

## Reminder of chapter 4

### Motion in two and three dimension

- 1- The position in three dimensions:  $\vec{r} = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$
- 2- The displacement:  $\Delta \vec{r} = \vec{r}_2 - \vec{r}_1 = \Delta x \mathbf{i} + \Delta y \mathbf{j} + \Delta z \mathbf{k}$
- 3- The average velocity:  $\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t} \mathbf{i} + \frac{\Delta y}{\Delta t} \mathbf{j} + \frac{\Delta z}{\Delta t} \mathbf{k}$
- 4- The instantaneous velocity:  $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = v_x \mathbf{i} + v_y \mathbf{j} + v_z \mathbf{k}$
- 5- The average acceleration:  $\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t}$
- 6- The instantaneous acceleration:  $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x \mathbf{i} + v_y \mathbf{j} + v_z \mathbf{k}) = a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k}$

### Projectile Motion

- 1- The magnitude of initial velocity on X axis, Y axis are respectively are  $v_o \cos \theta_o$ ,  $v_o \sin \theta_o$ , the angle:  $90 > \theta_o \geq 0$
- 2- The horizontal velocity at any time is constant:  $v_{ox} = v_o \cos \theta_o$ .
- 3- The horizontal displacement at any time:  $\Delta x = v_{ox} t = t v_o \cos \theta_o$

4- The horizontal range:  $R = \frac{v_o^2 \sin 2\theta_o}{g}$

5- The maximum horizontal range at  $\theta_o = 45^\circ$ :  $R_{\text{max}} = \frac{v_o^2}{g}$

- 6- The vertical velocity at any time:

$$v_y = v_o \sin \theta_o + a t$$

- 7- The vertical displacement at any time:

$$\Delta y = t v_o \sin \theta_o + \frac{1}{2} a t^2$$

- 8- The vertical velocity ant any displacement:

$$v_y^2 = (v_o \sin \theta_o)^2 + 2 a \Delta y$$

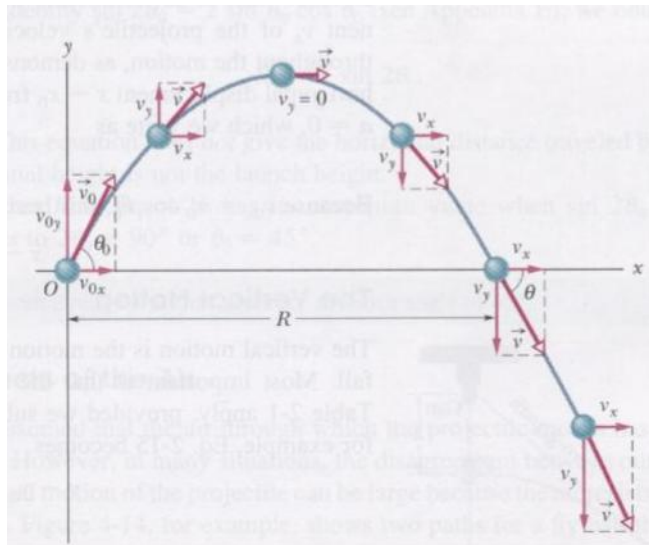
- 9- The vertical displacement ant any vertical

velocity:  $\Delta y = \frac{(v + v_o \sin \theta_o) t}{2}$

- 10- The final velocity:  $v = \pm \sqrt{v_x^2 + v_y^2}$

- 11- The angle of the final velocity respect to the

horizontal:  $\tan \theta = \frac{v_y}{v_x}$

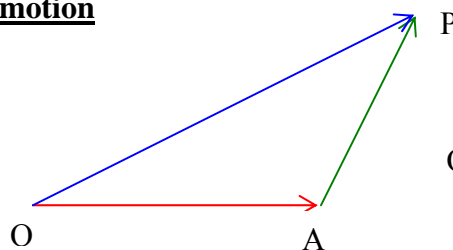


- 12- The equation of the path (parabolic path), assuming  $x_o = y_o = \text{zero}$  is:  $y = x \tan \theta_o + \frac{a x^2}{2 v_o^2 \cos^2 \theta_o}$

**Notice:**  $a = -g = -9.8 \text{ m/s}^2$ .

### Relative motion

- 1- The relative position:  $\vec{r}_{PO} = \vec{r}_{AO} + \vec{r}_{PA}$
- 2- The relative velocity:  $\vec{v}_{PO} = \vec{v}_{AO} + \vec{v}_{PA}$
- 3- The relative acceleration:  $\vec{a}_{PO} = \vec{a}_{AO} + \vec{a}_{PA}$



**Vectors sum**  
 $\vec{OP} = \vec{OA} + \vec{AP}$

### Uniform circular motion

- 1- The magnitude of the radial acceleration of a particle, moving with uniform circular motion (constant speed):  $a_r = \frac{v^2}{r}$
- 2- The periodic time of the motion:  $T = \frac{2\pi r}{v}$

