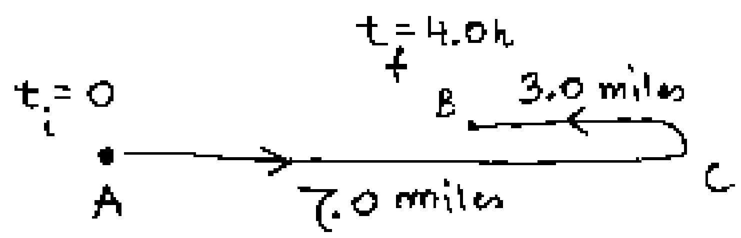
Chapter 2: Describing Motion: Kinematics in One Dimension

2-1 Reference Frames and Displacement
2-2 Average Velocity
2-3 Instantaneous Velocity
2-4 Acceleration
2-5 Motion at Constant Acceleration
2-6 Solving Problems
2-7 Falling Objects

EX. 1

Consider a walk $A \to C \to B$



- Distance travelled = length of the path = 7.0 miles + 3.0 miles = 10.0 miles

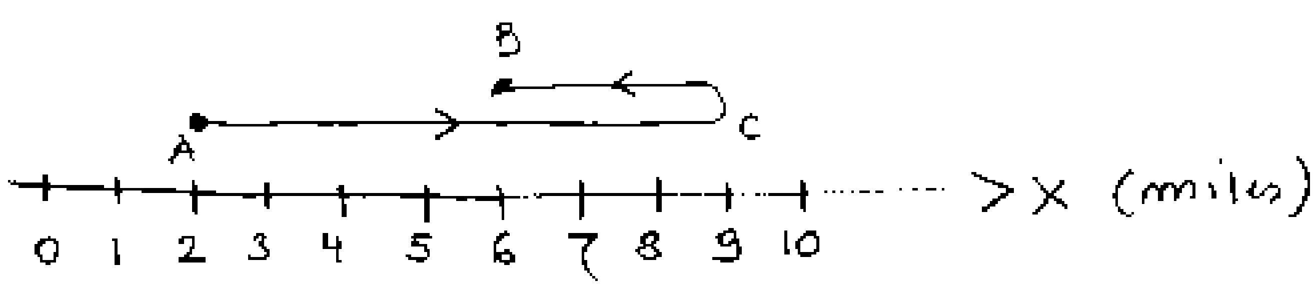
  Duration of the trip = 4.0 hours

☆ **Average speed** $\equiv \dfrac{\text{Distance}}{\text{Time}} = \dfrac{10.0 \text{ miles}}{4.0 \text{ hours}} = 2.5 \dfrac{\text{miles}}{\text{hour}} = 2.5$ mph

also

Distance = (Av. speed) x Time = $2.5 \dfrac{\text{mile}}{\text{hour}} \times 4.0$ hour = 10.0 mils

- Displacement – we need a reference frame.
  Let's choose a reference frame by drawing a x-axis and selecting an origin (this is up to you to choose)



In this reference frame $x_A = 2.0$ miles, $x_B = 6.0$ miles

- **Displacement** (Net change of position) $\Delta x \equiv x_{final} - x_{initial}$

  Here $\Delta x = 6.0 \text{ miles} - 2.0 \text{ miles} = +4.0$ miles

  ↑ towards the "+" end of the axis

☆ **Average velocity** $\overline{v_x} \equiv \dfrac{\text{Displacement}}{\text{Time}} = \dfrac{\Delta x}{\Delta t}$

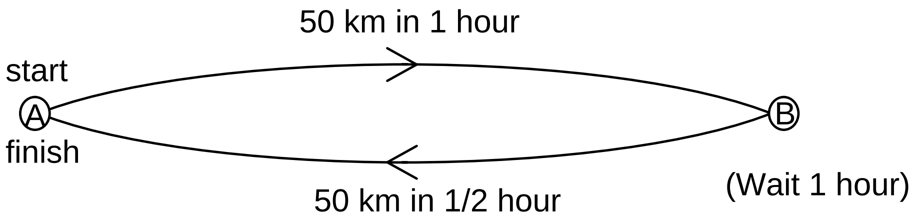
Here $\overline{v_x} = \dfrac{+4.0 \text{ miles}}{4.0 \text{ h}} = +1.0 \dfrac{\text{mile}}{\text{h}}$

↑ means toward the "+" end of the x-axis

Note: $\Delta x = \overline{v_x} \cdot \Delta t$

If you run at 8.0 m/s for 11 s then $\Delta x = (8.0 \frac{m}{s})(11 s) = 88$ m

A person starts in A, drives to B, 50 km away, in 1 hour, stays in B for 1 hour, then speeds back to A in 30 minutes.



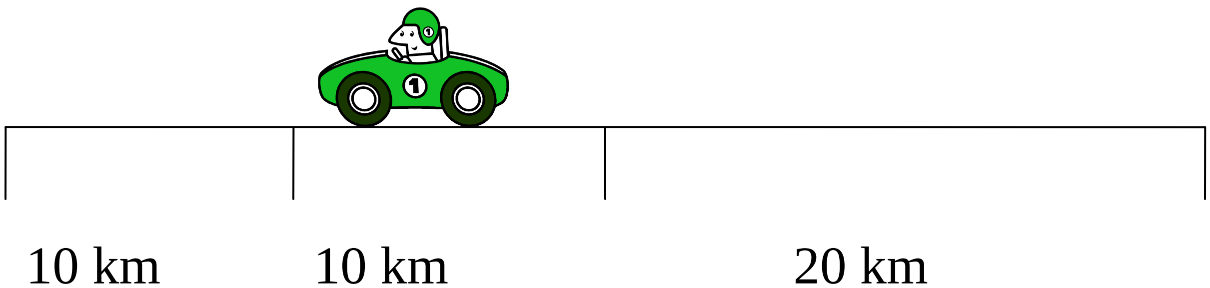
Q1. What is the average speed of this round trip?

a) zero
b) 40 km/hr
c) 67 km/hr
d) 75 km/hr
e) None of these.

Q2. What is the average velocity of this round trip?

a) zero
b) 40 km/hr
c) 67 km/hr
d) 75 km/hr
e) None of these.

A motorist wishes to travel 40 kilometers at an average speed of 40 km/h. During the first 20 kilometers he maintains an average speed of 40 km/h. No problem here. During the next 10 kilometers, however, the motorist goofs off and averages only 20 km/h.



With what speed must the motorist drive the last 10 km in order to achieve the average 40 km/h for the whole trip?

60 km/h
80 km/h
90 km/h
Faster than the speed of light
None of the above

Since $\overline{v}_x = \frac{\Delta x}{\Delta t}$

Then $\Delta x = \overline{v}_x \cdot \Delta t$

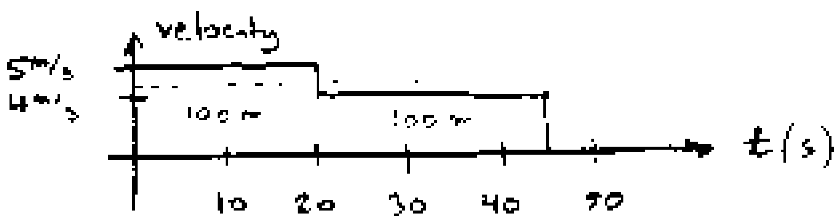
EX. 2 A jogger runs her first 100 m at 5 m/s and the second 100 m at 4 m/s. Find $\overline{v}$.

$\overline{v}_x = \frac{\text{Displacement}}{\text{Time}}$

Displacement = 100 m + 100 m = 200 m

Time = $\Delta t_1 + \Delta t_2$ = 20 s + 25 s = 45 s

$\overline{v}_x = \frac{200\,m}{45\,s} = 4.44\ m/s$

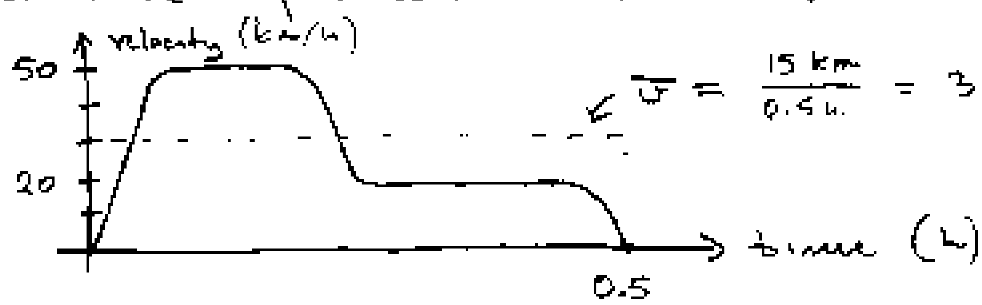


200 meters

Instantaneous velocity

What the speedometer shows

A car starts from rest, speeds up to 50 km/h, keeps going for a while, slows down to 20 km/h and finally gets to a final destination and stops. It travelled 15 km in 30 min.



$\overline{v} = \frac{15\,km}{0.5\,h} = 30\ \frac{km}{h}$

$v(t) \equiv$ { initially at the time t the object is at x; after a **short** period of time Δt the object is now at $x + \Delta x$ }

$$v(t) = \lim_{\Delta t \to 0} \frac{(x+\Delta x) - x}{\Delta t} = \frac{\Delta x}{\Delta t}$$

Meaning: take Δt

$\Delta t = 1$ min

$\Delta t = 1$ s

$\Delta t = \frac{1}{10}$ s

$\Delta t = \frac{1}{100}$ s etc.

Acceleration (One dimension)

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$$

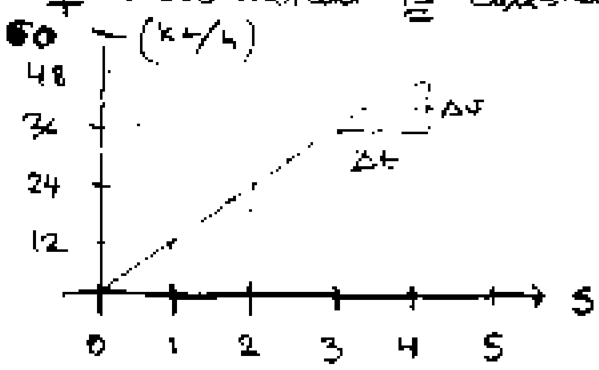
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \quad \text{instantaneous}$$

EX. 3

A car accelerates along a straight road from rest to 60 km/h in 5 s. Find average acceleration.

$$\bar{a} = \frac{60\ \text{km/h} - 0}{5.0\ \text{s}} = 12\ \frac{\text{km/h}}{\text{s}}$$

If acceleration is constant, then



"Rise": $\Delta v = 48 - 36 = 12$

"Run": $\Delta t = 4 - 3 = 1$

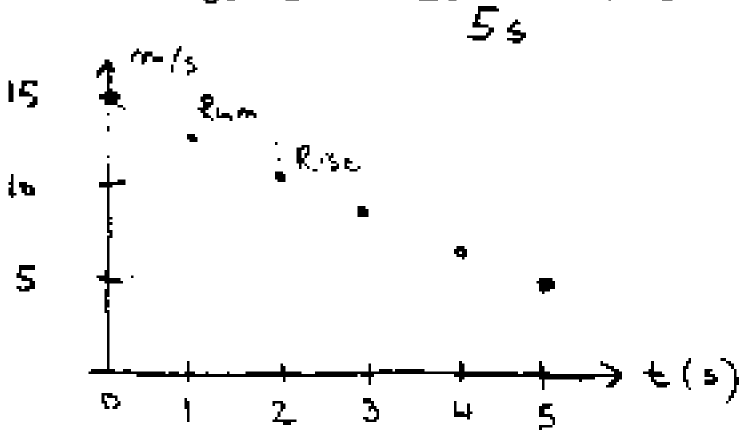
$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{12\ \text{km/h}}{1\ \text{s}} = 12\ \frac{\text{km/h}}{\text{s}}$$

<u>Note:</u> $60\ \frac{\text{km}}{\text{h}} = 60 \times \frac{1000\ \text{m}}{3600\ \text{s}} = 16.7\ \text{m/s}$

$$\bar{a} = \frac{16.7\ \text{m/s} - 0}{5\ \text{s}} = 3.33\ \frac{\text{m/s}}{\text{s}} = 3.33\ \frac{\text{m}}{\text{s}\cdot\text{s}} = 3.33\ \frac{\text{m}}{\text{s}^2}$$

EX. 4 You are going 15.0 m/s, then you slam on brakes and in 5.0 s your velocity drops to 5.0 m/s. What's your av. a?

$$\bar{a} = \frac{5.0\ \text{m/s} - 15.0\ \text{m/s}}{5\ \text{s}} = -2.0\ \frac{\text{m/s}}{\text{s}} = -2.0\ \text{m/s}^2$$



Run = 2 − 1 = 1 s

Rise = 11 − 13 = −2 (drop!)

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{-2}{1} = -2\ \text{m/s}$$

Name:
ID#:

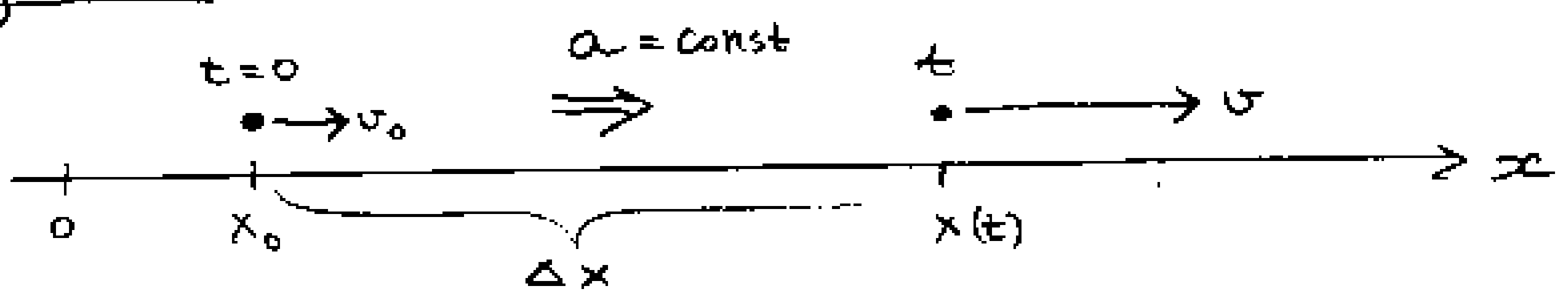
a = m/s^2

a = m/s^2

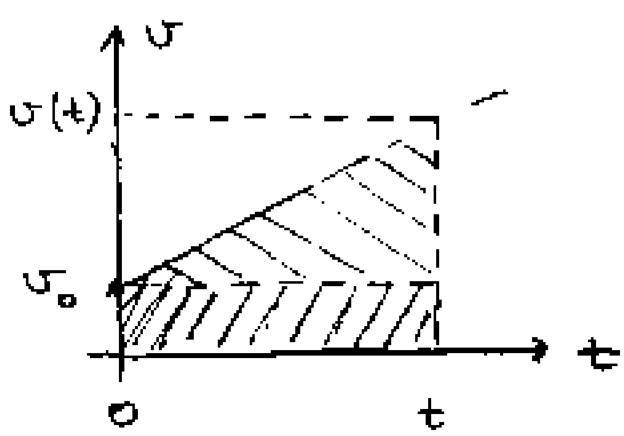
a = m/s^2

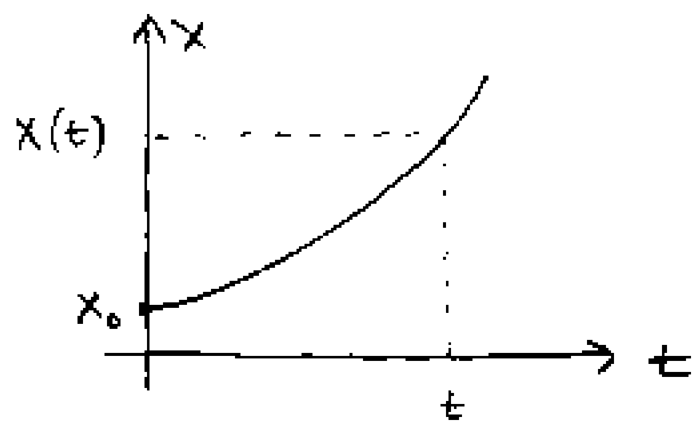
a = m/s^2

In general:



Object at $t=0$ is at x_0, has a velocity v_0, and accelerates at a rate a.





(*) $\boxed{v(t) = v_0 + at}$

What is $x(t)$?

$$x(t) = x_0 + \Delta x$$

↖ shaded area on v vs. t plot

$$\Delta x = v_{ave} \times t = \frac{v_0 + v(t)}{2} \cdot t$$

$$= \frac{v_0 + (v_0 + at)}{2} t = v_0 t + \frac{1}{2} a t^2$$

So we obtain:

(**) $\boxed{x(t) = x_0 + v_0 t + \frac{1}{2} a t^2}$

I've simplified $v(t)$ symbol

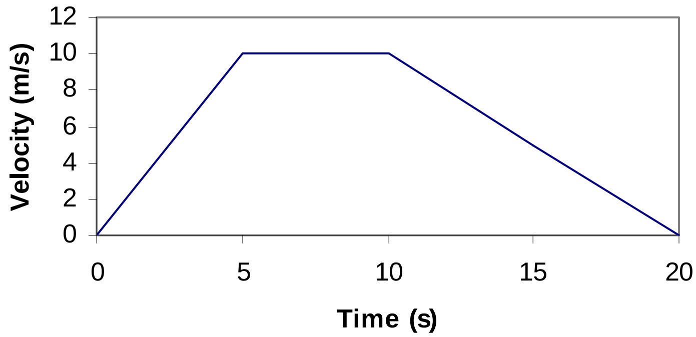
Note: calculate t from first equation: $t = \frac{v - v_0}{a}$

and plug it into second equation

$$x = x_0 + v_0 \frac{v - v_0}{a} + \frac{1}{2} a \frac{(v - v_0)^2}{a^2} = x_0 + \frac{v^2 - v_0^2}{2a}$$

(***) Hence $\boxed{v^2 = v_0^2 + 2a(x - x_0)}$ ← this equation does not involve time

Consider the following trip:



Q1. The total distance traveled by the object was…

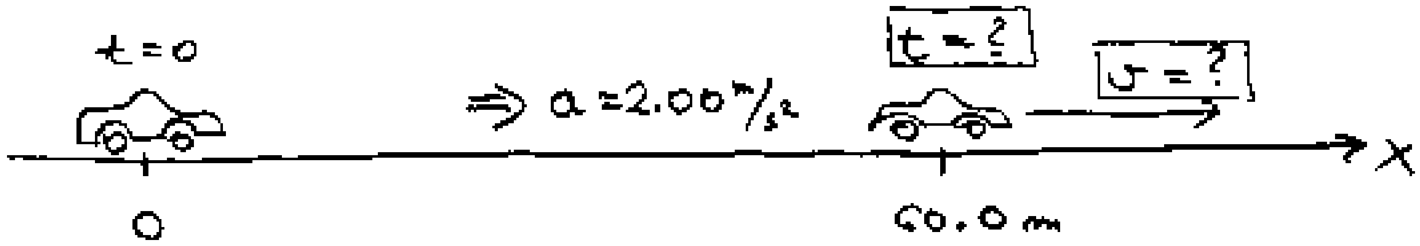
a) 50 m
b) 100 m
c) 125 m
d) 150 m
e) 200 m

Q2. And the average velocity of the object during the entire trip was…

a) 5.00 m/s
b) 6.25 m/s
c) 6.67 m/s
d) 7.50 m/s
e) none of the above

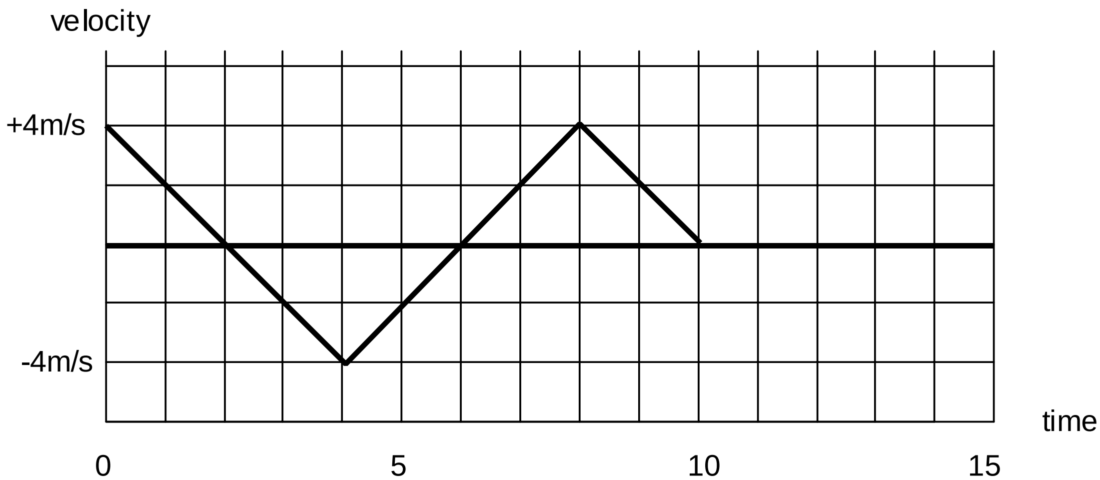
Examples:

① How long does it take a car to travel 60.0 m if it starts from rest and accelerates at a rate $a = 2.00\ m/s^2$?



quantity	value
$x - x_o$	
v_o	
v	
a	
t	

An objects moves along the x-axis with this velocity vs. time:



Q1: What is the displacement after 10 seconds?

a) 6m
b) 8m
c) 10m
d) 20 m
e) None of these.

Q2: What is the distance traveled after 10 seconds?

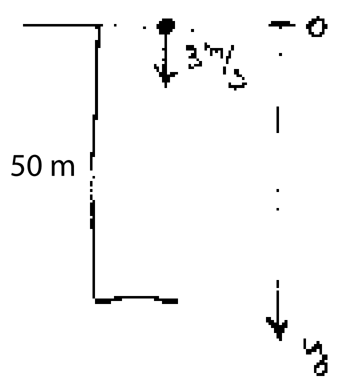
a) 6m
b) 8m
c) 10m
d) 20 m
e) None of these.

2.10 Falling bodies

In absence of air resistance all bodies fall with $g = 9.8\ m/s^2$ **downward**

$$a = -g = -9.80\ m/s^2$$

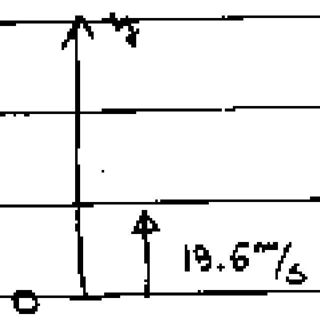
EX 1: Ball thrown down with $v = 3.0\ m/s$ from a cliff



(a) What is its velocity when it hits the ground?
(b) How long it takes to hit the ground?

EX 2 Ball thrown up with $v = 19.6\ m/s$

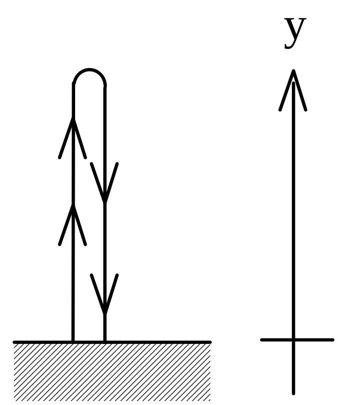
i) How high does it go?
ii) How long in the air before coming to hand
iii) its speed then



$y_0 = 0$
$v_0 = 19.6\ m/s$
$a = -9.8\ m/s^2$

On planet X, a cannon ball is fired straight upward. The position and velocity of the ball at many times are listed below. Note that we have chosen up as the positive direction.

Time (s)	Height (m)	Velocity (m/s)
0	0	20
1	17.5	15
2	30	10
3	37.5	5
4	40	0
5	37.5	-5
6	30	-10
7	17.5	-15
8	0	-20



What is the acceleration due to gravity on Planet X?

a) $-5 m/s^2$
b) $-10 m/s^2$
c) $-15 m/s^2$
d) $-20 m/s^2$
e) None of these.