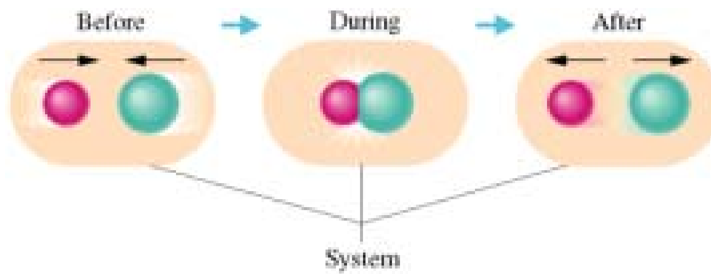
Collisions – Chapter 10

1. What is a Collision?

A **collision** is an isolated event in which two or more bodies (the colliding bodies) exert relatively strong forces on each other for a relatively short time.



2. Impulse and Linear Momentum

$$\vec{F} = d\,\vec{p}/d\,t$$

$$\Rightarrow\ d\,\vec{p} = \vec{F}(t)\,d\,t$$

$$\underbrace{\int_{\vec{p}_i}^{\vec{p}_f} d\,\vec{p}}_{\downarrow} = \int_{t_i}^{t_f} \vec{F}(t)\,d\,t$$

$$\vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}(t)\,d\,t$$

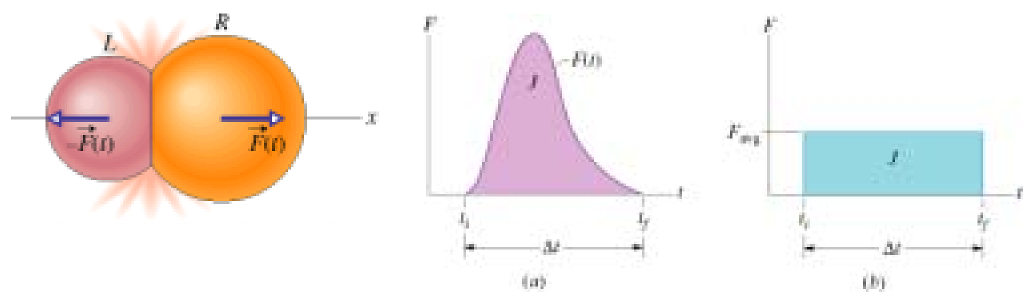
Definition of $\vec{J}$, Impulse

$$J = \int_{t_i}^{t_f} \vec{F}(t)\,d\,t$$

$$\Rightarrow\ \Delta\vec{p} = \vec{J}$$

impulse - linear momentum theorem

Single collision



Two particle-like bodies L and R collide with each other. During the collision, body L exert force $\vec{F}(t)$ on body R, and body R exerts force $-\vec{F}(t)$ on body L; a third-law force pair. Their magnitude vary with time during the collision.

J is the area under the curve of F vs t. If F_{avg} is the average magnitude of the varying force $F(t)$ such that it gives the same area (see graph (b)) for the same duration of the collision, then impulse can be equated to: $J = F_{avg}\,\Delta t$

- The impulse – momentum theory in component form:

$$p_{fx} - p_{ix} = \Delta p_x = J_x$$

$$p_{fy} - p_{iy} = \Delta p_y = J_y$$

$$p_{fz} - p_{iz} = \Delta p_z = J_z$$

- **Series of collision**

Impulse on target = - impulse on ***n*** projectiles

$J = -n\,\Delta p$ where Δp is the momentum change for each projectile.

$$F_{avg} = \frac{J}{\Delta t} = -\frac{n}{\Delta t}\Delta p = -\frac{n}{\Delta t} m \Delta v$$

- If the projectiles stop after impact, then $\Delta v = v_f - v_i = 0 - v = -v$
- If the projectiles bounce backward, with $v_f = -v$, then $\Delta v = v_f - v_i = -v - v = -2v$

In the time interval Δt, an amount of mass $nm = \Delta m$ collides with the target, therefore:

$$F_{avg} = -\frac{\Delta m}{\Delta t}\Delta v$$

3. <u>Momentum and Kinetic Energy in Collisions</u>

In the discussion that follows we consider **closed** (no mass enters or leaves them) and **isolated** (no net external forces act on the bodies within the system) **systems**.

- If the total kinetic energy of the system of two colliding bodies is unchanged by a collision, then the kinetic energy of the system is *conserved*. Such a collision is called an **elastic collision.**

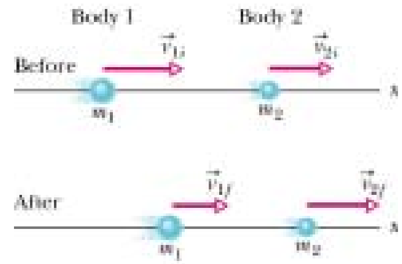
- If the kinetic energy of the system is *not* conserved, such a collision is called an **inelastic collision.** Most collisions are inelastic. A **completely inelastic collision** is where the colliding bodies stick together after collision.

- **The total linear momentum,** $\vec{P}$ of a closed, isolated system is always *conserved* in **any collision** whether the collision is elastic or inelastic. ***Conservation of Linear Momentum for Collision***

4. Inelastic Collision in One Dimension

Conservation of Linear Momentum

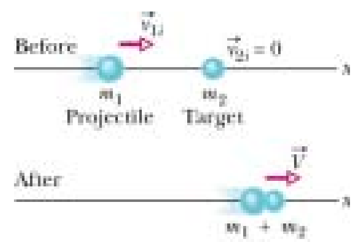
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$



Total Linear Momentum is conserved even when the collision is inelastic, in a ***closed***, ***isolated*** *system*

Completely Inelastic Collision

$$m_1 v_{1i} = (m_1 + m_2)V$$
$$\Rightarrow \quad V = \frac{m_1}{m_1 + m_2} v_{1i}$$

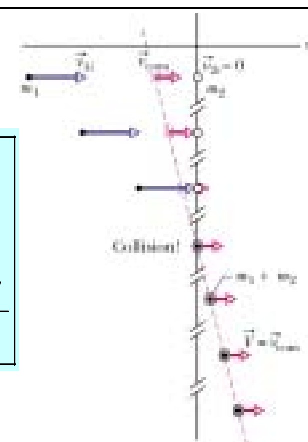


Special case in which $v_{2i} = 0$ *and the two bodies* ***stick together*** *after the collision.*
Note that the velocity ***V*** *after the collision depends on the ratio of* m_1 *and* $(m_1 + m_2)$.

Velocity of Center of Mass

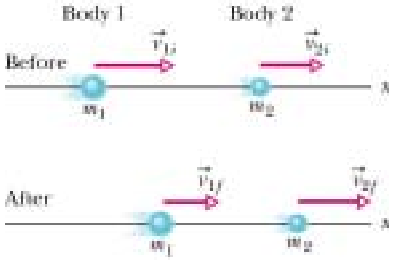
$$\vec{P} = M\,\vec{v}_{com} = (m_1 + m_2)\vec{v}_{com}$$

$$\Rightarrow \qquad \vec{v}_{com} = \frac{\vec{P}}{m_1 + m_2} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2} = \frac{\vec{p}_{1f} + \vec{p}_{2f}}{m_1 + m_2}$$



The right side of the equation is a constant before and after the collision. Therefore, $\vec{v}_{com}$ *does not change in a* ***closed***, ***isolated*** *system. There is no net external force to change its total momentum as* $F_{net} = d\vec{P}/dt$

5. Elastic Collision in One Dimension



In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change. The total KE is also conserved in addition to the total momentum.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\tfrac{1}{2} m_1 v_{1i}{}^2 + \tfrac{1}{2} m_2 v_{2i}{}^2 = \tfrac{1}{2} m_1 v_{1f}{}^2 + \tfrac{1}{2} m_2 v_{2f}{}^2$$

$m_1(v_{1i} - v_{1f}) = -m_2(v_{2i} - v_{2f})$	$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = -m_2(v_{2i} - v_{2f})(v_{2i} + v_{2f})$
$(v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$	

Moving Target

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

If $\boldsymbol{m_1 = m_2}$ $v_{1f} = v_{2i}$ and $v_{2f} = v_{1i}$

Stationary Target $(v_{2i} = 0)$

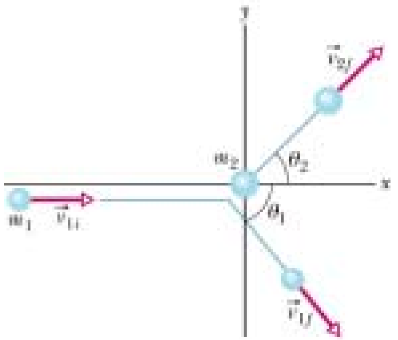
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \quad ; \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

In the special case that $\boldsymbol{m_1}$ is much, much larger than $\boldsymbol{m_2}$:

$$v_{1f} \approx v_{1i} \quad \text{and} \quad v_{2f} \approx 2v_{1i}$$

6. Collision in Two Dimension

Special case when target body is at rest before the collision: **(v_{2i}= 0)**



$$\vec{P}_{1i}+\vec{P}_{2i}=\vec{P}_{1f}+\vec{P}_{2f} \quad\rightarrow\quad \begin{cases} m_1 v_{1i}=m_1 v_{1f}\cos\theta_1+m_2 v_{2f}\cos\theta_2 \\ 0=-m_1 v_{1f}\sin\theta_1+m_2 v_{2f}\sin\theta_2 \end{cases}$$

$$K_{1i}+K_{2i}=K_{1f}+K_{2f} \quad\rightarrow\quad \tfrac{1}{2}m_1 v_{1i}{}^2=\tfrac{1}{2}m_1 v_{1f}{}^2+\tfrac{1}{2}m_2 v_{2f}{}^2$$

← *This is true only if the collision is ELASTIC*

In the special case of ELASTIC collision and $m_1 = m_2$, it can be proved that $\boldsymbol{\theta}_1+\boldsymbol{\theta}_2=90^{o}$