PHYS011 - Kariapper
Chapter 3 Lecture Notes

<u>Main Ideas:</u>
1. Vectors
2. Projectile Motion

1. Vectors

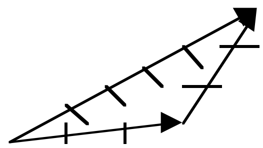
1.1 Vectors and Scalars
In our study of physics we come across two kinds of information. The first kind is something that can be described by a single number like the area of a circle or speed or distance. This is called a scalar. We will also come across quantities that have two pieces of information, a magnitude and a direction. Velocity is such a quantity. It has a magnitude, say 50 miles/hour, and a direction, say Northwest. We write scalars simply as numbers because that is what they are. We write vectors in print as bold face (**A**) or we write them in handwriting with an arrow over them ($\vec{A}$). Every vector quantity must have a magnitude and a direction. If you are asked to give an answer for a vector quantity on a problem, **both the magnitude and the direction must be given** or the problem is only half correct.

Vectors will be used in this class throughout the whole year, so it is very important that you learn this material and become comfortable with it. We draw vector quantities with an arrow. The magnitude of the vector is proportional to its length. The direction of the arrow points in direction of vector. Every time we encounter a vector, we must use its direction and it magnitude. Vectors do not have a particular location. I can move a vector which points up and is 2 units long anywhere I want as long as it is the same length and points in the same direction. The magnitude of a vector is a scalar and is written with a regular font. The magnitude of **v** is *v*.

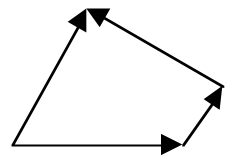
1.2 Multiplication by Constant
If I multiply a vector by a constant, that changes the magnitude of the vector. (For example, draw 2**v** and -**v**).

1.3 Addition and Subtraction
How do we add vectors? There is a way to draw vectors which adds them. We put the tail of one vector on the tip of the other and draw the resultant. If in the same direction, simply a linear addition. If I walk north for a distance, then north again, I can just add the two distances to see how far I am from where I

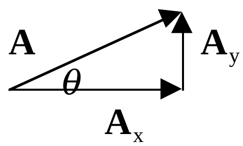
started. But if I walk north for a while then east, I can't just add the distances to see how far I am from where I started. But, if you draw the lines very accurately, then you can actually measure the result.



To add three:



Note that you must always put the tail of the one vector on the tip of the other. However, this is not always accurate enough so there is another way of adding vectors, using components. The components are the part of the vector along different axes, usually chosen to be the x and the y axis. Every vector can be written as the sum of its components: $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$. This is vector addition:

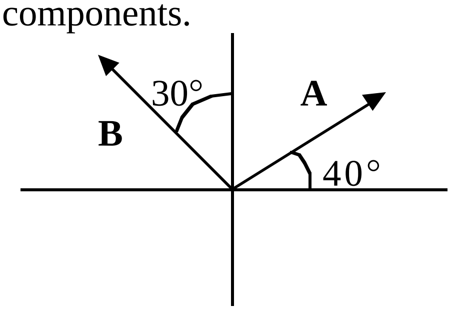


How long are the parts of the vector along the x axis and along the y axis, A_x and A_y? To answer that we use trigonometric relations. Recall that

$$\sin\theta = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{o}{h}\ ,\ \cos\theta = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{h}\ ,\ \ \tan\theta = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{o}{a}.$$

So look at the triangle. We see that $\cos\theta = A_x/A$ where A_x and A are the magnitude of the vectors $\mathbf{A}_x$ and $\mathbf{A}$. Then we get the length, $A_x = A\cos\theta$. In a similar manner $\sin\theta = A_y/A \Rightarrow A_y = A\sin\theta$. So there are two ways to specify a vector. We can give its magnitude A and its direction as an angle, or compass direction, or something, or we can give its x and y components. Suppose we have A_x and A_y, then what is the magnitude of the vector. From the pythagorean thereom, $A^2 = A_x^2 + A_y^2$.

<u>Problem:</u> Find the components of the following vectors **A** and **B** and draw the components.



Magnitude of A is 5.00 and magnitude of B is 7.00.

How do we add the vectors using components? It is very easy once you have found the components. You can see this by first drawing the addition of **A** and **B** graphically as already discussed. Then it is easy to see that to find the vector **C**, we just use $C_x = A_x + B_x$ and $C_y = A_y + B_y$.

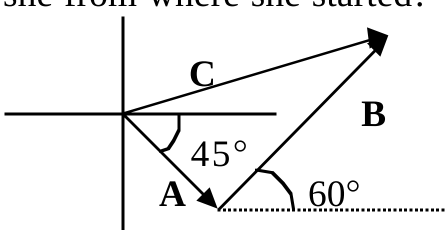
In order to solve problems with vectors, you must always follow these steps. If you don't you will almost always get the wrong answer.

<u>Solving Problems with Vectors</u>

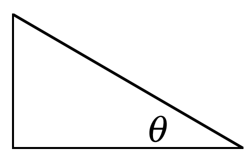
1. Determine which direction is the positive *x* direction, and which is the positive *y* direction. Draw and label the axes.
2. Draw and label all the vectors.
3. Determine the individual components of each vector.
   - The components of the vector must be along the *x* and *y* axis and add up as vectors to equal the original vector. (The length of each component may not be greater than the original vector).
   - Draw and label the components.
   - Use trigonometric functions to determine the length of each component.
   - The sign of the component is given by the direction of the component's arrow.
4. Do calculations in the *x* direction and in the *y* direction separately using only the *x* and *y* components of the vectors, respectively.
5. Combine the results from the *x* and *y* directions into one vector.

<u>Problem:</u> What is **A** + **B**?

<u>Problem:</u> A hiker walks 25.0 km due southeast, then 40.0 km in a direction 60° north of east. What are the component of her displacement vectors and how far is she from where she started?



<u>Problem:</u> A skier skis down a hill which is 770 m long. The vertical drop is 230 m. What is the angle of the hill relative to flat ground?



(You should know how to use $\sin^{-1}$ on calculator)

2. Projectile Motion

We have previously discussed motion in a constant gravitational field where the acceleration is constant. If I define the positive y direction as being up and g = 9.8 m/s^2, then we know that (writing the equations in the vertical y direction):

$$v_y = v_{y0} + gt$$
$$y = y_0 + v_{y0}t + (1/2)gt^2$$
$$y = y_0 + (1/2)(v_{y0} + v_y)t$$
$$v^2 = v_{y0}^2 + 2g(y - y_0)$$
$$\bar{v}_y = (v_{y_0} + v_y)/2$$

What about the motion in the x direction. Is there any acceleration? If we throw a ball across the room we might get some idea of what is going on. Is the particle changing velocity in the x direction. Amazingly, it is not. Galileo first showed this. The velocity and acceleration in the horizontal direction is independent of what is going on in the vertical direction. This is always true. The motion in each direction is independent of the motion in the other direction.

So we have general equations to use when the acceleration is constant, but not zero, and equations when there is no acceleration in the x direction which is the usual case for objects moving in a gravitational field.

General Equations	Equations for x acceleration = 0
$v_x = v_{x0} + a_x t$	$v_x = v_{x0}$
$x = x_0 + v_{x0}t + (1/2) a_x t^2$	$x = x_0 + v_{x0}t$
$x = x_0 + (1/2)(v_{x0} + v_x)t$	
$v^2 = v_{x0}^2 + 2 a_x (x - x_0)$	
$\bar{v}_x = (v_{x0} + v_x)/2$	

If the positive x direction is up, away from the center of the earth, then for objects in free fall, we find $a_x = -g$. See Problem Solving boxes on pages 56 and 61 in your textbook. Let's use these equations to solve some problems:

<u>Problem:</u> A plane must drop a package from 100 m above the ground. The plane is travelling at 40.0 m/s.

(a) Where does the package strike the ground?

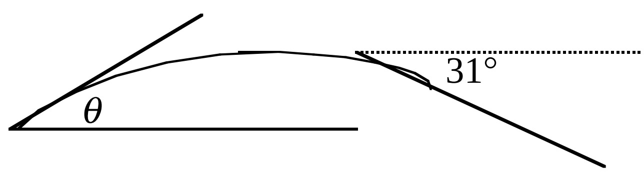
(b) What are the horizontal and vertical components of the velocity just before the package strikes the ground?

Problem: A golfer hits a ball with an initial speed of 40.3 m/s at an angle of 32.0° from the horizontal. (a) How far does the ball go and how long is it in the air? (b) What is its speed when it hits the ground? (Remember to draw a diagram)

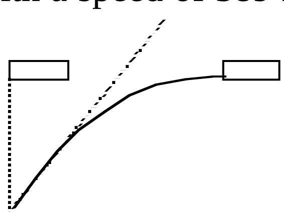
The final velocity is the same as the initial. This is true when the final position is at the same level as the initial position. (And of course there is no horizontal acceleration, and no air resistance).

Problem: (a) What angle should we launch something in order to make it go its maximum distance *on level ground*? (No air resistance). (b) What is the maximum distance the golf ball can go?

Problem: A baseball player hits a home run and the ball lands in the left field seats, 7.6 m above the point at which the ball was hit. The ball lands with a velocity of 49 m/s at an angle of 31° to the horizontal. What is the initial velocity of the ball when the ball leaves the bat?



Problem: A plane is flying horizontally at an altitude of 4.20 km with a speed of 225 m/s. When the plane is directly overhead, a projectile is fired at an angle θ with a speed of 389 m/s. The projectile hits the plane. What is the angle θ ?



Sometimes problems can be a little harder if the initial velocity is not given. Let's try a problem where no initial velocity is given.

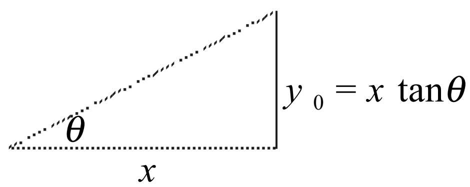
Problem: A golfer hits a ball 180 m on level ground at an angle of 60° above the horizontal. What was the initial speed of the ball?

The speed of the ball is the magnitude of the velocity.

We know $v_x = v_{x0} = v_0 \cos 60°$ and $x - x_0 = v_x t$

We also have $v_{y0} = v_0 \sin 60°$, $y - y_0 = 0$ and $a = -9.80 \text{ m/s}^2$. So we can write two equations with two unknowns (The two unknowns are t and v_0).

Demonstration/Problem: Monkey Gun



A hunter aims his gun at a monkey hanging on a branch. Just as the hunter shoots, the monkey drops from the branch to avoid the bullet. What happens?