~~Chapter~~

Exam 2 - 021

(~~#14~~) Detailed solution by Dr. M. S. Kariapper

(#14). $I_{cm} = 6.0 \times 10 \ kg \cdot m^2$

$\alpha = 2.0 \ rad/s^2$

Const α means the ~~for~~ net torque (τ_{net}) is const.

So $W = \int \tau_{net} d\theta = \underline{\underline{\tau_{net} \Delta\theta}}$

$$W = (I\alpha)\,\Delta\theta$$

Given the data $\omega_0 = 0$ (start from rest)

$\alpha = 2.0$ rad/s (~~and~~ constant α)

& $t = 5.0$ s

one can calculate $\Delta\theta$ using

$$\Delta\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\Delta\theta = 0 + \tfrac{1}{2}\alpha t^2$$

$$\Rightarrow \quad W = I\alpha\left(\tfrac{1}{2}\alpha t^2\right)$$

$$= (6)(2)\left(\tfrac{1}{2}(2)(5)^2\right)$$

$$\boxed{W = 300 \ J}$$

(#15) Consider the equation:

$$\underbrace{\sum \tau_{ext}}_{\substack{\| \\ 0}} = \frac{d\vec{L}}{dt}$$

0 means $\vec{L}$ is constant

$\therefore$ (1) ✓

(2) ×

~~(3) Also $\sum \vec{L} = I\omega$ for a fixed axis rotation~~

$\Rightarrow \sum \tau_{ext} = I\alpha \Rightarrow \alpha = 0$

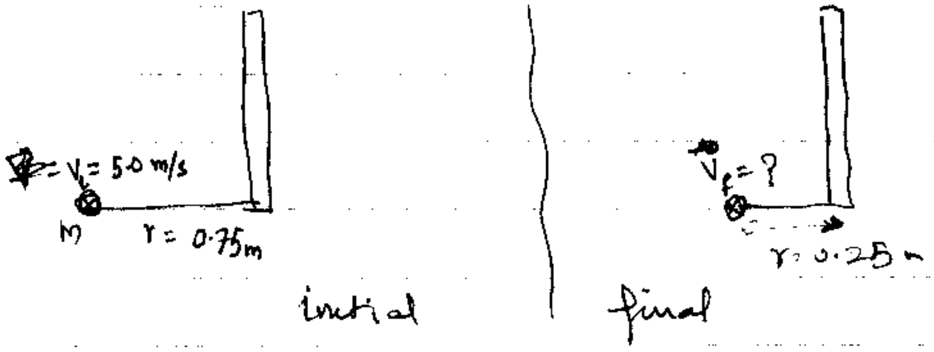
(3) ×

$K_R = \frac{1}{2} I\omega^2$ & ω is constant $\because \vec{L} = I\vec{\omega}$

$\therefore$ (4) ×

& (5) ×

(#16)



$\vec{L}_i = \vec{L}_f$

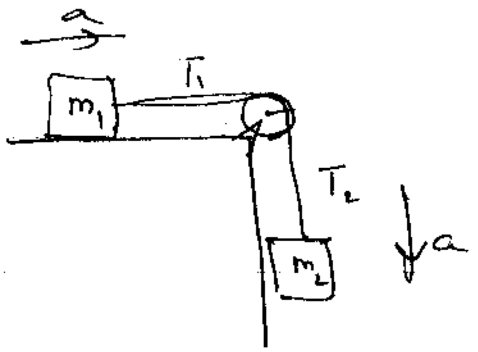
$L_i = L_f$

$\cancel{m} r_i v_i = \cancel{m} r_f v_f$

$(\cancel{0.75})^{3}(5) = (\cancel{0.25}) v_f$

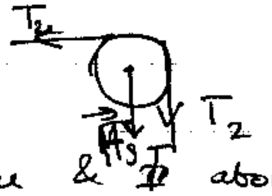
$\boxed{v_f = 15 \text{ m/s}}$

(#17)



$m_1 = 5.0$ kg
$m_2 = 4.0$ kg
$R = 0.20$ m
$a = 3.5$ m/s

N's 2nd law for the rotational motion of the pulley



$\underbrace{\sum \tau_{ext}} = I\alpha$ (torque & I about the axis of rotation)

$$\vec{\tau}_{T_1} + \vec{\tau}_{T_2} + \vec{\tau}_{F_g} = I\alpha$$

$$\alpha = -\frac{a_t}{R} = \frac{-3.5}{0.2}$$ $a_t = a$ & α is negative counterclockwise

$$= -17.5 \text{ rad/s}^2$$

$$\Rightarrow +T_1 R - T_2 R + 0 = I(-17.5)$$

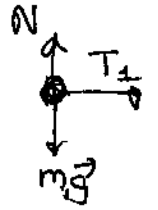
$\vec{\tau}_{F_g}$ is zero because ~~I~~ $r = 0$, ~~Fg~~ $\vec{F}_g$ $(= m\vec{g})$ goes through axis of rotation.

$$\frac{(T_2 - T_1)R}{17.5} = I$$

We still need to find T_1 & T_2. How? $\Rightarrow$

Apply N's 2nd law for m_1 & m_2

m_1

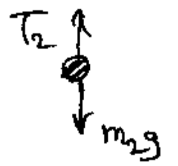


$$\underbrace{\sum F_x}_{} = ma_x$$

$$T_1 = m_1 a \quad \text{--- (1)}$$

~~$T_1 = (5)(3.5) =$~~

m_2



$$\Rightarrow T_2 - m_2 g = m_2(-a)$$

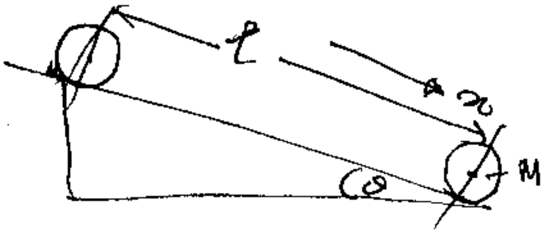
$$T_2 = m_2(g - a) \quad \text{--- (2)}$$

$$\therefore \quad I = \frac{\{m_2(g-a) - m_1 a\} R}{\alpha}$$

$$= \frac{\{4(9.8-3.5) - 5(3.5)\}(0.20)}{17.5}$$

$$\boxed{I = 0.088 \ \text{kg}\cdot\text{m}^2}$$

#18



$R = 10$ cm

$\ell = 5.0$ m

$M = 8.5$ kg

$\theta = 25°$

<u>Mtd 1</u>

Find $\vec{a}_{cm}$ & use the equation of kinematics with the known quantities a, V_0, Δx to find V (final).

Choosing down the incline as the +ve x-direction

$$\boxed{a_{com} = \frac{g\sin\theta}{1 + (I_{com}/mR^2)}}$$

This is an equation valid for any ~~smooth~~ ~~rolling~~, circularly symmetric objects of radius 'R' (such as sphere, ring, disk) rolling smoothly down an incline of angle θ.

$$\frac{I_{com}}{mR^2} = \begin{cases} 1 & (\text{ring}) \\ 1/2 & (\text{disk}) \\ 2/5 & (\text{sphere (ball)}) \end{cases}$$

$$\Rightarrow \quad a_{com} = \frac{(9.80)\sin 25}{1 + (2/5)} = 2.96 \ m/s^2$$

Now we have the linear motion of the COM of the object with $a = 2.96$

$v_0 = 0$ (starting from rest)

& $\Delta x = 5.0$ m

find v?

$$v^2 = v_0^2 + 2a\Delta x$$

$$v^2 = 0 + 2(2.96)(5.0)$$

$$\boxed{v = 5.4 \ m/s}$$

$\frac{2\sin\theta \ 5}{1.4}$

29.58 3.x

Mtd II Conservation of Energy Principle.

$$\Delta E_{mech} + \cancel{\Delta E_{th}} + \cancel{\Delta E_{int}} = \cancel{W_{ext}}$$

$$\Delta K + \Delta U = 0$$

$$\Delta K_{T} + \Delta K_{R} + \Delta U_g = 0$$

$$\frac{1}{2} m (v^2 - 0^2) + \frac{1}{2} I (\omega^2 - 0^2) + mg(-\ell \sin\theta) = 0$$

$$\frac{1}{2} \cancel{m} v^2 + \frac{1}{2}\left(\frac{2}{5} \cancel{m} R^2\right)\omega^2 - \cancel{m} g \ell \sin\theta = 0$$

$$\frac{1}{2} v^2 + \frac{1}{5} \underbrace{R^2\omega^2}_{= v^2} - g \ell \sin\theta = 0$$

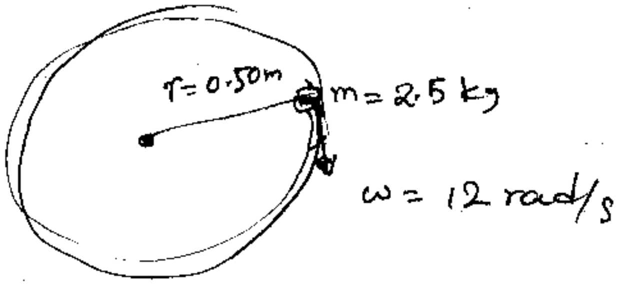
$$v^2\left(\frac{1}{2} + \frac{1}{5}\right) = g \ell \sin\theta$$

$$v^2 (0.5 + 0.2) = (9.8)(5) \sin 25$$

$$\boxed{v = 5.4 \, (m/s)}$$

(19)

$\cancel{\ell} =$



$\ell = m r v$ (magnitude

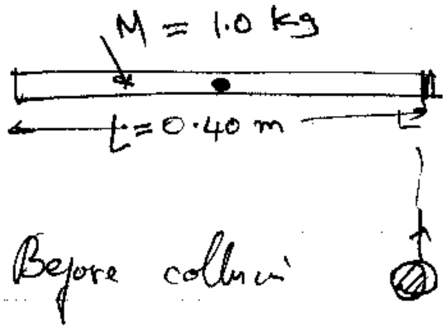
$= m r (r\omega)$ ← $v = r\omega$

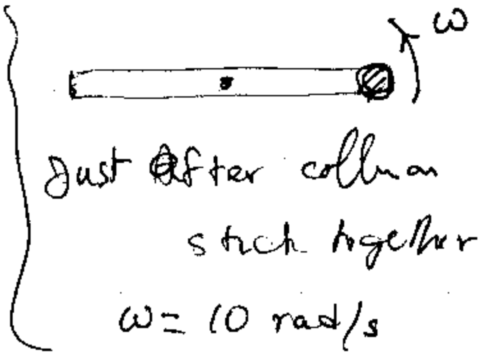
$= m r^2 \omega$

$= (2.5)(0.5)^2 (12)$

$\boxed{\ell = 7.5 \ \text{kg} \cdot \text{m}^2}$

(20)





v_0 ?

$\vec{L}_i = \vec{L}_f$

$\underbrace{\vec{L}_{rod,i}}_{0} + \vec{L}_{ball,i} = \cancel{\vec{L}_{rod,f}} \ \vec{L}_{(rod+ball),f}$

$0 + \underbrace{m r_\perp v_0}_{+ve} = (I_{rod} + I_{ball})\,\omega$

$r_\perp = L/2 = 0.20$ m & $\vec{L}_{ball,i}$ is counterclockwise out of the paper +ve

$$\underbrace{(0.100)}_{m}\underbrace{(0.20)}_{L/2} V_0 = \left(\frac{1}{12}ML^2 + m\left(\frac{L}{2}\right)^2\right)\omega$$

$$0.02\, V_0 = \left\{\frac{1}{12}(1.0)(0.4)^2 + (0.100)(0.20)^2\right\} 10$$

$$\boxed{V_0 = 8.7 \ \text{m/s}}$$