting the job to be done. "Sorry," said the foreman. "We're still working on it. Try next week." Finally the day came, and our friend got his cube. It looked very,

very smooth and shiny and beautiful in its velvet case. Seeing it, our hero had a premonition of disaster and became a bit nervous. But he summoned up enough courage to ask for the bill. "\$500, and cheap at the price. We had a terrific job getting it right -- had to make three before we got one right."

"But--but--my friend paid only \$35 for the same thing!"

"No. He wanted a cube 2.1 cm on an edge, and your specifications called for 2.097. We had yours roughed out to 2.1 that very afternoon, but it was the precision grinding and lapping to get it down to 2.097 which took so long and cost the big money. The first one we made was 2.089 on one edge when we got finished, so we had to scrap it. The second was closer, but still not what you specified. That's why the three tries."

"Oh!"

Rules for Rounding Off

Now that "everyone" has a calculator that will give a result to six or eight (or more) figures, it is important that we know how to round the answer off correctly. The typical rule taught is that you round up with five or more and round down with four or less.

THIS RULE IS WRONG!

However, please do not rush off to your elementary school teacher and read 'em the riot act!

The problem lies in rounding "up" (increasing) the number that is followed by a 5. For example, numbers like 3.65 or 3.75, where you are to round off to the nearest tenth.

OK, let's see if I can explain this. When you round off, you change the value of the number, except if you round off a zero. Following the old rules, you can round a number down in value four times (rounding with one, two, three, four) compared to rounding it upwards five times (five, six, seven, eight, nine). Remember that "rounding off" a zero does not change the value of the number being rounded off.

Suppose you had a very large sample of numbers to round off. On average you would be changing values in the sample downwards 4/9ths of the time, compared to changing values in the sample upward 5/9ths of the time.

This means the average of the values AFTER rounding off would be greater than the average of the values BEFORE rounding.

This is not acceptable.

We can correct for this problem by rounding "off" (keeping the number the same) in fifty percent of the roundings-even numbers followed by a 5. Then, on average, the roundings "off" will cancel out the roundings "up."

The following rules dictate the manner in which numbers are to be rounded to the number of figures indicated. The first two rules are more-or-less the old ones. Rule three is the change in the old way.

When rounding, examine the figure following (i.e., to the right of) the figure that is to be last. This figure you are examining is the first figure to be dropped.

1. If it is less than 5, drop it and all the figures to the right of it.
2. If it is more than 5, increase by 1 the number to be rounded, that is, the preceeding figure.
3. If it is 5, round the number so that it will be even. Keep in mind that zero is considered to be even when rounding off.

Example #1 - Suppose you wish to round 62.5347 to four significant figures. Look at the fifth figure. It is a 4, a number less than 5. Therefore, you will simply drop every figure after the fourth, and the original number rounds off to 62.53.

Example #2 - Round 3.78721 to three significant figures. Look at the fourth figure. It is 7, a number greater than 5, so you round the original number up to 3.79.

Example #3 - Round 726.835 to five significant figures. Look at the sixth figure. It is a 5, so now you must look at the fifth figure also. That is a 3, which is an odd number, so you round the original number up to 726.84.

Example #4 - Round 24.8514 to three significant figures. Look at the fourth figure. It is a 5, so now you must also look at the third figure. It is 8, an even number, so you simply drop the 5 and the figures that follow it. The original number becomes 24.8.

When the value you intend to round off is a five, you MUST look at the previous value ALSO. If it is even, you round down. If it is odd, you round up. A common question is "Is zero considered odd or even?" The answer is even.

Here are some more examples of the "five rule." Round off at the five.

3.075

3.85

22.73541

0.00565

2.0495

This last one is tricky (at least for high schoolers being exposed to this stuff for the first time!). The nine rounds off to a ten (not a zero), so the correct answer is 2.050, NOT 2.05.

Would your teacher be so mean as to include problems like this one on a test? In the ChemTeam classroom, the sufferers (oops, I mean students) have learned to shout "YES" in unison to such easy questions.

Lastly, before we get to the problems. Students, when they learn this rule, like to apply it across the board. For example, in 2.0495, let's say we want to round off to the nearest 0.01. Many times, a student will answer 2.04. When asked to explain, the rule concerning five will be cited. However, the important number in this problem is the nine, so the rule is to round up and the correct answer is 2.05.

Math With Significant Figures

Addition and Subtraction

In mathematical operations involving significant figures, the answer is reported in such a way that it reflects the reliability of the least precise operation. Let's state that another way: a chain is no stronger than its weakest link. An answer is no more precise than the least precise number used to get the answer. Let's do it one more time: imagine a team race where you and your team must finish together. Who dictates the speed of the team? Of course, the slowest member of the team. Your answer cannot be MORE precise than the least precise measurement.

For addition and subtraction, look at the decimal portion (i.e., to the right of the decimal point) of the numbers ONLY. Here is what to do:

- 1) Count the number of significant figures in the decimal portion of each number in the problem. (The digits to the left of the decimal place are not used to determine the number of decimal places in the final answer.)
- 2) Add or subtract in the normal fashion.
- 3) Round the answer to the LEAST number of places in the decimal portion of any number in the problem.

WARNING: the rules for add/subtract are different from multiply/divide. A very common student error is to swap the two sets of rules. Another common error is to use just one rule for both types of operations.

Multiplication and Division

The following rule applies for multiplication and division:

The LEAST number of significant figures in any number of the problem determines the number of significant figures in the answer.

This means you MUST know how to recognize significant figures in order to use this rule.

Example #1: 2.5×3.42 .

The answer to this problem would be 8.6 (which was rounded from the calculator reading of 8.55). Why?

2.5 has two significant figures while 3.42 has three. Two significant figures is less precise than three, so the answer has two significant figures.

Example #2: How many significant figures will the answer to 3.10×4.520 have?

You may have said two. This is too few. A common error is for the student to look at a number like 3.10 and think it has two significant figures. The zero in the hundredth's place is not recognized as significant when, in fact, it is. 3.10 has three significant figures.

Three is the correct answer. 14.0 has three significant figures. Note that the zero in the tenth's place is considered significant. All trailing zeros in the decimal portion are considered significant.

Another common error is for the student to think that 14 and 14.0 are the same thing. THEY ARE NOT. 14.0 is ten times more precise than 14. The two numbers have the same value, but they convey different meanings about how trustworthy they are.

Four is also an incorrect answer given by some ChemTeam students. It is too many significant figures. One possible reason for this answer lies in the number 4.520. This number has four significant figures while 3.10 has three. Somehow, the student (YOU!) maybe got the idea that it is the GREATEST number of significant figures in the problem that dictates the answer. It is the LEAST.

Sometimes student will answer this with five. Most likely you responded with this answer because it says 14.012 on your calculator. This answer would have been correct in your math class because mathematics does not have the significant figure concept.

Example #3: $2.33 \times 6.085 \times 2.1$. How many significant figures in the answer?

Answer - two.

Which number decides this?

Answer - the 2.1.

Why?

It has the least number of significant figures in the problem. It is, therefore, the least precise measurement.

Example #4: $(4.52 \times 10^{-4}) \div (3.980 \times 10^{-6})$.

How many significant figures in the answer?

Answer - three.

Which number decides this?

Answer - the 4.52×10^{-4} .

Why?

It has the least number of significant figures in the problem. It is, therefore, the least precise measurement. Notice it is the 4.52 portion that plays the role of determining significant figures; the exponential portion plays no role.

For practice with significant figures, go to:

<http://science.widener.edu/svb/tutorial/sigfigures.html>

From <http://chemed.chem.purdue.edu/genchem/topicreview/bp/ch1/sigfigs.html>

It is important to be honest when reporting a measurement, so that it does not appear to be more accurate than the equipment used to make the measurement allows. We can achieve this by controlling the number of digits, or **significant figures**, used to report the measurement.

Determining the Number of Significant Figures

The number of significant figures in a measurement, such as 2.531, is equal to the number of digits that are known with some degree of confidence (2, 5, and 3) plus the last digit (1), which is an estimate or approximation. As we improve the sensitivity of the equipment used to make a measurement, the number of significant figures increases.

Postage Scale	3 ± 1 g	1 significant figure
Two-pan balance	2.53 ± 0.01 g	3 significant figures
Analytical balance	2.531 ± 0.001 g	4 significant figures

Rules for counting significant figures are summarized below.

Zeros *within* a number are always significant. Both 4308 and 40.05 contain four significant figures.

Zeros that do nothing but set the decimal point are not significant. Thus, 470,000 has two significant figures.

Trailing zeros that aren't needed to hold the decimal point are significant. For example, 4.00 has three significant figures.

If you are not sure whether a digit is significant, assume that it isn't. For example, if the directions for an experiment read: "Add the sample to 400 mL of water," assume the volume of water is known to one significant figure.

Addition and Subtraction with Significant Figures

When combining measurements with different degrees of accuracy and precision, *the accuracy of the final answer can be no greater than the least accurate measurement*. This principle can be translated into a simple rule for addition and subtraction: **When measurements are added or subtracted, the answer can contain no more decimal places than the least accurate measurement.**

150.0 g H₂O (using significant figures)

+ 0.507 g salt

150.5 g solution

Multiplication and Division With Significant Figures

The same principle governs the use of significant figures in multiplication and division: the final result can be no more accurate than the least accurate measurement. In this case, however, we count the significant figures in each measurement, not the number of decimal places: **When measurements are multiplied or divided, the answer can contain no more significant figures than the least accurate measurement.**

Example: To illustrate this rule, let's calculate the cost of the copper in an old penny that is pure copper. Let's assume that the penny has a mass of 2.531 grams, that it is essentially pure copper, and that the price of copper is 67 cents per pound. We can start by from grams to pounds.

$$2.531 \text{ g} \times \frac{1 \text{ lb}}{453.6 \text{ g}} = 0.005580 \text{ lb}$$

We then use the price of a pound of copper to calculate the cost of the copper metal.

$$0.005580 \text{ lb} \times \frac{67¢}{1 \text{ lb}} = 0.3749 \text{ ¢}$$

There are four significant figures in both the mass of the penny (2.531) and the number of grams in a pound (453.6). But there are only two significant figures in the price of copper, so the final answer can only have two significant figures.

Rounding Off

When the answer to a calculation contains too many significant figures, it must be rounded off.

There are 10 digits that can occur in the last decimal place in a calculation. One way of rounding off involves *underestimating* the answer for five of these digits (0, 1, 2, 3, and 4) and *overestimating* the answer for the other five (5, 6, 7, 8, and 9). This approach to rounding off is summarized as follows.

If the digit is smaller than 5, drop this digit and leave the remaining number unchanged. Thus, 1.684 becomes 1.68.

If the digit is 5 or larger, drop this digit and add 1 to the preceding digit. Thus, 1.247 becomes 1.25.