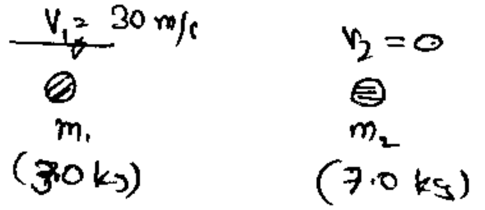
Major 2 _ 021
solutions
Q7 → Q13
ch 8 & ch 9
prepared by Dr M. S. Kariapper

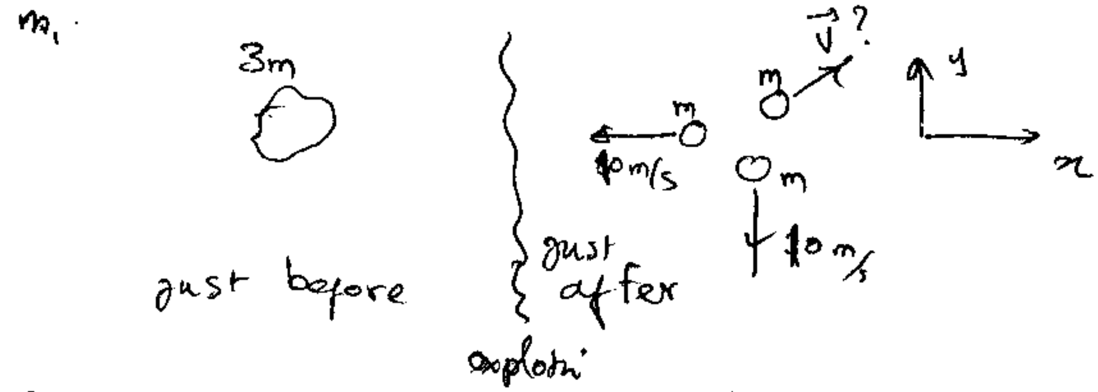
Q7.



$$V_{cm} = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2} = \frac{(3.0)(30) + (7.0)(0)}{3.0 + 7.0}$$

$$\boxed{V_{cm} = 9.0 \text{ m/s}}$$

Q8



Let's choose the two pieces traveling perpendicular to be in the −ve−x & −ve y directions as shown in the figure.

One can apply the conservation of total momentum immediately before explosion to immediately after explosion(Any effect due to any external forces (if they exist) can be neglected as the period is too short and the internal forces are much greater)

$$\vec{P}_i = \vec{P}_f$$

$$(3m)0 = m(-10\hat{i}) + m(-10\hat{j}) + m(\vec{v})$$

$m = 1.0$ kg

$$\vec{v} = 10\hat{i} + 10\hat{j}$$

$$\Rightarrow v = \sqrt{10^2 + 10^2} = 10\sqrt{2} = \boxed{14 \text{ m/s}}$$

Mtd 2

If you prefer you can solve it by applying conservation of momentum in x & y directions separately:

$$(P_i)_x = (P_f)_x$$

$$\rightarrow \quad 0 = 1(-10) + 0 + (1)V_x$$

$$(P_i)_y = (P_f)_y$$

$$\uparrow \quad 0 = 0 + 1(-10) + (1)V_y$$

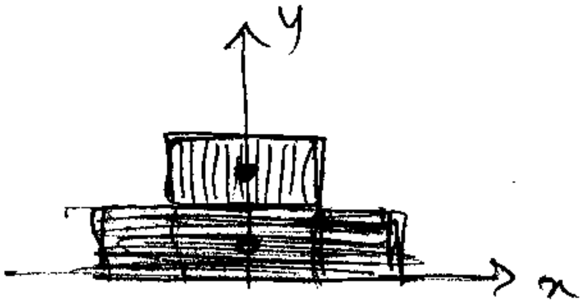
$$V_x = 10 \quad \& \quad V_y = 10$$

the same result!

$$V = \sqrt{V_x^2 + V_y^2} = 10\sqrt{2} = 14 \text{ m/s}$$

And, of course, you can shorten the calculation (therefore save time) by considering just two pieces as shown below:

II



piece 1 with mass ~~4m~~ $m_1 = 4m$
(bottom row) & $(x_1, y_1) = (0, +1)$

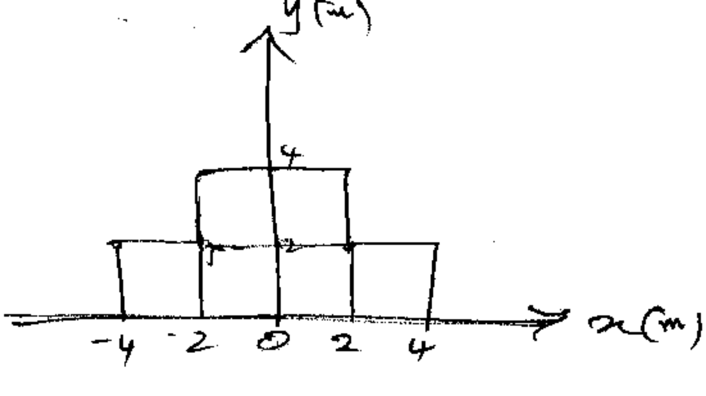
piece 2 with mass $m_2 = 2m$
(top row) & $(x_2, y_2) = (0, +3)$

$$y_{cm} = \frac{(4m)(+1) + (2m)(+3)}{4m + 2m}$$

$$= \frac{4 + 6}{6} = \frac{10}{6} = \underline{\underline{1.67}}$$

III Also one can replace the m by A (area) as the density is uniform. But the above method ~~is m~~ is shorter!

(Q9)

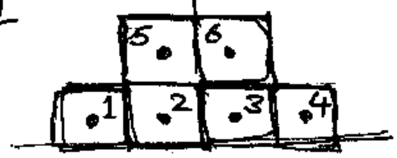


From symmetry one can see that the center of mass (COM) must lie in the y-axis this means $\boxed{x_{cm} = 0}$. We still have to find y_{cm}.

The trick is to devide this plat into smaller pieces of known center of mass positions. The ~~cm (COM)~~ COM of squares & rectangle can be found very easily

<u>Many possibilities:</u>

I



6 squares with COM given as

$(x_1, y_1) = (-3, +1)$

$(x_2, y_2) = (-1, +1)$

$(x_3, y_3) = (+1, +1)$

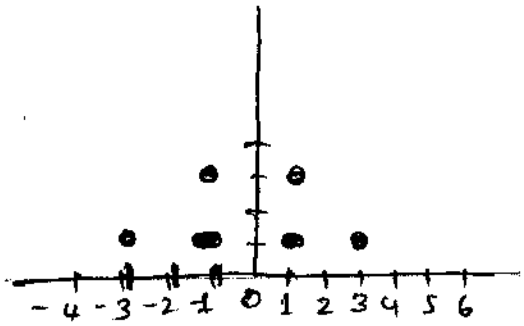
$(x_4, y_4) = (+3, +1)$

$(x_5, y_5) = (-1, +3)$

$(x_6, y_6) = (+1, +3)$

All of them have the same mass, say m (uniform density)

We can now consider them as point masses situated at their respective com positions.



$$\Rightarrow \quad y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4 + m_5 y_5 + m_6 y_6}{M}$$

$m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m$

& $M = 6m$

$$y_{cm} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6}{6}$$

$$= \frac{+1 + 1 + 1 + 1 + 3 + 3}{6}$$

$$= \frac{10}{6} = \frac{5}{3} = \boxed{1.67 \text{ m}}$$

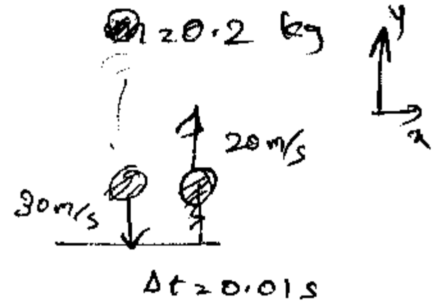
$$\boxed{(x_{cm}, y_{cm}) = (0, 1.67) \text{ m}}$$

(Q10) The dog + sled system does not have ~~not~~ net external force acting on it. ~~in the x direction~~ This means:

$$\left(\sum F_{ext}\right)_x \left(\frac{d\vec{P}}{dt}\right)_x = 0 \Rightarrow (\vec{P})_x \text{ is const}$$
$$\Rightarrow (M\vec{v}_{cm})_x \text{ is const}$$
$$\Rightarrow \vec{v}_{cm,x} \text{ is const.}$$
$$\cancel{\vec{v}_{cm} \text{ (before initial)}} = 0$$

v_{cm} initially is zero, this means that $v_{cm,x}$ should continue to be zero even when the dog starts to walk. As the dog walks towards edge B (that is away from O), the sled must move toward O to keep the x_{cm} at the same place.

Q11



$$\vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$

$$= \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$$

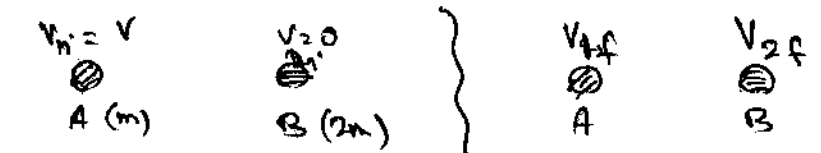
$$= \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t}$$

$$= \frac{(0.2)\left(20\hat{j} - (-30\hat{j})\right)}{0.01}$$

$$= \frac{(0.2)(50\hat{j})}{0.01}$$

$$\boxed{\vec{F}_{avg} = 1000\,\hat{j}\ (N)}$$

Q12



The equation for 2-body head-on elastic collision:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \left(\frac{?}{?}\right) v_{2i}$$

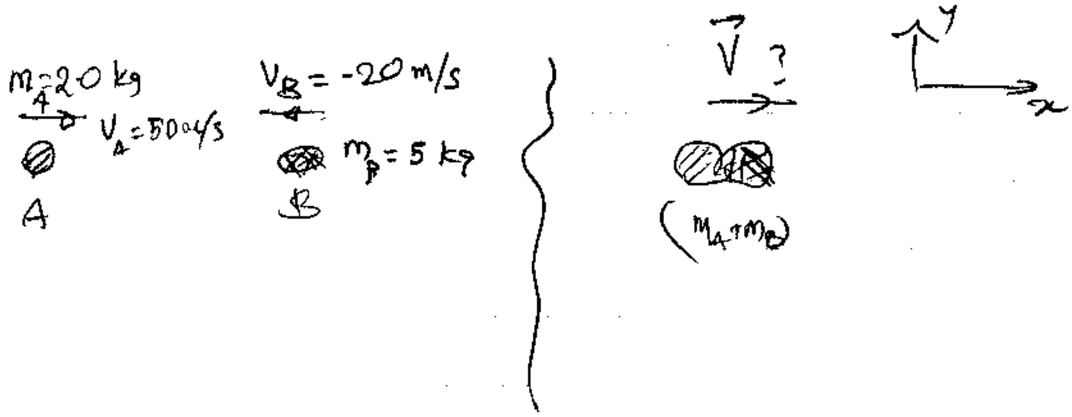
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \left(\frac{?}{?}\right) v_{2i}$$

($v_{2i} = 0$ anyway)

$$\Rightarrow \quad V_{1f} = \frac{m - 2m}{m + 2m} V = \frac{-\cancel{m}}{3\cancel{m}} V = \boxed{-\tfrac{1}{3} V}$$

$$V_{2f} = \frac{2m}{m + 2m} V = \frac{2\cancel{m}}{3\cancel{m}} V = \boxed{+\tfrac{2}{3} V}$$

13



$$\underbrace{\vec{P}_i}_{} = \vec{P}_f$$

$$m_A \vec{V}_A + m_B \vec{V}_B = (m_A + m_B)\vec{V}$$

$$(2)(50) + (5)(-20) = (7)\,\vec{V}$$

$\vec{V} =$ zero.

$$\sum K_i = \tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2$$

$$= \tfrac{1}{2}(2)(50)^2 + \tfrac{1}{2}(-20)^2(5)$$

$$= 2500 + 1000 = 3500\ \text{J}$$

$$\sum K_f = 0$$

energy lost $= \sum K_i - \sum K_f$

$\cancel{\Delta K} = K_i -$

$= 3500$ J

it lost all its KE!