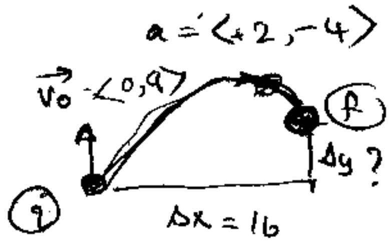
Exam 022 - Solutions

Q10 → Q13

Q10

list	x	y
Δr	✓ $\Delta x = 16$ (m)	$\Delta y = ?$
v_0	✓ $v_{x0} = 0$	✓ $v_{y0} = 9.0$
v	$v_x = ?$	$v_y = ?$
a	✓ $a_x = 2.0$	✓ $a_y = -4.0$
t	? t	$= t$?



We know 3 quantities ~~for~~ in the x-direction & only 2 " in the y-direction. However, note "t" is common for both directions.

x// find v_x

$$v_x^2 = v_{x0}^2 + 2a_x(\Delta x)$$

$$v_x^2 = 0 + 2(2)(16)$$

$$\boxed{v_x = 8 \text{ m/s}}$$

let us find t also, and take it the y-direction.

$$v_x = v_{x0} + a_x t \Rightarrow t = \frac{v_x - v_{x0}}{a_x} = \frac{8-0}{2} = 4.0 \text{ (s)}$$

y- direct

$$v_y = v_{y0} + a_y t$$

$$= 9 + (-4)(4)$$

$$= 9 - 16 = -7$$

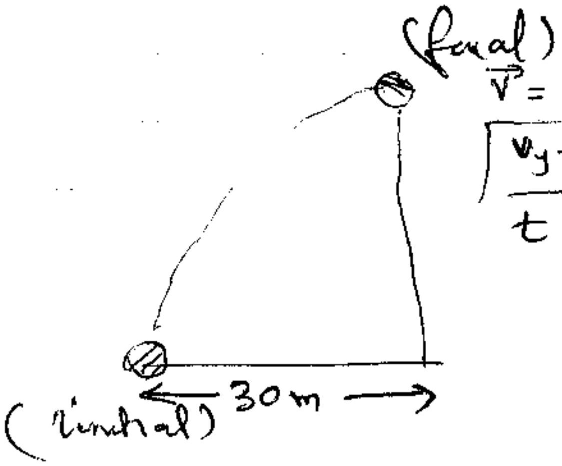
$$\boxed{v_y = -7 \ m/s}$$

$$\Rightarrow \quad \vec{v} = \langle v_x , v_y \rangle$$

$$= \langle 8, -7 \rangle$$

$$\boxed{\vec{v} = 8\hat{i} - 7\hat{j} \ (m/s)}$$

Q11



$\vec{v} = \langle v_x , 0 \rangle$

$\boxed{v_y = 0 \text{ , } \because \text{ maximum height}}$

$t = 3.0 \ s$

$\Rightarrow$

	x	y
Δr	✓ $\Delta x = 30$	$\Delta y = ?$
v_0	$v_{x0} = ?$	$v_{y0} = ?$
v	$v_x = ?$	✓ $v_y = 0$
a	✓ $a_x = 0$	✓ -9.8
t	✓ 3.0	= ✓ 3.0

$$\vec{v}_0 = \langle v_{x0}, v_{y0} \rangle = ?$$

x-dir

$$\Delta x = v_{x0} t + \cancel{\tfrac{1}{2} a_x t^2}$$

$$v_{x0} = \frac{\Delta x}{t} = \frac{30}{3.0} = \cancel{40 \text{ m/s}}$$

$$\boxed{v_{x0} = 10 \text{ m/s}}$$

y-dir

$$v_y = v_{y0} + a_y t$$

$$0 = v_{y0} + (-9.8)(3.0)$$

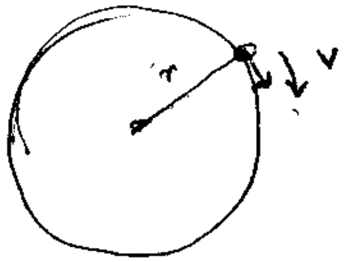
$$\boxed{v_{y0} = -29.4 \text{ m/s}}$$

$$\vec{v}_0 = \langle 10, -29.4 \rangle$$

$$v_0 = \sqrt{10^2 + (-29.4)^2}$$

$$\boxed{v_0 = 31 \text{ m/s}}$$

(Q12)



$a_r = \frac{v^2}{r}$ $r = 15$ m

$v = ?$

We know $v = \frac{2\pi r}{T}$ for uniform circular motion

& ~~pe~~ it is given $5T = 1$ (min)

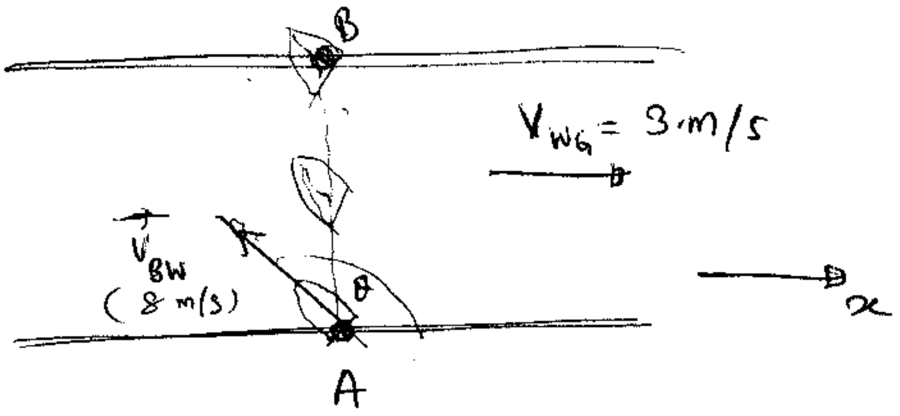
$T = \frac{60}{5}$ (s) $= 12$ s

$\Rightarrow$ $v = \frac{2\pi(15)}{12} = 7.85$ m/s

Now $a_r = \frac{(7.85)^2}{15}$

$\boxed{a_r = 4.1 \text{ m/s}^2}$

(Q13)



The velocity of the boat (B) is being measured in two frames; the ground (G) & ~~coal~~ river or water (W)

V_{BG} = velocity of boat relative to ground

V_{BW} = velocity of boat relative to water

In the lectures remember the equation

$$\vec{V}_{PB} = \vec{V}_{PA} + \vec{V}_{AB}$$

for this question P is the boat → B

frame A is ground → G

frame B is water → W

So $\vec{V}_{BW} = \vec{V}_{BG} + \vec{V}_{GW}$

but $\vec{V}_{GW} = -\vec{V}_{WG} =$

$$\vec{V}_{BW} = \vec{V}_{BG} - \vec{V}_{WG} \qquad (\vec{v'} = \vec{v} - \vec{u})$$

let us now take each of those 3 velocities one at a time & look at it

V_{BW}

$|\vec{V}_{BW}| = 8$ m/s & the direction is the unknown we have to find. We call it θ (with the east.

If we choose East - +ve x-axis & North - +ve y-axis

$$\vec{V}_{BW} = 8\,(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) \quad (m/s)$$

$$= 8\cos\theta\,\hat{i} + 8\sin\theta\,\hat{j} \quad (m/s)$$

$$\vec{V}_{BW} = \langle \underbrace{8\cos\theta}_{(V_{BW})_x},\ \underbrace{8\sin\theta}_{(V_{BW})_y} \rangle \quad (m/s)$$

V_{BG} Relative to the ground observer the boat starts from A and heads directly ~~northward~~. NORTH.

⇒ direction is north & magnitude unknown. let us denote the magnitude as v $\{|V_{BG}| = v\}$

then $\vec{V}_{BG} = v\,\hat{j}$ (m/s) $= \langle 0, v \rangle$ (m/s)

V_{WG}

$V_{WG} = 3\,\hat{i}$ (m/s) $= \langle 3, 0 \rangle$ m/s

Put them back into the equation

$$\underbrace{\vec{V}_{BW}}_{8} = \underbrace{\vec{V}_{BG}} - \underbrace{\vec{V}_{WG}}$$

$$(8\cos\theta\,\hat{i} + 8\sin\theta\,\hat{j}) = (v\,\hat{j}) - (3\,\hat{i})$$

Equating the x-components & the y-components separately:

$$8\cos\theta = -3 \quad \text{——} (1)$$

& $$8\sin\theta = v \quad \text{——} (2)$$

We are asked to find only θ

$$\cos\theta = -3/8$$

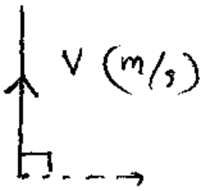
$$\boxed{\theta = 112^\circ}$$

Another method (Geometrically)

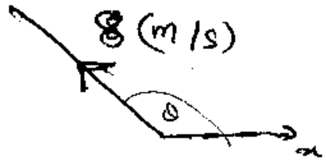
$$\vec{V}_{BW} = \vec{V}_{BG} - \vec{V}_{WG}$$

$$\vec{V}_{BG} = \vec{V}_{BW} + \vec{V}_{WG}$$

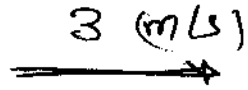
$\vec{V}_{BG} =$

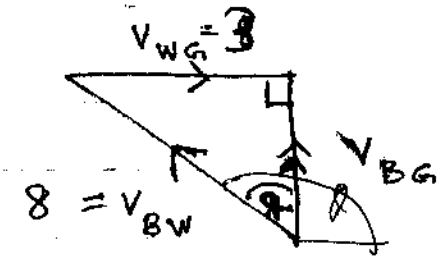


$\vec{V}_{BW} =$



$\vec{V}_{WG} =$





from the figure $\sin\alpha = 3/8$

$$\alpha = 22^\circ$$

but $\theta = 90 + \alpha = 90 + 22 = \underline{\underline{112^\circ}}$