The purpose of a model is to give us an approximate mental or visual picture—something to hold onto—when we cannot see or understand what actually is happening. Models often give us a deeper understanding: the analogy to a known system (for instance, water waves in the above example) can suggest new experiments to perform and can provide ideas about what other related phenomena might occur.

You may wonder what the difference is between a theory and a model. Usually, a model is relatively simple and provides a structural similarity to the phenomena being studied. A **theory** is broader, more detailed, and can give quantitatively testable predictions, often with great precision. It is important, however, not to confuse a model or a theory with the real system or the phenomena themselves.

Theories (vs. models)

Scientists give the title **law** to certain concise but general statements about how nature behaves (that energy is conserved, for example). Sometimes the statement takes the form of a relationship or equation between quantities (such as Newton's second law, $F = ma$).

Laws and principles

To be called a law, a statement must be found experimentally valid over a wide range of observed phenomena. For less general statements, the term **principle** is often used (such as Archimedes' principle).

Scientific laws are different from political laws in that the latter are *prescriptive*: they tell us how we ought to behave. Scientific laws are *descriptive*: they do not say how nature *should* behave, but rather are meant to describe how nature *does* behave. As with theories, laws cannot be tested in the infinite variety of cases possible. So we cannot be sure that any law is absolutely true. We use the term "law" when its validity has been tested over a wide range of cases, and when any limitations and the range of validity are clearly understood.

Scientists normally do their work as if the accepted laws and theories were true. But they are obliged to keep an open mind in case new information should alter the validity of any given law or theory.

1–4 Measurement and Uncertainty; Significant Figures

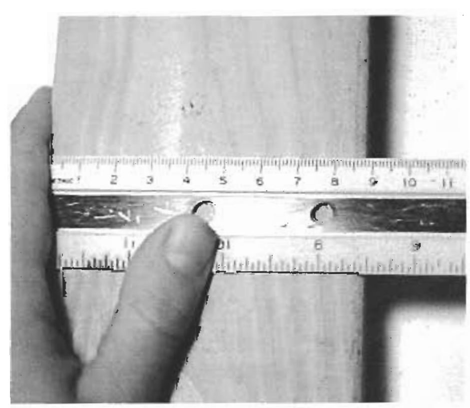
In the quest to understand the world around us, scientists try to work out relationships among physical quantities that can be measured.

Uncertainty

Every measurement has an uncertainty

Accurate, precise measurements are an important part of physics. But no measurement is absolutely precise. There is an uncertainty associated with every measurement. Among the most important sources of uncertainty, other than blunders, are the limited accuracy of every measuring instrument and the inability to read an instrument beyond some fraction of the smallest division shown. For example, if you were to use a centimeter ruler to measure the width of a board (Fig. 1–5), the result could be claimed to be precise to about 0.1 cm (1 mm), the smallest division on the ruler, although half of this value might be a valid claim as well. The reason for this is that it is difficult for the observer to estimate between the smallest divisions. Furthermore, the ruler itself may not have been manufactured to an accuracy any better than this.†

FIGURE 1–5 Measuring the width of a board with a centimeter ruler. Accuracy is about ± 1 mm.



†There is a technical difference between "precision" and "accuracy." **Precision** in a strict sense refers to the repeatability of the measurement using a given instrument. For example, if you measure the width of a board many times, getting results like 8.81 cm, 8.85 cm, 8.78 cm, 8.82 cm (estimating between the 0.1-cm marks as best as possible each time), you could say the measurements give a *precision* a bit better than 0.1 cm. **Accuracy** refers to how close a measurement is to the true value. For example, if the ruler shown in Fig. 1–5 was manufactured with a 2% error, the accuracy of its measurement of the board's width (about 8.8 cm) would be about 2% of 8.8 cm, or about ± 0.2 cm. Estimated uncertainty is meant to take both accuracy and precision into account.

Stating the uncertainty

When giving the result of a measurement, it is important to state the **estimated uncertainty** in the measurement. For example, the width of a board might be written as 8.8 ± 0.1 cm. The ± 0.1 cm ("plus or minus 0.1 cm") represents the estimated uncertainty in the measurement, so that the actual width most likely lies between 8.7 and 8.9 cm. The **percent uncertainty** is simply the ratio of the uncertainty to the measured value, multiplied by 100. For example, if the measurement is 8.8 and the uncertainty about 0.1 cm, the percent uncertainty is

$$\frac{0.1}{8.8} \times 100\% \approx 1\%$$

where $\approx$ means "is roughly equal to."

Assumed uncertainty

Often the uncertainty in a measured value is not specified explicitly. In such cases, the uncertainty is generally assumed to be one or a few units in the last digit specified. For example, if a length is given as 8.8 cm, the uncertainty is assumed to be about 0.1 cm or 0.2 cm. It is important in this case that you do not write 8.80 cm, for this implies an uncertainty on the order of 0.01 cm; it assumes that the length is probably between 8.79 cm and 8.81 cm, when actually you believe it is between 8.7 and 8.9 cm.

CONCEPTUAL EXAMPLE 1–1 **Is the diamond yours?** A friend asks to borrow your precious diamond for a day to show her family. You are a bit worried, so you carefully have your diamond weighed on a scale which reads 8.17 grams. The scale's accuracy is claimed to be ± 0.05 gram. The next day you weigh the returned diamond again, getting 8.09 grams. Is this your diamond?

RESPONSE The scale readings are measurements and do not necessarily give the "true" value of the mass. Each measurement could have been high or low by up to 0.05 gram or so. The actual mass of your diamond lies most likely between 8.12 grams and 8.22 grams. The actual mass of the returned diamond is most likely between 8.04 grams and 8.14 grams. These two ranges overlap, so there is not a strong reason to doubt that the returned diamond is yours, at least based on the scale readings.

Significant Figures

Which digits are significant?

The number of reliably known digits in a number is called the number of **significant figures**. Thus there are four significant figures in the number 23.21 cm and two in the number 0.062 cm (the zeros in the latter are merely place holders that show where the decimal point goes). The number of significant figures may not always be clear. Take, for example, the number 80. Are there one or two significant figures? If we say it is *about* 80 km between two cities, there is only one significant figure (the 8) since the zero is merely a place holder. If it is *exactly* 80 km within an accuracy of 1 or 2 km, then the 80 has two significant figures.[†] If it is precisely 80 km, to within ± 0.1 km, then we write 80.0 km.

When making measurements, or when doing calculations, you should avoid the temptation to keep more digits in the final answer than is justified. For example, to calculate the area of a rectangle 11.3 cm by 6.8 cm, the result of multiplication would be 76.84 cm^2. But this answer is clearly not accurate to 0.01 cm^2, since (using the outer limits of the assumed uncertainty for each measurement) the result could be between $11.2\text{ cm} \times 6.7\text{ cm} = 75.04\text{ cm}^2$ and $11.4\text{ cm} \times 6.9\text{ cm} = 78.66\text{ cm}^2$. At best, we can quote the answer as 77 cm^2, which implies an uncertainty of about 1 or 2 cm^2. The other two digits (in the number 76.84 cm^2) must be dropped since they are not significant. As a rough general rule (i.e., in the absence of a detailed consideration of uncertainties), we can say that *the final result of a multiplication or division should have only as many digits as the number with the least number of significant figures used in the calculation*. In our example, 6.8 cm has the least number of significant figures, namely two. Thus the result 76.84 cm^2 needs to be rounded off to 77 cm^2.

➡ PROBLEM SOLVING

Number of significant figures in final result should be same as least significant input value

[†] If the 80 has two significant figures, some people prefer to write it 80., with a decimal point. This is not usually done, so the number of significant figures in 80 can be ambiguous unless something is said about it such as "about" (meaning 80 ± 10), or "very nearly" or "precisely" (meaning 80 ± 1).

EXERCISE A The area of a rectangle 4.5 cm by 3.25 cm is correctly given by (*a*) 14.625 cm^2; (*b*) 14.63 cm^2; (*c*) 14.6 cm^2; (*d*) 15 cm^2.

When adding or subtracting numbers, the final result is no more accurate than the least accurate number used. For example, the result of subtracting 0.57 from 3.6 is 3.0 (and not 3.03).

Keep in mind when you use a calculator that all the digits it produces may not be significant. When you divide 2.0 by 3.0, the proper answer is 0.67, and not some such thing as 0.666666666. Digits should not be quoted in a result, unless they are truly significant figures. However, to obtain the most accurate result, you should normally *keep one or more extra significant figures throughout a calculation, and round off only in the final result.* (With a calculator, you can keep all its digits in intermediate results.) Note also that calculators sometimes give too few significant figures. For example, when you multiply 2.5 × 3.2, a calculator may give the answer as simply 8. But the answer is good to two significant figures, so the proper answer is 8.0. See Fig. 1–6.

CAUTION

Calculators err with significant figures

PROBLEM SOLVING

Report only the proper number of significant figures in the final result. Keep extra digits during the calculation.

(a) (b)

FIGURE 1–6 These two calculators show the wrong number of significant figures. In (a), 2.0 was divided by 3.0. The correct final result would be 0.67. In (b), 2.5 was multiplied by 3.2. The correct result is 8.0.

EXERCISE B Do 0.00324 and 0.00056 have the same number of significant figures?

Be careful not to confuse significant figures with the number of decimal places.

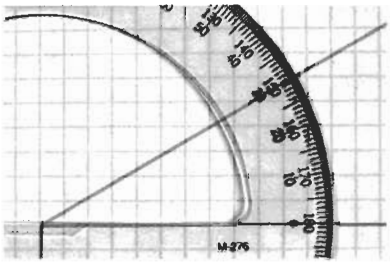
EXERCISE C For each of the following numbers, state the number of significant figures and the number of decimal places: (*a*) 1.23; (*b*) 0.123; (*c*) 0.0123.

CONCEPTUAL EXAMPLE 1–2 **Significant figures.** Using a protractor (Fig. 1–7), you measure an angle to be 30°. (*a*) How many significant figures should you quote in this measurement? (*b*) Use a calculator to find the cosine of the angle you measured.

RESPONSE (*a*) If you look at a protractor, you will see that the precision with which you can measure an angle is about one degree (certainly not 0.1°). So you can quote two significant figures, namely, 30° (not 30.0°). (*b*) If you enter cos 30° in your calculator, you will get a number like 0.866025403. However, the angle you entered is known only to two significant figures, so its cosine is correctly given by 0.87; i.e., you must round your answer to two significant figures.

NOTE We discuss trigonometric functions like cosine in Chapter 3.

FIGURE 1–7 Example 1–2. A protractor used to measure an angle.



Scientific Notation

We commonly write numbers in "powers of ten," or "scientific" notation—for instance 36,900 as 3.69×10^4, or 0.0021 as 2.1×10^{-3}. One advantage of scientific notation (discussed in Appendix A) is that it allows the number of significant figures to be clearly expressed. For example, it is not clear whether 36,900 has three, four, or five significant figures. With powers of ten notation the ambiguity can be avoided: if the number is known to an accuracy of three significant figures, we write 3.69×10^4, but if it is known to four, we write 3.690×10^4.

* Percent Error

The significant figures rule is only approximate, and in some cases may underestimate the precision of the answer. Suppose for example we divide 97 by 92:

$$\frac{97}{92} = 1.05 \approx 1.1.$$

Both 97 and 92 have two significant figures, so the rule says to give the answer as 1.1. Yet the numbers 97 and 92 both imply an uncertainty of ± 1 if no other uncertainty is stated. Now 92 ± 1 and 97 ± 1 both imply an accuracy of about 1% ($1/92 \approx 0.01 = 1\%$). But the final result to two significant figures is 1.1, with an implied uncertainty of ± 0.1, which is an uncertainty of $(0.1/1.1) \times 100\% \approx 10\%$. In this case it is better to give the answer as 1.05 (which is three significant figures). Why? Because 1.05 implies an uncertainty of ± 0.01 which is $(0.01/1.05) \times 100\% \approx 1\%$, just like the uncertainty in the original numbers 92 and 97.

SUGGESTION: Use the significant figures rule, but consider the % uncertainty too, and add an extra digit if it gives a more realistic estimate of uncertainty.

1-5 Units, Standards, and the SI System

The measurement of any quantity is made relative to a particular standard or **unit**, and this unit must be specified along with the numerical value of the quantity. For example, we can measure length in units such as inches, feet, or miles, or in the metric system in centimeters, meters, or kilometers. To specify that the length of a particular object is 18.6 is meaningless. The unit *must* be given; for clearly, 18.6 meters is very different from 18.6 inches or 18.6 millimeters.

For any unit we use, such as the meter for distance or the second for time, we need to define a **standard** which defines exactly how long one meter or one second is. It is important that standards be chosen that are readily reproducible so that anyone needing to make a very accurate measurement can refer to the standard in the laboratory.

Length

Standard of length (meter)

The first truly international standard was the **meter** (abbreviated m) established as the standard of **length** by the French Academy of Sciences in the 1790s. The standard meter was originally chosen to be one ten-millionth of the distance from the Earth's equator to either pole,† and a platinum rod to represent this length was made. (One meter is, very roughly, the distance from the tip of your nose to the tip of your finger, with arm and hand stretched out to the side.) In 1889, the meter was defined more precisely as the distance between two finely engraved marks on a particular bar of platinum–iridium alloy. In 1960, to provide greater precision and reproducibility, the meter was redefined as 1,650,763.73 wavelengths of a particular orange light emitted by the gas krypton-86. In 1983 the meter was again redefined, this time in terms of the speed of light (whose best measured value in terms of the older definition of the meter was 299,792,458 m/s, with an uncertainty of 1 m/s). The new definition reads: "The meter is the length of path traveled by light in vacuum during a time interval of 1/299,792,458 of a second." ‡

British units of length (inch, foot, mile) are now defined in terms of the meter. The inch (in.) is defined as precisely 2.54 centimeters (cm; 1 cm = 0.01 m). Other conversion factors are given in the Table on the inside of the front cover of this book. Table 1–1 presents some typical lengths, from very small to very large, rounded off to the nearest power of ten. See also Fig. 1–8. [Note that the abbreviation for inches (in.) is the only one with a period, to distinguish it from the word "in".]

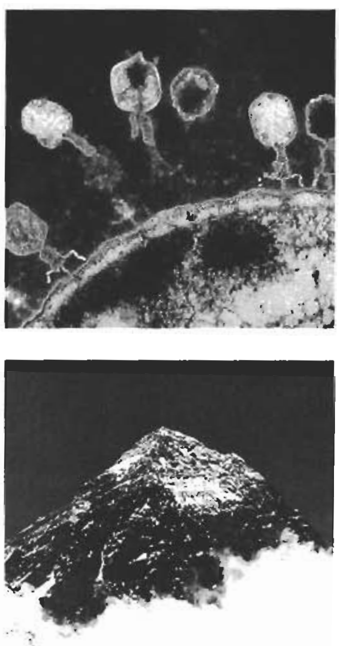


FIGURE 1–8 Some lengths: (a) viruses (about 10^{-7} m long) attacking a cell; (b) Mt. Everest's height is on the order of 10^4 m (8850 m, to be precise).

(a)

(b)

TABLE 1–1 Some Typical Lengths or Distances (order of magnitude)

Length (or Distance)	Meters (approximate)
Neutron or proton (radius)	10^{-15} m
Atom	10^{-10} m
Virus [see Fig. 1–8a]	10^{-7} m
Sheet of paper (thickness)	10^{-4} m
Finger width	10^{-2} m
Football field length	10^{2} m
Height of Mt. Everest [see Fig. 1–8b]	10^{4} m
Earth diameter	10^{7} m
Earth to Sun	10^{11} m
Earth to nearest star	10^{16} m
Earth to nearest galaxy	10^{22} m
Earth to farthest galaxy visible	10^{26} m

† Modern measurements of the Earth's circumference reveal that the intended length is off by about one-fiftieth of 1%. Not bad!

‡ The new definition of the meter has the effect of giving the speed of light the exact value of 299,792,458 m/s.

TABLE 1–2 Some Typical Time Intervals

Time Interval	Seconds (approximate)
Lifetime of very unstable subatomic particle	10^{-23} s
Lifetime of radioactive elements	10^{-22} s to 10^{28} s
Lifetime of muon	10^{-6} s
Time between human heartbeats	10^{0} s (= 1 s)
One day	10^{5} s
One year	3×10^{7} s
Human life span	2×10^{9} s
Length of recorded history	10^{11} s
Humans on Earth	10^{14} s
Life on Earth	10^{17} s
Age of Universe	10^{18} s

TABLE 1–3 Some Masses

Object	Kilograms (approximate)
Electron	10^{-30} kg
Proton, neutron	10^{-27} kg
DNA molecule	10^{-17} kg
Bacterium	10^{-15} kg
Mosquito	10^{-5} kg
Plum	10^{-1} kg
Human	10^{2} kg
Ship	10^{8} kg
Earth	6×10^{24} kg
Sun	2×10^{30} kg
Galaxy	10^{41} kg

Time

The standard unit of **time** is the **second** (s). For many years, the second was defined as 1/86,400 of a mean solar day. The standard second is now defined more precisely in terms of the frequency of radiation emitted by cesium atoms when they pass between two particular states. [Specifically, one second is defined as the time required for 9,192,631,770 periods of this radiation.] There are, by definition, 60 s in one minute (min) and 60 minutes in one hour (h). Table 1–2 presents a range of measured time intervals, rounded off to the nearest power of ten.

Mass

The standard unit of **mass** is the **kilogram** (kg). The standard mass is a particular platinum–iridium cylinder, kept at the International Bureau of Weights and Measures near Paris, France, whose mass is defined as exactly 1 kg. A range of masses is presented in Table 1–3. [For practical purposes, 1 kg weighs about 2.2 pounds on Earth.]

When dealing with atoms and molecules, we usually use the **unified atomic mass unit** (u). In terms of the kilogram,

$$1\ \text{u} = 1.6605 \times 10^{-27}\ \text{kg}.$$

The definitions of other standard units for other quantities will be given as we encounter them in later Chapters.

Unit Prefixes

In the metric system, the larger and smaller units are defined in multiples of 10 from the standard unit, and this makes calculation particularly easy. Thus 1 kilometer (km) is 1000 m, 1 centimeter is $\frac{1}{100}$ m, 1 millimeter (mm) is $\frac{1}{1000}$ m or $\frac{1}{10}$ cm, and so on. The prefixes "centi-," "kilo-," and others are listed in Table 1–4 and can be applied not only to units of length, but to units of volume, mass, or any other metric unit. For example, a centiliter (cL) is $\frac{1}{100}$ liter (L), and a kilogram (kg) is 1000 grams (g).

TABLE 1–4 Metric (SI) Prefixes

Prefix	Abbreviation	Value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro†	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

† μ is the Greek letter "mu."

Systems of Units

PROBLEM SOLVING

Always use a consistent set of units

When dealing with the laws and equations of physics it is very important to use a consistent set of units. Several systems of units have been in use over the years. Today the most important is the **Système International** (French for International System), which is abbreviated SI. In SI units, the standard of length is the meter, the standard for time is the second, and the standard for mass is the kilogram. This system used to be called the MKS (meter-kilogram-second) system.

SI units

A second metric system is the **cgs system**, in which the centimeter, gram, and second are the standard units of length, mass, and time, as abbreviated in the title. The **British engineering system** takes as its standards the foot for length, the pound for force, and the second for time.

TABLE 1–5 SI Base Quantities and Units

Quantity	Unit	Unit Abbreviation
Length	meter	m
Time	second	s
Mass	kilogram	kg
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

SI units are the principal ones used today in scientific work. We will therefore use SI units almost exclusively in this book, although we will give the cgs and British units for various quantities when introduced.

Base vs. Derived Quantities

Physical quantities can be divided into two categories: *base quantities* and *derived quantities*. The corresponding units for these quantities are called *base units* and *derived units*. A **base quantity** must be defined in terms of a standard. Scientists, in the interest of simplicity, want the smallest number of base quantities possible consistent with a full description of the physical world. This number turns out to be seven, and those used in the SI are given in Table 1–5. All other quantities can be defined in terms of these seven base quantities,[†] and hence are referred to as **derived quantities**. An example of a derived quantity is speed, which is defined as distance divided by the time it takes to travel that distance. A Table inside the front cover lists many derived quantities and their units in terms of base units. To define any quantity, whether base or derived, we can specify a rule or procedure, and this is called an **operational definition**.

1–6 Converting Units

Any quantity we measure, such as a length, a speed, or an electric current, consists of a number *and* a unit. Often we are given a quantity in one set of units, but we want it expressed in another set of units. For example, suppose we measure that a table is 21.5 inches wide, and we want to express this in centimeters. We must use a **conversion factor**, which in this case is

$$1 \text{ in.} = 2.54 \text{ cm}$$

or, written another way,

$$1 = 2.54 \text{ cm/in.}$$

Since multiplying by one does not change anything, the width of our table, in cm, is

$$21.5 \text{ inches} = (21.5 \text{ in.}) \times \left(2.54 \frac{\text{cm}}{\text{in.}}\right) = 54.6 \text{ cm}.$$

Note how the units (inches in this case) cancelled out. A Table containing many unit conversions is found inside the front cover of this book. Let's take some Examples.

FIGURE 1–9 The world's second highest peak, K2, whose summit is considered the most difficult of the 8,000-ers. K2 is seen here from the north (China). Our cover shows K2 from the south (Pakistan). Example 1–3.

PHYSICS APPLIED
The world's tallest peaks

EXAMPLE 1–3 The 8000-m peaks. The fourteen tallest peaks in the world (Fig. 1–9 and Table 1–6) are referred to as "eight-thousanders," meaning their summits are over 8000 m above sea level. What is the elevation, in feet, of an elevation of 8000 m?

APPROACH We need simply to convert meters to feet, and we can start with the conversion factor $1 \text{ in.} = 2.54 \text{ cm}$, which is exact. That is, $1 \text{ in.} = 2.5400 \text{ cm}$ to any number of significant figures.

SOLUTION One foot is 12 in., so we can write

$$1 \text{ ft} = (12 \text{ in.})\left(2.54 \frac{\text{cm}}{\text{in.}}\right) = 30.48 \text{ cm} = 0.3048 \text{ m}.$$

The units cancel (colored slashes), and our result is exact. We can rewrite this

[†] The only exceptions are for angle (radians—see Chapter 8) and solid angle (steradian). No general agreement has been reached as to whether these are base or derived quantities.

equation to find the number of feet in 1 meter:

$$1\text{ m} = \frac{1\text{ ft}}{0.3048} = 3.28084\text{ ft}.$$

We multiply this equation by 8,000.0 (to have five significant figures):

$$8{,}000.0\text{ m} = (8{,}000.0\text{ m})\left(3.28084\,\frac{\text{ft}}{\text{m}}\right) = 26{,}247\text{ ft}.$$

An elevation of 8000 m is 26,247 ft above sea level.

NOTE We could have done the conversion all in one line:

$$8000\text{ m} = (8000\text{ m})\left(\frac{100\text{ cm}}{1\text{ m}}\right)\left(\frac{1\text{ in.}}{2.54\text{ cm}}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right) = 26{,}247\text{ ft}.$$

The key is to multiply conversion factors, each equal to one (= 1.0000), and to make sure the units cancel.

EXERCISE D There are only 14 eight-thousand-meter peaks in the world (see Example 1–3) and their names and elevations are given in Table 1–6. They are all in the Himalaya mountain range in India, Pakistan, Tibet, and China. Determine the elevation of the world's three highest peaks in feet.

TABLE 1–6 The 8000-m Peaks

Peak	Height (m)
Mt. Everest	8850
K2	8611
Kangchenjunga	8586
Lhotse	8516
Makalu	8462
Cho Oyu	8201
Dhaulagiri	8167
Manaslu	8156
Nanga Parbat	8125
Annapurna	8091
Gasherbrum I	8068
Broad Peak	8047
Gasherbrum II	8035
Shisha Pangma	8013

EXAMPLE 1–4 **Area of a semiconductor chip.** A silicon chip has an area of 1.25 square inches. Express this in square centimeters.

APPROACH We use the same conversion factor, 1 in. = 2.54 cm, but this time we have to use it twice.

SOLUTION Because 1 in. = 2.54 cm, then $1\text{ in.}^2 = (2.54\text{ cm})^2 = 6.45\text{ cm}^2$. So

$$1.25\text{ in.}^2 = (1.25\text{ in.}^2)\left(2.54\,\frac{\text{cm}}{\text{in.}}\right)^2 = (1.25\text{ in.}^2)\left(6.45\,\frac{\text{cm}^2}{\text{in.}^2}\right) = 8.06\text{ cm}^2.$$

EXAMPLE 1–5 **Speeds.** Where the posted speed limit is 55 miles per hour (mi/h or mph), what is this speed (*a*) in meters per second (m/s) and (*b*) in kilometers per hour (km/h)?

APPROACH We again use the conversion factor 1 in. = 2.54 cm, and we recall that there are 5280 ft in a mile and 12 inches in a foot; also, one hour contains $(60\text{ min/h}) \times (60\text{ s/min}) = 3600\text{ s/h}$.

SOLUTION (*a*) We can write 1 mile as

$$1\text{ mi} = (5280\text{ ft})\left(12\,\frac{\text{in.}}{\text{ft}}\right)\left(2.54\,\frac{\text{cm}}{\text{in.}}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right) = 1609\text{ m}.$$

Note that each conversion factor is equal to one. We also know that 1 hour contains 3600 s, so

Conversion factors = 1

$$55\,\frac{\text{mi}}{\text{h}} = \left(55\,\frac{\text{mi}}{\text{h}}\right)\left(1609\,\frac{\text{m}}{\text{mi}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 25\,\frac{\text{m}}{\text{s}},$$

where we rounded off to two significant figures.

(*b*) Now we use 1 mi = 1609 m = 1.609 km; then

$$55\,\frac{\text{mi}}{\text{h}} = \left(55\,\frac{\text{mi}}{\text{h}}\right)\left(1.609\,\frac{\text{km}}{\text{mi}}\right) = 88\,\frac{\text{km}}{\text{h}}.$$

NOTE These unit conversions are very handy. You can always look them up in the Table inside the front cover.

EXERCISE E Would a driver traveling at 15 m/s in a 35 mi/h zone be exceeding the speed limit?

When changing units, you can avoid making an error in the use of conversion factors by checking that units cancel out properly. For example, in our conversion of 1 mi to 1609 m in Example 1–5(*a*), if we had incorrectly used the factor $\left(\frac{100\text{ cm}}{1\text{ m}}\right)$ instead of $\left(\frac{1\text{ m}}{100\text{ cm}}\right)$, the meter units would not have cancelled out; we would not have ended up with meters.

PROBLEM SOLVING

Unit conversion is wrong if units do not cancel

1–7 Order of Magnitude: Rapid Estimating

We are sometimes interested only in an approximate value for a quantity. This might be because an accurate calculation would take more time than it is worth or would require additional data that are not available. In other cases, we may want to make a rough estimate in order to check an accurate calculation made on a calculator, to make sure that no blunders were made when the numbers were entered.

PROBLEM SOLVING
How to make a rough estimate

A rough estimate is made by rounding off all numbers to one significant figure and its power of 10, and after the calculation is made, again only one significant figure is kept. Such an estimate is called an **order-of-magnitude estimate** and can be accurate within a factor of 10, and often better. In fact, the phrase "order of magnitude" is sometimes used to refer simply to the power of 10.

To give you some idea of how useful and powerful rough estimates can be, let us do a few "worked-out Examples."

PHYSICS APPLIED
Estimating the volume (or mass) of a lake; see also Fig. 1–10

EXAMPLE 1–6 ESTIMATE Volume of a lake. Estimate how much water there is in a particular lake, Fig. 1–10a, which is roughly circular, about 1 km across, and you guess it has an average depth of about 10 m.

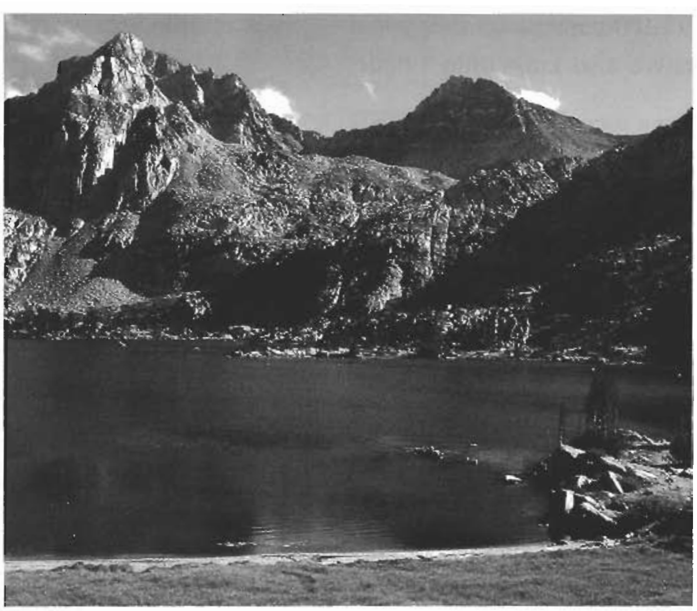
APPROACH No lake is a perfect circle, nor can lakes be expected to have a perfectly flat bottom. We are only estimating here. To estimate the volume, we can use a simple model of the lake as a cylinder: we multiply the average depth of the lake times its roughly circular surface area, as if the lake were a cylinder (Fig. 1–10b).

SOLUTION The volume V of a cylinder is the product of its height h times the area of its base: $V = h\pi r^2$, where r is the radius of the circular base.† The radius r is $\frac{1}{2}$km $= 500$ m, so the volume is approximately

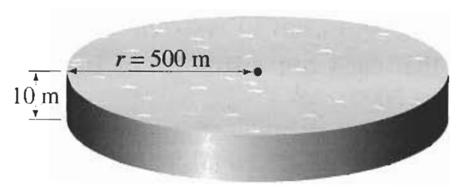
$$V = h\pi r^2 \approx (10\,\text{m}) \times (3) \times \left(5 \times 10^2\,\text{m}\right)^2 \approx 8 \times 10^6\,\text{m}^3 \approx 10^7\,\text{m}^3,$$

where π was rounded off to 3. So the volume is on the order of $10^7\,\text{m}^3$, ten million cubic meters. Because of all the estimates that went into this calculation, the order-of-magnitude estimate $\left(10^7\,\text{m}^3\right)$ is probably better to quote than the $8 \times 10^6\,\text{m}^3$ figure.

† Formulas like this for volume, area, etc., are found inside the back cover of this book.



(a)



(b)

FIGURE 1–10 Example 1–6. (a) How much water is in this lake? (Photo is of one of the Rae Lakes in the Sierra Nevada of California.) (b) Model of the lake as a cylinder. [We could go one step further and estimate the mass or weight of this lake. We will see later that water has a density of $1000\,\text{kg/m}^3$, so this lake has a mass of about $\left(10^3\,\text{kg/m}^3\right)\left(10^7\,\text{m}^3\right) \approx 10^{10}\,\text{kg}$, which is about 10 billion kg or 10 million metric tons. (A metric ton is 1000 kg, about 2200 lbs, slightly larger than a British ton, 2000 lbs.)]

NOTE To express our result in U.S. gallons, we see in the Table on the inside front cover that 1 liter $= 10^{-3}\,\text{m}^3 \approx \frac{1}{4}$ gallon. Hence, the lake contains $(10^7\,\text{m}^3)(1\text{ gallon}/4 \times 10^{-3}\,\text{m}^3) \approx 2 \times 10^9$ gallons of water.

EXAMPLE 1–7 ESTIMATE **Thickness of a page.** Estimate the thickness of a page of this book.

PROBLEM SOLVING
Use symmetry when possible

APPROACH At first you might think that a special measuring device, a micrometer (Fig. 1–11), is needed to measure the thickness of one page since an ordinary ruler clearly won't do. But we can use a trick or, to put it in physics terms, make use of a *symmetry*: we can make the reasonable assumption that all the pages of this book are equal in thickness.

SOLUTION We can use a ruler to measure hundreds of pages at once. If you measure the thickness of the first 500 pages of this book (page 1 to page 500), you might get something like 1.5 cm. Note that 500 pages counted front and back is 250 separate pieces of paper. So one page must have a thickness of about

$$\frac{1.5\text{ cm}}{250\text{ pages}} \approx 6 \times 10^{-3}\,\text{cm} = 6 \times 10^{-2}\,\text{mm},$$

or less than a tenth of a millimeter (0.1 mm).

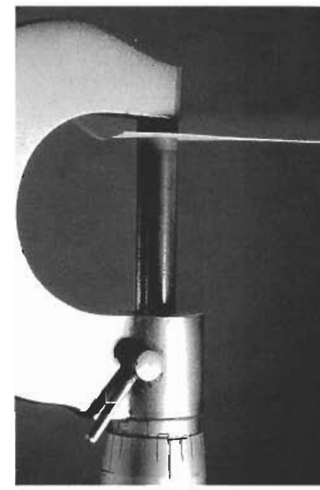


FIGURE 1–11 Example 1–7. A micrometer, which is used for measuring small thicknesses.

EXAMPLE 1–8 ESTIMATE **Total number of heartbeats.** Estimate the total number of beats a typical human heart makes in a lifetime.

APPROACH A typical resting heart rate is 70 beats/min. But during exercise it can be a lot higher. A reasonable average might be 80 beats/min.

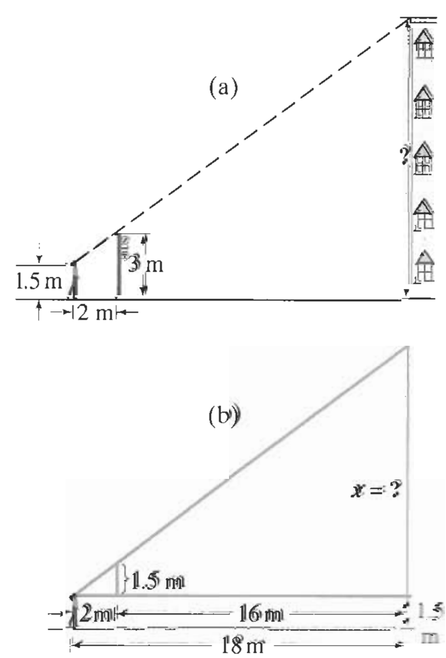
SOLUTION If an average person lives 70 years $\approx 2 \times 10^9$ s (see Table 1–2),

$$\left(80\,\frac{\text{beats}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)(2 \times 10^9\,\text{s}) \approx 3 \times 10^9,$$

or 3 trillion.

Now let's take a simple Example of how a diagram can be useful for making an estimate. It cannot be emphasized enough how important it is to draw a diagram when trying to solve a physics problem.

FIGURE 1–12 Example 1–9. Diagrams are really useful!



EXAMPLE 1–9 ESTIMATE **Height by triangulation.** Estimate the height of the building shown in Fig. 1–12, by "triangulation," with the help of a bus-stop pole and a friend.

APPROACH By standing your friend next to the pole, you estimate the height of the pole to be 3 m. You next step away from the pole until the top of the pole is in line with the top of the building, Fig. 1–12a. You are 5 ft 6 in. tall, so your eyes are about 1.5 m above the ground. Your friend is taller, and when she stretches out her arms, one hand touches you, and the other touches the pole, so you estimate that distance as 2 m (Fig. 1–12a). You then pace off the distance from the pole to the base of the building with big, 1-m-long, steps, and you get a total of 16 steps or 16 m.

SOLUTION Now you draw, to scale, the diagram shown in Fig. 1–12b using these measurements. You can measure, right on the diagram, the last side of the triangle to be about $x = 13$ m. Alternatively, you can use similar triangles to obtain the height x:

$$\frac{1.5\text{ m}}{2\text{ m}} = \frac{x}{18\text{ m}}, \quad \text{so} \quad x \approx 13\tfrac{1}{2}\,\text{m}.$$

Finally you add in your eye height of 1.5 m above the ground to get your final result: the building is about 15 m tall.