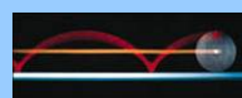
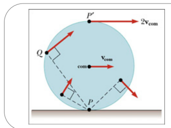
Rolling, Torque and Angular Momentum





Torque:

$\vec{\tau} = \vec{r} \times \vec{F}$

Smooth Rolling (no slipping):

$v_{com} = R\,\omega$

$a_{com} = R\,\alpha$

$K_{rolling} = \frac{1}{2} I_{com}\,\omega^2 + \frac{1}{2} M v_{com}^{2}$

Conservation of Mechanical Energy for Rolling down a ramp:

$\Delta K_R + \Delta K_T + \Delta U + \cancel{\Delta E_{th}} = \cancel{W}$

$\frac{1}{2} I_{com}\left(\omega_f^2 - \omega_i^2\right) + \frac{1}{2} m\left(v_f^2 - v_i^2\right) + mg\,\Delta y = 0$

$\frac{1}{2}\beta\left(v_f^2 - v_i^2\right) + \frac{1}{2}\left(v_f^2 - v_i^2\right) + g\,\Delta y = 0$

Rolling down a ramp:

$$a_{com} = \frac{g\sin\theta}{1 + I_{com}/M R^2}$$

$$= \frac{g\sin\theta}{1+\beta} \quad \text{where } \beta = {I_{com}}/{MR^2} = \begin{matrix} 1 & \text{for ring} \\ 1/2 & \text{for cylinder} \\ 2/5 & \text{for sphere} \end{matrix}$$

Newton's Second Law in Angular Form:

$\vec{\tau}_{net} = \frac{d\,\vec{l}}{d\,t}$ single particle

$\vec{\tau}_{net} = \frac{d\,\vec{L}}{d\,t}$ system of particle

Angular momentum of a particle:

$\vec{l} = \vec{r} \times \vec{p}$

$= m\,(\vec{r} \times \vec{v})$

magnitude:

$l = r\,p_{\perp} = r\,m v_{\perp}$

$l = r_{\perp} p = r_{\perp} m v$

Conservation of angular momentum (when τ_{net}=0)

$\vec{L}$ = constant

$\vec{L}_i = \vec{L}_f$ = constant

$I_i\,\omega_i = I_f\,\omega_f$ = constant

Angular momentum of a rigid object rotating about a fixed axis:

$L_z = I \cdot \omega$

Dr. Kariapper