Chapter 4 Homework Solution

1. A particle starts from the origin at t=0 with a velocity of 8.0 j m/s and moves in the xy plane with constant acceleration (4.0 i+ 2.0 j) m/s^2. When the particle x-coordinate is 29 m, what is its y coordinate? Give the answers to three significant figures.

Solution:
Since the *x* and *y* components of the acceleration are constants, we can use Table 2-1 (textbook) for the motion along both axes. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr). Where units are not shown, SI units are to be understood.

(a) Since $\vec{r}_0 = 0$, the position vector of the particle is (adapting Eq. 2-15 refer to textbook)

$$\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = \left(8.0\,\hat{\jmath}\right)t + \frac{1}{2}\left(4.0\,\hat{\imath} + 2.0\,\hat{\jmath}\right)t^2 = \left(2.0t^2\right)\hat{\imath} + \left(8.0t + 1.0t^2\right)\hat{\jmath}.$$

Therefore, we find when x = 29 m, by solving $2.0t^2 = 29$, which leads to t = 3.8 s. The y coordinate at that time is $y = 8.0(3.8) + 1.0(3.8)^2 = \boxed{45.0}$ m.

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2. A moderate wind accelerates a pebble over a horizontal xy plane with a constant acceleration a =(5.00 m/s^2) i + (7.00 m/s^2) j . At time t=0, the velocity is 4.00 (m/s) i. What is magnitude of its velocity when it has been displaced by 12.0 m parallel to x-axis? Give three significant figures.

Solution

We find t by solving $\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$:

$$12.0 = (4.00)t + \frac{1}{2}(5.00)t^2$$

where Δx = 12.0 m, v_x = 4.00 m/s, and $a_x = 5.00\ m/s^2$. We use the quadratic formula and find t = 1.53 s. Then, Eq. 2-11 (actually, its analog in two dimensions) applies with this value of t. Therefore, its velocity (when Δx = 12.00 m) is

$$\begin{aligned}\vec{v} = \vec{v}_0 + \vec{a}t &= (4.00\text{ m/s})\hat{\imath} + (5.00\text{ m/s}^2)(1.53\text{ s})\hat{\imath} + (7.00\text{ m/s}^2)(1.53\text{ s})\hat{\jmath} \\ &= (11.7\text{ m/s})\,\hat{\imath} + (10.7\text{ m/s})\,\hat{\jmath}.\end{aligned}$$

Thus, the magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{(11.7)^2 + (10.7)^2} = \boxed{15.8}$ m/s.

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3. A cart is propelled over an xy plane with an acceleration a_x = - 2.00 m/s^2 and a_y = 1.00 m/s^2 . Its velocity has components v_{ox}= 11.2 m/s and v_{oy}= 12.0 m/s. What is magnitude of cart's velocity when it

reaches its greatest x-coordinate? *Give three significant figures.*

Solution

We find t by applying Eq. 2-11 to motion along the x axis (with $v_x = 0$ characterizing $x = x_{max}$): $0 = (11.2 \text{ m/s}) + (-2.0 \text{ m/s}^2)t \Rightarrow t = 5.6$ s. Then, Eq. 2-11 applies to motion along the y axis to determine the answer: $v_y = (12.0 \text{ m/s}) + (1.00 \text{ m/s}^2)(5.6 \text{ s}) = 17.6$ m/s. Therefore, the velocity of the cart, when it reaches $x = x_{max}$, is $\sqrt{v_x^{\ 2} + v_y^{\ 2}} = \sqrt{0^2 + 17.6^2} = \boxed{17.6}\ m/s$

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4. A ball is shot from ground into the air. At a height of 9.1 m, its velocity is v=(7.6 i + 6.1 j), with i horizontal and j upward. What total horizontal distance the ball travel? *Give two significant figures*

We designate the given velocity $\vec{v} = 7.6\,\hat{\text{i}} + 6.1\,\hat{\text{j}}$ (SI units understood) as $\vec{v}_1 -$ as opposed to the velocity when it reaches the max height $\vec{v}_2$ or the velocity when it returns to the ground $\vec{v}_3 -$ and take $\vec{v}_0$ as the launch velocity, as usual. The origin is at its launch point on the ground.

First find the initial y velocity, that is how we will proceed. Using Eq. 2-16, we have

$$v_{1\,y}^2 = v_{0y}^2 - 2g\Delta y \;\Rightarrow\; (6.1)^2 = v_{0\,y}^2 - 2(9.8)(9.1)$$

which yields $v_{0\,y} = 14.7$ m/s. And that $v_{0x} = v_{1x} = 7.6$ m/s (because a_x=0)

Now take the motion from initial (shooting the ball up from the ground) to final (returning to the ground). For this motion Δy= 0, and substituting data into the equation $\Delta\vec{r} = \vec{v}_o t + \frac{1}{2}\vec{a}\,t^2$ in both x and y direction, we have

$$\begin{aligned} 0 &= v_{0y}t - \frac{1}{2}gt^2 \\ R &= v_{0x}t \end{aligned}$$

which leads to $R = 2v_{0x}v_{0y}/g$. We plug in values and obtain $R = 2(7.6)(14.7)/9.8 = 23$ m.

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5. A plane **diving** with constant speed at an angle of 53.0 degrees with the vertical (an angle of (90 - 53.0) **below** the horizontal x-axis), releases a projectile at an altitude of 730 m. The projectile hit the ground 5.00 s later. What is the angle of the projectile velocity with the x-axis when it hit the ground? Give three significant figures and to match the answer stored in the computer, please provide the answer with a negative sign - measured below the clockwise.

Solution

We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_0 = -37.0°$ for the angle measured from $+x$, since the angle given in the problem is measured from the $-y$ direction. We note that the initial speed of the projectile is the plane's speed at the moment of release.

We use Eq. 4-22 to find v_0 (SI units are understood).

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \quad \Rightarrow \quad 0 - 730 = v_0 \sin(-37.0°)(5.00) - \frac{1}{2}(9.80)(5.00)^2$$

which yields v_0 = 202 m/s.

The x component of the velocity (just before impact) is

$$v_x = v_0 \cos\theta_0 = (202)\cos(-37.0°) = 161 \text{ m/s}.$$

The y component of the velocity (just before impact) is

$$v_y = v_0 \sin\theta_0 - gt = (202)\sin(-37.0°) - (9.80)(5.00) = -171 \text{ m/s}.$$

The angle is $\tan^{-1}(vy/vx)=\tan^{-1}(-171/161)=-46.7°$

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6. A woman rides a carnival Ferris wheel with radius 15.0 m, completing six revolutions about its horizontal axis turns every minute. What is magnitude of her centripetal acceleration at her highest point? Give the answer to three significant figures.

Since the wheel completes 6 turns each minute, its period is one-sixth of a minute, or 10 s.

The magnitude of the centripetal acceleration is given by $a = v^2/R$, where R is the radius of the wheel, and v is the speed of the passenger. Since the passenger goes a distance $2\pi R$ for each revolution, his speed is

$$v = \frac{2\pi(15\text{m})}{10\text{s}} = 9.4248\text{m/s}$$

and his centripetal acceleration is

$$a = \frac{(9.4248\text{ m/s})^2}{15\text{m}} = 5.92\text{m/s}^2.$$

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7. A particle moves horizontally in uniform circular motion over a xy plane. At one instant, it moves through the point at coordinates (4.00 m, 4.00 m) with a velocity of -5.00 **i** m/s and an acceleration of +12.5 **j** m/s^2. What is y coordinates of the center of the circular path? Give three significant figures.

<u>Solution</u>
When traveling in circular motion with constant speed, the instantaneous acceleration vector necessarily points towards the center. Thus, the center is "straight up" from the cited point.

Since the center is "straight up" from (4.00 m, 4.00 m), the *x* coordinate of the center is 4.00 m.

To find out "how far up" we need to know the radius. Using Eq. 4-34 we find

$$r = \frac{v^2}{a} = \frac{5.00^2}{12.5} = 2.00 \text{ m.}$$

Thus, the *y* coordinate of the center is 2.00 + 4.00 = $\boxed{6.00}$ m. Thus, the center may be written as (*x*, *y*) = (4.00 m, 6.00 m).

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8. A person walks up a stalled 14.4-m long escalator in 90 s. When standing on the same escalator, now moving, the person is carried up in 60 s. How long would it take the person to walkup the moving escalator?

When the escalator is stalled the speed of the person is $v_p = \ell/t$, where ℓ is the length of the escalator and *t* is the time the person takes to walk up it. This is v_p = (15 m)/(90 s) = 0.167 m/s. The escalator moves at v_e = (15 m)/(60 s) = 0.250 m/s. The speed of the person walking up the moving escalator is $v = v_p + v_e$ = 0.167 m/s + 0.250 m/s =
0.417 m/s and the time taken to move the length of the escalator is

$$t = \ell / v = (15 \text{ m})/(0.417 \text{ m/s}) = \boxed{36} \text{ s.}$$

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9. A train travels due south at 30 m/s (relative to the ground) in a rain that is blown towards the south by the wind. The path of each raindrop makes an angle of 70.0 degree with the vertical, as measured by an observer stationary on the ground. An observer on the train, however, sees the drop falls perfectly vertically. Determine the speed of the raindrop relative to the ground. Give three significant figures.

Solution

Take south to be the positive x-direction and downward vertical to be the positive y-direction. With this convention:

$\mathbf{v}_{rt}$ = <0,v_{rt}>; the raindrop has only vertical component relative to train
$\mathbf{v}_{tg}$ = <30,0>;
$\mathbf{v}_{rg}$ = <v_{rg}Cos20,v_{rg}Sin20>; vrg makes an angle of (90-70) with the positive x-axis.

$\mathbf{v}_{rg} = \mathbf{v}_{rt} + \mathbf{v}_{tg}$

Substituting in components form:

<v_{rg} Cos20,v_{rg} Sin20> = <0,v_{rt}> + <30,0>

Equating x and y components separately:

→ v_{rg}Cos20 = 30 & v_{rg}Sin20 = v_{rt}

The first of the equation gives vrg = 30/Cos20 = $\boxed{31.9}$ m/s