Chapter 11- Reminder

1- The definition of angular position (**vector**): $\vec{\theta} = \frac{\vec{s}}{r}$ (The SI measurement of it is radian [**rad**])

2- Relationship between revolution, radian, and degree is: **1 rev = 2 π rad = 360 deg**

3- Definition of the angular displacement (**vector**) is: $\Delta\vec{\theta} = \vec{\theta}_2 - \vec{\theta}_1$ (The SI measurement of it is **rad**)

4- Definition of average angular velocity (**vector**) is: $\vec{w}_{avg} = \frac{\Delta\vec{\theta}}{\Delta t} = \frac{\vec{\theta}_2 - \vec{\theta}_1}{t_2 - t_1}$ (The SI measurement of it is **rad/sec**)

5- Instantaneous angular velocity is: $\vec{w} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t} = \frac{d\vec{\theta}}{dt}$

6- Instantaneous angular speed is: $w = |\vec{w}|$

7- Definition of average angular acceleration is: $\vec{\alpha}_{avg} = \frac{\Delta\vec{w}}{\Delta t} = \frac{\vec{w}_2 - \vec{w}_1}{t_2 - t_1}$ (The SI measurement of it is **rad/sec²**)

8- Instantaneous angular acceleration is: $\vec{\alpha} = \lim \frac{\Delta\vec{w}}{\Delta t} = \frac{d\vec{w}}{dt}$

Rotation with constant Angular Acceleration

1- The angular velocity at any time is given by: $w = w_o - \alpha t$

2- The angular displacement at any time is given by: $\Delta\theta = \theta_2 - \theta_1 = w_o t + \frac{1}{2}\alpha t^2$

3- The angular velocity ant any angular displacement is given by: $w^2 = w_o^2 + 2\alpha\Delta\theta$

4- The angular displacement at any angular velocity and time is given by: $\Delta\theta = \theta_2 - \theta_1 = \frac{1}{2}(w + w_o)t$

Comparison between Linear Motion with Constant Acceleration, and Angular Motion with Constant Acceleration

Linear Motion	Angular Motion
$v = v_o + a t$	$w = w_o - \alpha t$
$\Delta s = s_2 - s_1 = v_o t + \frac{1}{2} t^2$	$\Delta\theta = \theta_2 - \theta_1 = w_o t + \frac{1}{2}\alpha t^2$
$v^2 = v_o^2 + 2 a \Delta s$	$w^2 = w_o^2 + 2\alpha\Delta\theta$
$\Delta s = s_2 - s_1 = \frac{1}{2}(v + v_o)t$	$\Delta\theta = \theta_2 - \theta_1 = \frac{1}{2}(w + w_o)t$

Relating the Linear and Angular variables

1- The linear position, and angular: $s = \theta r$ (Radian measure)
2- The linear velocity, and angular velocity: $v = w r$ (Radian measure)
3- The linear tangent acceleration component, and angular acceleration: $a_t = \alpha r$ (Radian measure)
4- The linear radian acceleration, and angular velocity: $a_r = \frac{v^2}{r} = w^2 r$ (Radian measure)
5- For Uniform Circular Motion (**Constant linear Speed**): $T = \frac{2\pi r}{v} = \frac{2\pi}{w}$ (Radian measure)
6- Moment of Inertia of n particle is define as: $I = \sum_{i=1}^{n} m_i r_i^2$ (The SI measurement of it is **Kg.m²**)
7- Moment of Inertia of a Body is define as: $I = \int_{r=a}^{b} r^2\, dm$
8- Parallel-Axis Theorem: $I = I_{com} + M h^2$
9- Torque of force is define as: $\vec{\tau} = \vec{r} \times \vec{F} = r F \sin\theta\, \vec{u}$ (The SI measurement of it is **N.m**)
10- Newton's Second Law for Rotational Motion is: $\vec{\tau}_{net} = I \vec{\alpha}$
11- The Rotational Kinetic Energy is define as: $K = \frac{1}{2} I w^2$ (The SI measurement of it is J)
12- The Work-Kinetic Energy Theorem is: $\Delta K = K_f - K_i = \frac{1}{2} I w_f^2 - \frac{1}{2} I w_i^2 = W$
13- The Work for varied torque is: $W = \int_{\theta=\theta_i}^{\theta_f} \tau\, d\theta$
14- The Work for constant torque is: $W = \tau(\theta_f - \theta_i)$
15- The Instantaneous Power is define as: $P = \frac{dW}{dt} = \tau w$

Comparison between Pure Translation and Pure Rotational Motion

Pure Translation (Fixed Direction)	Pure Rotation (Fixed Direction)
Position s	Angular position θ
Velocity $v = \frac{ds}{dt}$	Angular velocity $w = \frac{d\theta}{dt}$
Acceleration $a = \frac{dv}{dt}$	Angular acceleration $\alpha = \frac{dw}{dt}$
Mass m	Rotational inertia $I = \sum_{i=1}^{n} m_i r_i^2$
Newton's Second Law $F_{net} = m a$	Newton's Second Law $\tau_{net} = I \alpha$
Work $W = \int_{s=s_i}^{s_f} F\, ds$	Work $W = \int_{\theta=\theta_i}^{\theta_f} \tau\, d\theta$
Kinetic Energy $K = \frac{1}{2} m v^2$	Kinetic Energy $K = \frac{1}{2} I w^2$
Work-Kinetic Energy Theorem $W = \Delta K$	Work-Kinetic Energy Theorem $W = \Delta K$
Power (Constant force) $P = F v$	Power (Constant torque) $P = \tau w$