

Potential Energy and Conservation of Energy – Chapter 8

1. Potential Energy

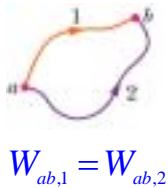
- **Potential Energy U** is a form of stored energy that can be associated with the configuration (or arrangement) of a *system of objects* that exert *certain types* of forces (*conservative*) on one another.
- Forms of potential energy:
 - *Gravitational* – associated with the gravitational attraction between objects, e.g. an apple and the earth
 - *Elastic* - associated with Hooke's law e.g. stretched or compressed springs
- We know that the work done result in a change in kinetic energy ($W=\Delta K$). Now we can ask the question: where did the kinetic energy go (if it is decreased) or where did it come from (if it increased)! Note that the force only function as the agent which rearranges the configuration of the system (by displacing one or more of the object in the system). Assuming that our system is *isolated* (no external force acting on it) the answer, as you have already guessed, is to (or from) the *potential energy of the system..*
- When one of these special forces (let us label it F_c) does some work (W_c) by changing the system configuration, the force derives the energy from the stored potential energy associated with that force:

$\Delta U_c = -W_c$	$W > 0 \Rightarrow U$ decreases
	$W < 0 \Rightarrow U$ increases

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2. Conservative and Nonconservative forces

- Let us say the work done by a force F when a system is changed from configuration 1 to configuration 2 is W_{12} . We now reverse the process, i.e. take the system back to 1 from 2 and let us say that we measure the work done by the same force now to be W_{21} . It is obvious that we can define a potential energy for this force by the equation $\Delta U = -W$ only if $W_{12} = -W_{21}$.
- A force for which $W_{12} = -W_{21}$ is called a conservative forces. This is same as saying that the net work done by a conservative force around any closed path (return back to the initial configuration) is zero. A force that is not conservative is called a nonconservative force. We cannot define potential energy associated with a nonconservative forces.
- The gravitational force and the spring force are examples of conservative forces. The frictional force and fluid drag force are examples of nonconservative forces.
- The work done by a conservative force on a particle does not depend by the path taken.



path independance of conservative forces

The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

*Explain the proof &
DO SP 8-1*

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3. Determining Potential Energy Values

- In general we can write:
$$\Delta U = -W = -\int_{x_i}^{x_f} F_x dx - \int_{y_i}^{y_f} F_y dy - \int_{z_i}^{z_f} F_z dz$$

Gravitational potential energy, ΔU_g

$$\begin{aligned}\Delta U_g &= -W_g = -\int_{y_i}^{y_f} F_g dy = -\int_{y_i}^{y_f} (-mg) dy \quad \because \vec{F}_g \text{ has only } y\text{-component: } (-mg)\hat{j} \\ &= mg[y]_{y_i}^{y_f}\end{aligned}$$

$$\Rightarrow \boxed{\Delta U_g = mg(y_f - y_i) = mg \Delta y}$$

Rewrite U_f as U and y_f as y and take U_i to be the reference level and assign the values $U_i=0$ and $y_i=0$.

$$\Rightarrow \boxed{U(y) = mg y}$$

DO SP 8-2

Elastic Potential Energy, ΔU_s

$$\boxed{\Delta U_s = -W_s = \frac{1}{2}k(x_f^2 - x_i^2)}$$

Choosing the reference level $U_i=0$, when the spring is at its relaxed length and the block is at $x_i=0$

$$\Rightarrow \boxed{U(x) = \frac{1}{2}kx^2}$$

4. Conservation of Mechanical Energy

- The **mechanical energy** E_{mec} of a system is the sum of its potential energy and the kinetic energy K of the objects within it:

$$E_{mec} = K + U \quad (\text{mechanical energy})$$

- Consider an isolated system, i.e. no external force causes energy changes inside the system. When a conservative force does work on an object within the system, it transfers energy between kinetic energy K of the object and potential energy U of the system.

from the two equations:

$$\Delta K = W \quad \text{and} \quad \Delta U = -W \quad \Rightarrow \quad \begin{aligned} \Delta K &= -\Delta U \\ \Delta K + \Delta U &= 0 \end{aligned}$$

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

or

$$K_2 + U_2 = K_1 + U_1$$

Principle of Conservation of mechanical energy

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum the mechanical energy E_{mec} of the system cannot change

6. Work done on a System by an External Force

- When we stated the conservation of mechanical energy for a system in the previous section, we specified two conditions:

- *Isolated system (no external forces)*
- *Only conservative forces* in the system.

- Let us now introduce *external forces* doing work on the system, then:

$$W_{ex} = \underbrace{\Delta K + \Delta U}_{\downarrow}$$
$$W_{ex} = \Delta E_{mec} \quad (\text{work done on the system, no friction involved})$$

- And also add *nonconservative forces* (friction involved) in the system:

$$W_{ex} = \Delta E_{mec} + \Delta E_{th}$$

(work done on the system, friction involved)

$$E_{th} = f_k d = \mu_k N d$$

(increase in thermal energy by sliding)

7. Conservation of Energy

- The total energy of a system can change only by amounts of energy W_{ex} that are transferred to or from the system.

$$W_{ex} = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$$

- ΔE_{int} acknowledges the fact that thermal energy is not the only other form of energy that a system can have which is not mechanical energy, e.g. chemical energy in your muscles or in a battery, or nuclear energy.
- The total energy of an *isolated* system cannot change.

$$\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0$$

- Power as the rate at which energy is transferred from one form to another

$$P_{avg} = \frac{\Delta E}{\Delta t}$$

Average power

$$P = \frac{dE}{dt}$$

Instantaneous power

The rate at which the work is done is a special case of the rate at which energy is being transferred to (or from) kinetic energy (one form of energy) from (or to) other forms of energy.