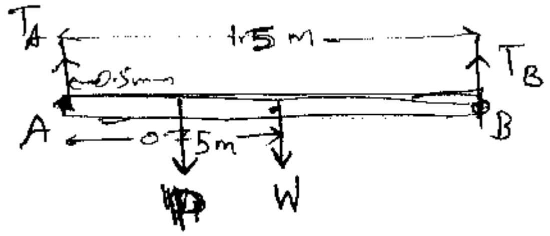
Final Exam - 021 (Q1–Q16)
solutions by Dr. M. S. Kanapper

(Q 1)
(ch 13)



$\Sigma \tau_A = 0$ will find T_B as the other unknown T_A will not get into the equation.

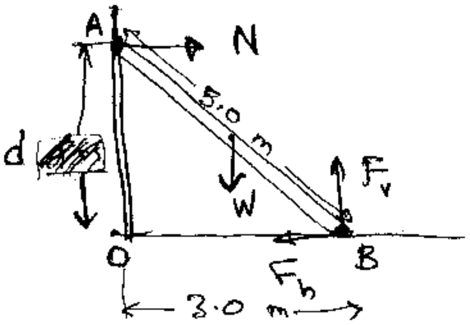
$\Downarrow$

$-P(0.5) - W(0.75) + T_B(1.5) = 0$

$T_B = \frac{80(0.5) + (20)(0.5)}{(1.5)}$

$\boxed{T_B = 37 \text{ N}}$

(Q 2)
(ch 13)



1) Where the ladder touch the floor (point B) we have F_h (friction) & F_v (normal force)

2) By looking at the length of the ladder AB = 5.0 m & OB = 3.0, it is obvious OA = d = 4.0 m ! This information is

very useful & saves time as we don't have to calculate any angles!!

8) Since we need to find only N, take the torque equation ($\Sigma \tau = 0$) about ~~an perpen~~ an axis (perpendicular to the paper) through point B; thus eliminating the unknowns F_v & F_h in the equation,

$$\Sigma \tau_B = 0$$

$$\downarrow$$

$$-N(OA) + W(1.5) = 0$$

Note W is only the weight of the man, not the weight of the ladder which we assume to be zero (light ladder)

$$N = \frac{W(1.5)}{(OA)} = \frac{(720)(1.5)}{4}$$

$$\boxed{N = 270 \text{ N}}$$

Q3
(ch 13)

$F = 6.0 \times 10^4$ N

$A = \pi(8.5 \times 10^{-3})^2$ m^2

$E = 11 \times 10^{11}$ N/m^2

$\ell = 100$ cm $= 1.00$ m

$\Delta \ell = ?$

stress = (modulus) strain

$$\left(\frac{F}{A}\right) = E\left(\frac{\Delta \ell}{\ell}\right)$$

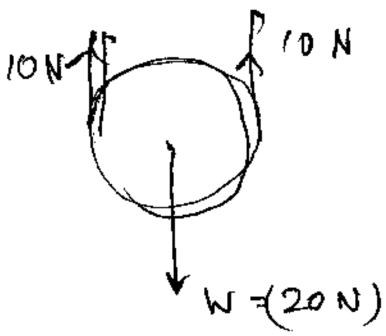
$$\Delta \ell = \left(\frac{F}{A}\right) \ell \left(\frac{1}{E}\right)$$

$$= \frac{6.0 \times 10^4}{\pi(8.5 \times 10^{-3})^2} (1.00) \frac{1}{11 \times 10^{11}}$$

$$= 2.4 \times 10^{-4} \text{ m}$$

$$\boxed{\Delta \ell = 0.24 \text{ mm}}$$

Q 4
(ch 13)



$\Sigma F_x = 0$ ✓

$\Sigma F_y = 0$ ✓

$\Sigma \tau = 0$ about any z-axis perpendicular to the plan of paper.

(Q5)
ch14

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$ K's "law of periods"

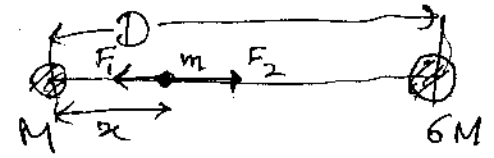
$$M = \frac{4\pi^2 r^3}{GT^2}$$

$r = 1.2 \times 10^7$ m , $G = 6.67 \times 10^{-11}$ (.)
$T = 2.8$ h $= (2.8 \times 3600)$ s

$$M = \frac{4\pi^2 (1.2 \times 10^7)^3}{(6.67 \times 10^{-11})(2.8 \times 3600)^3}$$

$$\boxed{M = 1.0 \times 10^{25} \text{ kg}}$$

(Q6)
ch 14



at x , $F_1 = F_2$

$$\frac{\cancel{G}M\cancel{m}}{x^2} = \cancel{\frac{G(6M)m}{(D-x)^2}} = \frac{\cancel{G}(6M)\cancel{m}}{(D-x)^2}$$

$$\frac{1}{x^2} = \frac{6}{(D-x)^2}$$

$$\frac{1}{x} = \frac{\pm\sqrt{6}}{D-x}$$

$D - x = \pm\sqrt{6}\, x$

$x(1 \pm \sqrt{6}) = D$

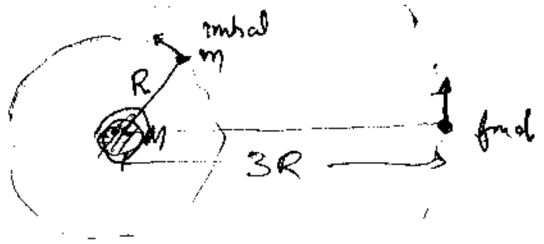
$x = \dfrac{D}{1+\sqrt{6}}$ or $\dfrac{D}{1-\sqrt{6}}$ this is −ve

$x = 0.29D$ or $-0.69D$

The answer −0.69D implies the neutral point lies outside the line joining M & 6M (to the left). This is not possible as both M & 6M will be attracting 'm'; there the forces are equal but in the same direction!

So $\boxed{x = +0.29D}$

(Q7)
(ch13)



i) $E_1 = -\dfrac{GMm}{2R}$

This is because the total energy E ~~of a satell~~ (K + U) of a satallite (m) going round a planet (M) is given by $-\dfrac{GMm}{2R}$ where R is ~~radius~~ orbital radius of the satellite.

Similarly, when it is taken to radius 3R

$$2)\quad E_2 = -\frac{GMm}{2(3R)}$$

~~Now cons~~

We need to provide $\Delta E = E_2 - E_1$ amount of energy to take it to the new orbit

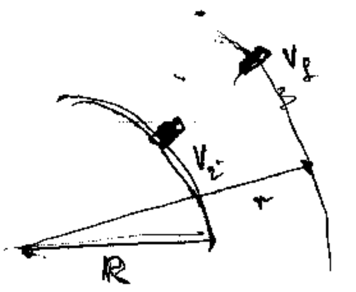
$$\Delta E = \frac{-GMm}{2(3R)} - \left(-\frac{GMm}{2R}\right)$$

$$= \frac{GMm}{2R}\left[1 - \frac{1}{3}\right]$$

$$= \frac{GMm}{2R}\left(\frac{2}{3}\right)$$

$$\boxed{\Delta E = \frac{GMm}{3R}}$$

(Q8)
(ch14)



Assuming the engine is shut off after acquiring the initial speed $v_i = 4.0$ km/s & ignoring air drag ~~on the~~ if any on that planet, we can write conservation of mechanical energy as

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2}mv_1^2 - \frac{GMm}{R} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2}$$

$$\frac{1}{2}v_1^2 - \frac{GM}{R} + \frac{GM}{r_2} = \frac{1}{2}v_2^2$$

$$v_1^2 - \frac{2GM}{R} + \frac{2GM}{r_2} = v_2^2$$

$v_1 = 4.0 \frac{km}{s} = 4.0 \times 10^3 \ m/s$

$G = 6.67 \times 10^{-11} \ (\ldots)$

$M = 5.0 \times 10^{23} \ kg$

$r_1 = R = 2.0 \times 10^6 \ m$

$r_2 = R + 1.0 \times 10^6 = 3.0 \times 10^6 \ m$

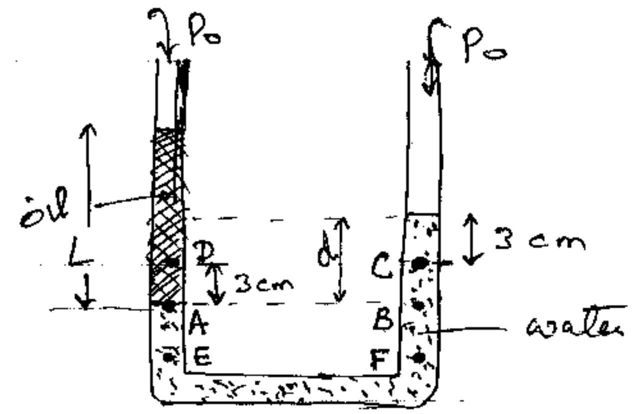
$$\Rightarrow v_2 = \sqrt{(4.0\times10^3)^2 - \frac{2\times(6.67\times10^{-11})\times(5.0\times10^{23})}{2.0\times10^6} + \frac{2(6.67\times10^{-11})(5.0\times10^{23})}{3.0\times10^6}}$$

$v_2 = 2210 \ m/s$

$$\boxed{v_2 = 2.2 \ \frac{km}{s}}$$

Q9

ch 15



The KEY IDEA here is that since the water ~~liquid~~ column is in equilibrium, the pressure at the same level ~~inside~~ anywhere in that liquid (water) is the same.

Consider these:

$P_D = P_C$? No wrong — (1)

$P_A = P_B$ ✓ yes — (2)

$P_E = P_F$ ✓ yes — (3)

(2) & (3) are both correct, but choosing (3) will bring you to (2) with one extra step. So we go straight to (2), the level at the interface.

$$\underbrace{P_A}_{\downarrow} = P_B$$

$$\overbrace{\cancel{P_0} + L\rho_{oil}\, g} = \overbrace{\cancel{P_0} + d\rho_w\, g}$$

where $d = 3 + 3 = 6$ cm

$$L\rho_{oil}\cancel{g} = d\rho_w\cancel{g}$$

$$L\rho_{oil} = d\rho_w$$

$$L = d\,\frac{\rho_w}{\rho_{oil}}$$

$$= 6 \text{ cm } \frac{1\,(g/cm^3)}{0.75\,(g/cm^3)}$$

$$\boxed{L = 8 \text{ cm}}$$

Q10
ch 15

$$\underbrace{\text{Apparent weight}}_{F_g'} = \underbrace{\text{Real weight}}_{F_g} - \underbrace{\text{Buoyant force}}_{F_b}$$

$$F_b = F_g - F_g'$$

$$F_b\,(\text{water}) = 30 - 20 = 10\ \text{N} \quad \text{— (1)}$$

From Archemedes' ~~law~~ principle

$$F_b\,(\text{water}) = \rho_w V g \quad \text{— (2)}$$

[V is the volume of the object as it is completely submerged]

∴ from (1) & (2)

∴ $$\rho_w V g = 10\ (\text{N}) \quad \text{— (3)}$$

Similarly $$F_b\,(\text{unknown}) = 30 - 24 = 6\ (\text{N}) \quad \text{— (4)}$$

From Archemedes' ~~law~~ principle

$$F_b\,(\text{unknown}) = \rho_u V g \quad \text{— (5)}$$

from (4) & (5)

$$\rho_u V g = 6\ (\text{N}) \quad \text{— (6)}$$

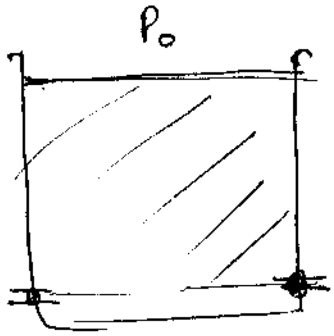
$$\frac{(6)}{(3)} \Rightarrow \quad \frac{\rho_u}{\rho_w} = \frac{6}{10} = 0.6$$

$$\rho_u = 0.6\,\rho_w$$

$$\rho_w = 1\ \text{g/cm}^3$$

$$\boxed{\rho_u = 0.6\ \text{g/cm}^3}$$

Q11
ch 15



~~Please~~ Reed sample problem 15-9 (page 338) in your text book. There it is shown that the ~~velocity of the~~ speed of water leaking from a large tank depends only in the depth below the surface of water.

$$v = \sqrt{2gh}$$

Therefore $\boxed{v_1 = v_2}$

Q12
ch 15

Equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$\left\{\pi \left(\frac{d_1}{2}\right)^2\right\} v_1 = \left\{\pi \left(\frac{d_2}{2}\right)^2\right\} v_2$$

$$d_1^2 v_1 = d_2^2 v_2$$

$d_1 = 6.0$ cm, $d_2 = 8.0$ cm

$v_1 = 5.0$ m/s, $v_2 = ?$

$$v_2 = \left(\frac{d_1}{d_2}\right)^2 v_1 = \left(\frac{6}{8}\right)^2 5.0$$

$$v_2 = 2.8 \text{ m/s}$$

Note how I postponed the substitution of numbers →

so that final calculations become much ~~simpler.~~ easier.

Q13
ch 16

$$x = \underbrace{0.02}\cos(\underbrace{300}t - \underbrace{\pi/3})$$

$x_m = 0.02$ $\quad \omega = 300$ $\quad \phi = (-\pi/3)$

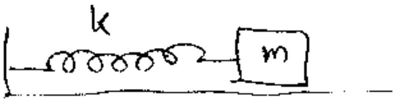
differentiate

$$V = -\underbrace{\omega x_m}_{V_m}\sin(\omega t + \phi)$$

$\therefore \quad V_m = +\omega x_m = +(300)(0.02)$

$\boxed{V_m = 6\ m/s}$ magnitude

Q.14
ch 16



linear Harmonic Oscillator

$$k = m\omega^2$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65}{0.65}} = 10\ rad/s$$

$\boxed{\omega = 10\ rad/s}$

Q15
ch 16

$$U = \frac{1}{2} k x^2$$

$$E = K + U = \frac{1}{2} k x_m^2$$

$U(x = 2\ \text{cm}) = \cancel{\frac{4}{2}}\ 2$ & $K(x = 2\ \text{cm}) = 6$

$\Rightarrow E = 8$ at all times & positions because E is conserved

$$\Rightarrow \quad \frac{1}{2} k x_m^2 = 8$$

$$x_m^2 = \frac{16}{k}$$

but how do find k?

$$U(x = 2\ \text{cm}) = \frac{1}{2} k x^2 = 2$$

$$\frac{1}{2} k (0.02)^2 = 2$$

$$\boxed{k = 10000\ N/m}$$

$$x_m = \sqrt{\frac{16}{10000}} = \frac{4}{100}$$

$$\boxed{x_m = 0.04\ m = 4\ cm}$$

Q16 The equation of SHM is

ch 16

$$a = -\omega^2 x$$

which means the force acting on it must be of the form

$$\boxed{F = -kx}$$ k - a constant equal to $m\omega^2$

only $F = -5x$ satisfy this with $k = 5$