Major 2 - 012

Solutions prepared by Dr. M. S. Kariapper

Q.1. The work done W is given in general by:

$$W = \int \vec{F} \cdot d\vec{s}$$

(this is equal to $\vec{F} \cdot \vec{d}$ only when F is constant)

F here being a function only of x

$$W = \int_{x=1} F_x \, dx = \int_{x=1}^{x=3} (4x) \, dx$$

$$= \left[\frac{\cancel{4}^2 x^2}{\cancel{2}} \right]_{x=1}^{x=3}$$

$$= \left[2x^2 \right]_1^3$$

$$= 2(3)^2 - 2(1)^2$$

$$= 18 - 2$$

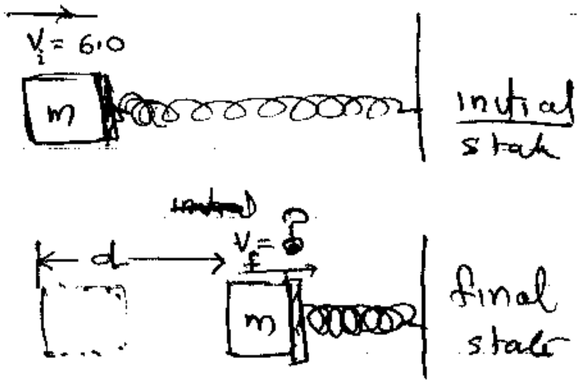
$$\boxed{W = 16 \text{ J}}$$

2.

Q2. First thing to do is to understand the problem. Drawing a figure always helps. to show the initial & final states of the problem.

$m = 2.0$ kg

$d = 15$ cm.
0.15 m

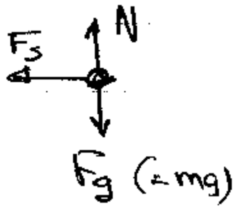


Method 1 (ch 7)

Treating this as ch7 problem (as it was meant to be) ~~we can~~

Concentrate on the one object (block, a particle-like) and ~~the~~ all the forces acting on it.

FBD for block →



F_s = spring force

Each of these forces does a work on the block, W_s, W_g & W_N. In chapter 7 we have 'Work - Kinetic Energy theorem'

$\boxed{W_{net} = \Delta K}$ → which applies to a single particle-like body which gives

$$W_g + W_N + W_s = \frac{1}{2} m \left(v_f^2 - v_i^2\right)$$

Let us look at each term on the left separately

$$W_g = \vec{F}_g \cdot \vec{d} = (mg)(d) \underbrace{\cos 90}_{0} = 0$$

W_N also zero

$$W_s = \frac{1}{2} k \left(x_i^2 - x_f^2\right)$$

So

$$0 + 0 + \frac{1}{2} k \left(x_i^2 - x_f^2\right) = \frac{1}{2} m \left(v^2 - 6^2\right)$$

$$\frac{1}{2}(2000)\left(0^2 - (0.15)^2\right) = \frac{1}{2}(2.0)\left(v^2 - 6^2\right)$$

$$-22.5 = v^2 - 36$$

$$\boxed{v = 3.7 \text{ m/s}}$$

Method 2 (ch 8)

Here instead of focussing on one object we choose the system

block + spring + earth

& try to apply the "Law of conservation of energy", which in its most general form:

$$\boxed{W = \Delta E_{mech} + \Delta E_{th} + \Delta E_{int}}$$

* The "W" here is the work done on the system (energy transferred to) via any net external force.

In this problem and for the system we chose (block + spring + earth) there are no external forces whatsoever

$\Rightarrow W = 0$

$\Delta E_{int} = 0$ (this can be taken zero most of the time.

$\Delta E_{th} = 0$ because no friction

$\Delta E_{mech} = \Delta K + \Delta U.$

Therefore

$$W = \Delta E_{mech} + \Delta E_{th} + \Delta E_{int}$$

$$0 = (\Delta K + \Delta U) + 0 + 0$$

reduces to

$$\Delta K + \underbrace{\Delta U}_{\Delta U_g + \Delta U_s} = 0$$

(the law of conservation of mechanical energy)

In this problem we have both a gravitational force & elastic (spring) force, two conservative forces in the system

$$\underbrace{\Delta K}_{} + \underbrace{\Delta U_g}_{} + \Delta U_s = 0$$

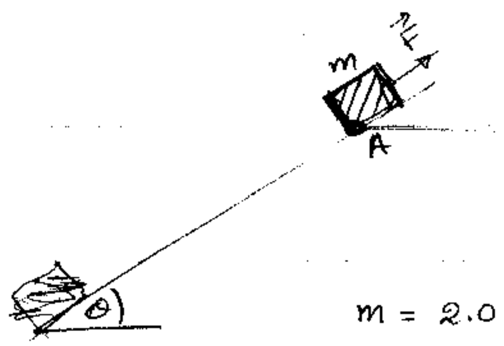
$$\frac{1}{2}m\left(v_f^2 - v_i^2\right) + mg\,\Delta y + \frac{1}{2}k\left(x_f^2 - x_i^2\right) = 0$$

$$\frac{1}{2}(2.0)\left(v^2 - 6^2\right) + mg(0) + \frac{1}{2}(2000)\left((0.15)^2 - 0^2\right) = 0$$

$\Delta y = 0$ as the block moves only horizontally.

and we get the same answer $\boxed{V = 3.7\ m/s}$

Q3. As before (Q2) draw the figures for initial & final for this problem.



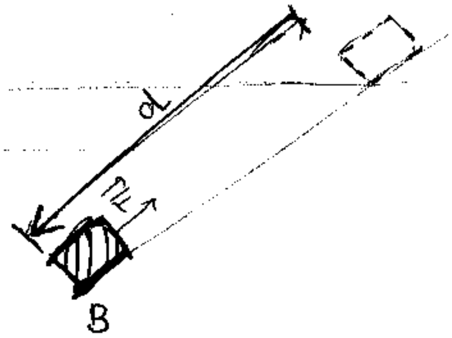
initial

$m = 2.0$ kg

$\theta = 30°$

$F = 3$ N

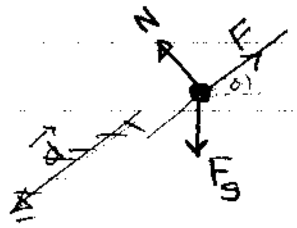
$d = 2.0$ m



final

Method 1 (ch 7 Work - KE theorem)
focus on just the block

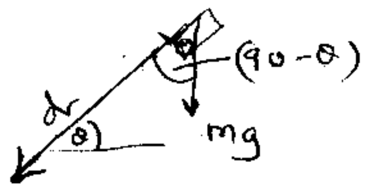
FBD →



$$\underbrace{W_{net}}_{W_g + W_F + W_N} = \Delta K = K_B - K_A$$

F · $W_g = \vec{F}_g \cdot \vec{d} = (mg)(d)\cos(90-\theta)$

$= mgd\sin\theta$



you can also use the fact

$$W_g = mg\,(\underbrace{y_i - y_f}_{=(-\Delta y) = -(y_f - y_i)})$$

$$W_F = \vec{F}\cdot\vec{d} = F\,d\,\cos(180) = -Fd$$

$$W_N = N\,d\,\cos 90 = 0$$

So $\Rightarrow$ $mgd\sin\theta + (-Fd) + 0 = K_B - K_A$

but $K_A = 10\text{ J}$ (given)

$$\Rightarrow \quad K_B = mgd\sin\theta - Fd + 10$$

$$= (2.0)(9.8)(2.0)\sin 30 - (3)(2) + 10$$

$$= 19.6 - 6 + 10 = 23.6$$

$$\boxed{K_B = 24\text{ J}}$$

Method II (Ch 8 - law of conservation of energy)

$$W_{ext} = \Delta E_{mech} + \Delta E_{th} + \cancel{\Delta E_{int}} \to 0$$

Choose block + incline + earth as our system.

W (~~work~~ done by ~~all~~ net external force) $= 0$ ~~No external forces.~~

as there are no forces external to the system we have chosen.

$$\Delta E_{mech} = \Delta K + \Delta U$$

$$= \cancel{\Delta K + \Delta U +}$$

& $\Delta E_{th} = 0$ as there ~~is~~ no friction

$$\Rightarrow \quad 0 = (\Delta K + \Delta U) + 0 + 0$$

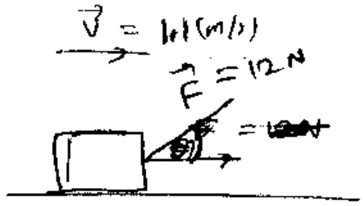
$$\cancel{0 = \frac{1}{2} m (v_f^2 - v_i^2)}$$

$$0 = (K_B - K_A) + mg\underbrace{(y_B - y_A)}$$

$$0 = (K_B - 10) + mg(-d \sin\theta)$$

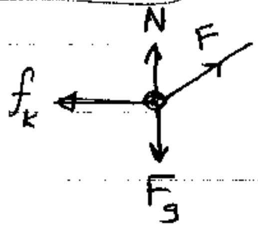
$$\boxed{K_B = 24\,J}$$

Q4.



$$\boxed{W_{net} = \Delta K}$$

FBD



let's first find the work done by friction, W_f. by definition

$$W_f = \vec{f} \cdot \vec{d} = (\mu N)\, d \cos(180)$$
$$= -\mu N d$$

but μ is not given! So it is clear we have to find W_f using an equation (a law or theorem). let's use Work - ~~KE~~ theorem $W_{net} = \Delta K$ on the block as it moves say a distance 'd' horizontally

$$\boxed{W_f + W_g + W_F + W_N = \Delta K} \Longleftarrow W_{net} = \Delta K$$

$$W_f + 0 + F d \cos\theta + 0 = 0$$

Now why $\Delta K = 0$? because the block moves with const velocity $V_f = V_i$!

$$W_f = - F d \cos\theta$$

~~He~~ let us say the block moves the distance 'd' in ~~Δt~~ time Δt. Then the rate at which the work done by f is

$$P = \frac{W_f}{\Delta t}$$

$$= -F\left(\frac{d}{\Delta t}\right)\cos\theta$$

but $d/\Delta t = v = 1.1$ m/s

$$\therefore \quad P = -(12)(1.1)\cos 30$$

$$P = -11.4\,(J/s) \approx W$$

$$\boxed{P = -11.4 \text{ W}}$$

Q5.

$E = K + U$

$$\boxed{W = \Delta E_{mech} + \Delta E_{th} + \cancel{\Delta E_{int}} \rightarrow 0}$$

$0 = \Delta E + \Delta E_{th}$

$0 = \Delta K + \Delta U + \Delta E_{th}$

$\Delta E_{th} = -(W_{nc}) = +40$

$\Delta U = -(W_c) = -60$

or you can consider the non-conservative force as external in such case $W = -40J$, $\Delta E_{th} = 0$

$\Rightarrow \quad \Delta K - 60 + 40 = 0$

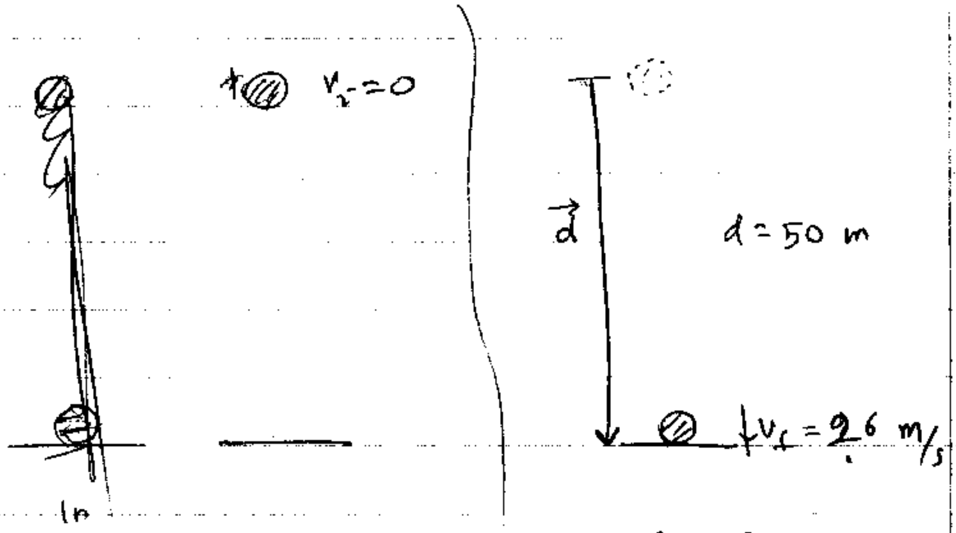
$$\boxed{\Delta K = +20}$$

$\Delta E = \Delta K + \Delta U = 20 + (-60) = -40$

$$\boxed{\Delta E = -40}$$

~~A is~~ so $\boxed{K \uparrow \ \& \ E \downarrow}$

Q6.



initial final

system $\longrightarrow$ (object + earth)

$W = \Delta E_{mech} + \Delta E_{th} + \cancel{\Delta E_{int}} \rightarrow 0$

$0 = \Delta K + \Delta U + \Delta E_{th}$

$$0 = \frac{1}{2}(10)\left(26^2 - 0^2\right) + mg(-50) + \Delta E_{th}$$

$$\Delta E_{th} = (10)(9.8)(50) - 5(26)^2$$

$$= 4900 - 3380$$

$$\Delta E_{th} = +1520 \text{ J}$$

~~ΔE_{th} is really the energy transferred~~

$$W(\text{air resistance}) = -\Delta E_{th}$$

$$\boxed{W_{ar} = -1500 \text{ J}}$$ for two sig digits.

Q7.