Rotation about a fixed axis

Rotational Energy

$K_R = \frac{1}{2} I \omega^2$

where $I = \sum m_i r_i^2$ is called the Moment of Inertia or the Rotational Inertia of the object about the rotation axis

$I \equiv \lim_{\Delta m_i \to 0} \sum_i r_i^2 \cdot \Delta m_i = \int r^2 dm = \int r^2 \rho dV$

Parallel Axis Theorem

$I = I_{com} + M h^2$

Torque

$\tau = r \cdot F \cdot \sin\phi \quad (magnitude)$

$\tau = (r)(F \sin\phi) = r F_t$

$\tau = (r \sin\phi)(F) = r_\perp F$

$\vec{\tau} = \vec{r} \times \vec{F}$

Newton's Second Law for Rotation

$\tau_{net} = I \alpha$

Work done:

$W = \tau \Delta\theta$

Power:

$P = \frac{dW}{dt} = \tau \omega$

Work-Kinetic Energy Theorem for Rotation (about a fixed axis)

$W_{net} = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$

Relating linear variables to angular variables:

$s = r \cdot \theta$

$v = r \cdot \omega$

$a_t = r \cdot \alpha$

Note this is NOT the centripetal (radial) acceleration $a_r = \frac{v^2}{r}$

Angular Variables

angular postion θ:

$\theta = \frac{s}{r} \quad (radian\ measure)$

angular displacement $\Delta\theta$:

$\Delta\theta = \theta - \theta_o$

Average angular speed ω_{avg}:

$\omega_{avg} = \frac{\theta - \theta_o}{t - t_o} = \frac{\Delta\theta}{\Delta t}$

Instantanious angular speed ω:

$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

Average angular acceleration α_{avg} :

$\alpha_{avg} = \frac{\omega - \omega_o}{t - t_o} = \frac{\Delta\omega}{\Delta t}$

Instantaneous angular acceleration α :

$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$

When angular acceleration is constant:

$\omega = \omega_o + \alpha t$ compare with $\Rightarrow$ $v = v_o + at$

$\theta - \theta_o = \frac{1}{2}(\omega + \omega_o)t$ — $x - x_o = \frac{1}{2}(v + v_o)t$

$\theta - \theta_o = \omega_o t + \frac{1}{2}\alpha t^2$ — $x - x_o = v_o t + \frac{1}{2} a t^2$

$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$ — $v^2 = v_o^2 + 2a(x - x_o)$

Dr. M. S. Kariapper