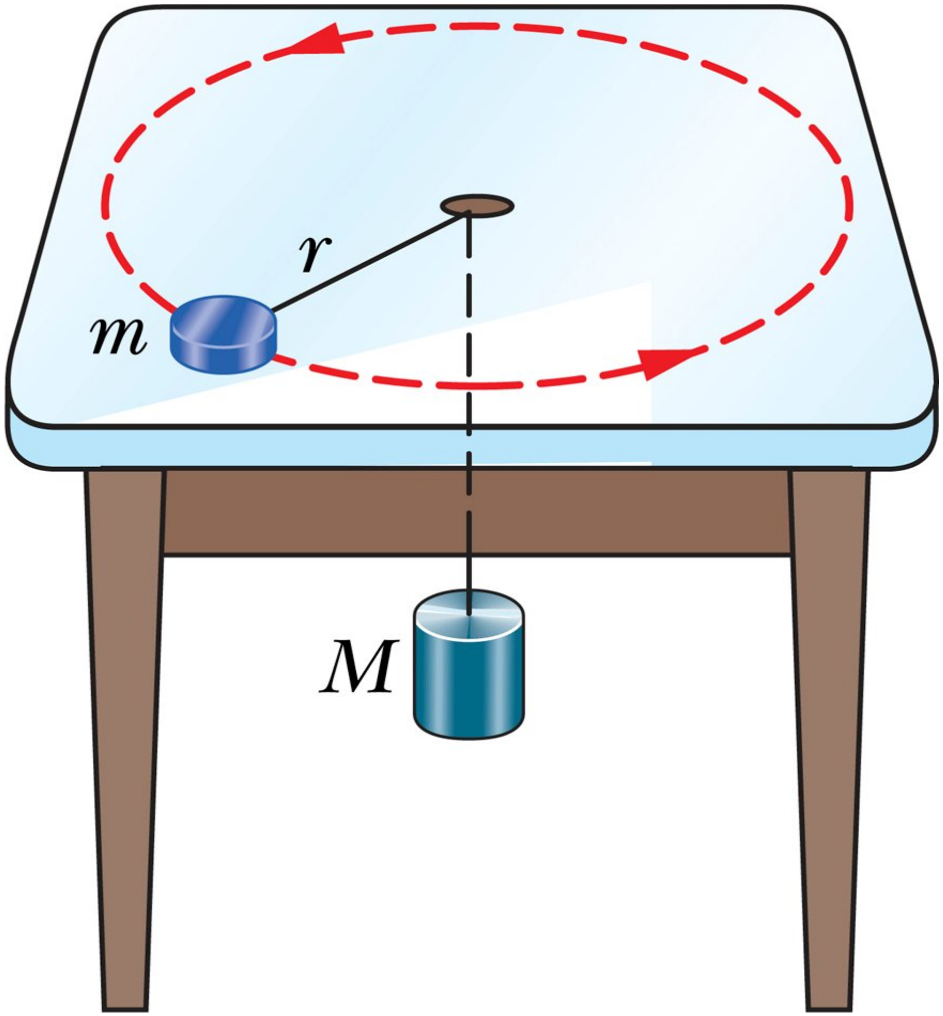
55. A puck of mass m=1.5kg slides in a circle of radius r=20.0cm on a frictionless table while attached to hanging cylinder of mass M=2.5kg by a cord a hole in the table . What speed keeps the cylinder at rest?



55. For the puck to remain at rest the magnitude of the tension force T of the cord must equal the gravitational force Mg on the cylinder. The tension force supplies the centripetal force that keeps the puck in its circular orbit, so $T = mv^2/r$. Thus $Mg = mv^2/r$. We solve for the speed:

$$v = \sqrt{\frac{Mgr}{m}} = \sqrt{\frac{(2.50\ \text{kg})(9.80\ \text{m/s}^2)(0.200\ \text{m})}{1.50\ \text{kg}}} = 1.81\ \text{m/s}.$$

Tarzan (m = 85.0 kg) tries to cross a river by swinging from a vine. The vine is 10.0 m long, and his speed at the bottom of the swing (as he just clears the water) will be 8.00 m/s. Tarzan doesn't know that the vine has a breaking strength of 1 000 N. Does he make it safely across the river?

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Let the tension at the lowest point be T.

$$\sum F = ma: \quad T - mg = ma_c = \frac{mv^2}{r}$$

$$T = m\left(g + \frac{v^2}{r}\right)$$

$$T = (85.0\ \text{kg})\left[9.80\ \text{m/s}^2 + \frac{(8.00\ \text{m/s})^2}{10.0\ \text{m}}\right] = 1.38\ \text{kN} > 1\,000\ \text{N}$$

He doesn't make it across the river because the vine breaks.

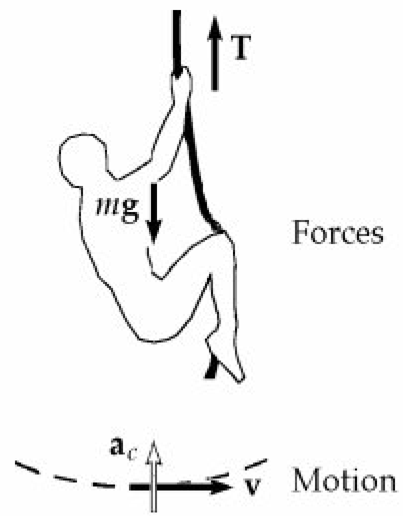

FIG. P6.15

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. Determine the values of n and m and write the simplest form of an equation for the acceleration.

Solution Let us take a to be

$$a = kr^n v^m$$

where k is a dimensionless constant of proportionality. Knowing the dimensions of a, r, and v, we see that the dimensional equation must be

$$\frac{\mathrm{L}}{\mathrm{T}^2} = \mathrm{L}^n \left(\frac{\mathrm{L}}{\mathrm{T}}\right)^m = \frac{\mathrm{L}^{n+m}}{\mathrm{T}^m}$$

This dimensional equation is balanced under the conditions

$$n + m = 1 \qquad \text{and} \qquad m = 2$$

Therefore $n = -1$, and we can write the acceleration expression as

$$a = kr^{-1}v^2 = k\frac{v^2}{r}$$

A particle moves along the x axis. Its position is given by the equation $x = 2 + 3t - 4t^2$ with x in meters and t in seconds. Determine (a) its position when it changes direction and (b) its velocity when it returns to the position it had at $t = 0$.

(a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to recognize that $x_i = 2.00\ \text{m}$, $v_i = 3.00\ \text{m/s}$, and $a = -8.00\ \text{m/s}^2$. The velocity equation, $v_f = v_i + at$, is then

$$v_f = 3.00\ \text{m/s} - (8.00\ \text{m/s}^2)t.$$

The particle changes direction when $v_f = 0$, which occurs at $t = \frac{3}{8}\ \text{s}$. The position at this time is:

$$x = 2.00\ \text{m} + (3.00\ \text{m/s})\left(\frac{3}{8}\ \text{s}\right) - (4.00\ \text{m/s}^2)\left(\frac{3}{8}\ \text{s}\right)^2 = \boxed{2.56\ \text{m}}.$$

(b) From $x_f = x_i + v_i t + \frac{1}{2}at^2$, observe that when $x_f = x_i$, the time is given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00\ \text{m/s})}{-8.00\ \text{m/s}^2} = \frac{3}{4}\ \text{s}$$

and the velocity is $v_f = 3.00\ \text{m/s} - (8.00\ \text{m/s}^2)\left(\frac{3}{4}\ \text{s}\right) = \boxed{-3.00\ \text{m/s}}$.

A ball starts from rest and accelerates at 0.500 m/s^2 while moving down an inclined plane 9.00 m long. When it reaches the bottom, the ball rolls up another plane, where, after moving 15.0 m, it comes to rest. (a) What is the speed of the ball at the bottom of the first plane? (b) How long does it take to roll down the first plane? (c) What is the acceleration along the second plane? (d) What is the ball's speed 8.00 m along the second plane?

(a) Take initial and final points at top and bottom of the incline. If the ball starts from rest,

$$v_i = 0,\ a = 0.500\ \text{m/s}^2,\ x_f - x_i = 9.00\ \text{m}.$$

Then

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = 0^2 + 2(0.500\ \text{m/s}^2)(9.00\ \text{m})$$

$$v_f = \boxed{3.00\ \text{m/s}}.$$

(b) $x_f - x_i = v_i t + \frac{1}{2}at^2$

$$9.00 = 0 + \frac{1}{2}(0.500\ \text{m/s}^2)t^2$$

$$t = \boxed{6.00\ \text{s}}$$

(c) Take initial and final points at the bottom of the planes and the top of the second plane, respectively:

$$v_i = 3.00\ \text{m/s},\ v_f = 0,\ x_f - x_i = 15.00\ \text{m}.$$

$v_f^2 = v_i^2 + 2a(x_f - x_i)$ gives

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{\left[0 - (3.00\ \text{m/s})^2\right]}{2(15.0\ \text{m})} = \boxed{-0.300\ \text{m/s}^2}.$$

(d) Take the initial point at the bottom of the planes and the final point 8.00 m along the second:
$v_i = 3.00\ \text{m/s},\ x_f - x_i = 8.00\ \text{m},\ a = -0.300\ \text{m/s}^2$

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = (3.00\ \text{m/s})^2 + 2(-0.300\ \text{m/s}^2)(8.00\ \text{m}) = 4.20\ \text{m}^2/\text{s}^2$$

$$v_f = \boxed{2.05\ \text{m/s}}.$$

. Heather in her Corvette accelerates at the rate of $(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}})$ m/s^2, while Jill in her Jaguar accelerates at $(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}})$ m/s^2. They both start from rest at the origin of an *xy* coordinate system. After 5.00 s, (a) what is Heather's speed with respect to Jill, (b) how far apart are they, and (c) what is Heather's acceleration relative to Jill?

(a)

$$\mathbf{v}_H = 0 + \mathbf{a}_H t = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right)\ \text{m/s}^2\,(5.00\ \text{s})$$

$$\mathbf{v}_H = \left(15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$\mathbf{v}_J = 0 + \mathbf{a}_J t = \left(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\ \text{m/s}^2\,(5.00\ \text{s})$$

$$\mathbf{v}_J = \left(5.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$\mathbf{v}_{HJ} = v_H - v_J = \left(15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 5.00\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$\mathbf{v}_{HJ} = \left(10.0\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$\left|\mathbf{v}_{HJ}\right| = \sqrt{(10.0)^2 + (25.0)^2}\ \text{m/s} = \boxed{26.9\ \text{m/s}}$$

(b)

$$\mathbf{r}_H = 0 + 0 + \frac{1}{2}\mathbf{a}_H t^2 = \frac{1}{2}\left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right)\ \text{m/s}^2\,(5.00\ \text{s})^2$$

$$\mathbf{r}_H = \left(37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}\right)\ \text{m}$$

$$\mathbf{r}_J = \frac{1}{2}\left(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\ \text{m/s}^2\,(5.00\text{s})^2 = \left(12.5\hat{\mathbf{i}} + 37.5\hat{\mathbf{j}}\right)\ \text{m}$$

$$\mathbf{r}_{HJ} = \mathbf{r}_H - \mathbf{r}_J = \left(37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}} - 12.5\hat{\mathbf{i}} - 37.5\hat{\mathbf{j}}\right)\ \text{m}$$

$$\mathbf{r}_{HJ} = \left(25.0\hat{\mathbf{i}} - 62.5\hat{\mathbf{j}}\right)\ \text{m}$$

$$\left|\mathbf{r}_{HJ}\right| = \sqrt{(25.0)^2 + (62.5)^2}\,\text{m} = \boxed{67.3\ \text{m}}$$

(c)

$$\mathbf{a}_{HJ} = \mathbf{a}_H - \mathbf{a}_J = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}}\right)\ \text{m/s}^2$$

$$\mathbf{a}_{HJ} = \boxed{\left(2.00\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)\ \text{m/s}^2}$$

A car travels due east with a speed of 50.0 km/h. Raindrops are falling at a constant speed vertically with respect to the Earth. The traces of the rain on the side windows of the car make an angle of 60.0° with the vertical. Find the velocity of the rain with respect to (a) the car and (b) the Earth.

$\mathbf{v}_{ce}$ = the velocity of the car relative to the earth.
$\mathbf{v}_{wc}$ = the velocity of the water relative to the car.
$\mathbf{v}_{we}$ = the velocity of the water relative to the earth.

These velocities are related as shown in the diagram at the right.

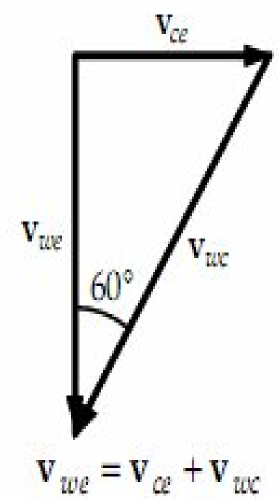

FIG. P4.39

(a) Since $\mathbf{v}_{we}$ is vertical, $v_{wc}\sin 60.0° = v_{ce} = 50.0$ km/h or

$\mathbf{v}_{wc} = \boxed{57.7\ \text{km/h at } 60.0°\ \text{west of vertical}}$.

(b) Since $\mathbf{v}_{ce}$ has zero vertical component,

$$v_{we} = v_{wc}\cos 60.0° = (57.7\ \text{km/h})\cos 60.0° = \boxed{28.9\ \text{km/h downward}}.$$

The pilot of an airplane notes that the compass indicates a heading due west. The airplane's speed relative to the air is 150 km/h. If there is a wind of 30.0 km/h toward the north, find the velocity of the airplane relative to the ground.

$$v = \sqrt{150^2 + 30.0^2} = \boxed{153\ \text{km/h}}$$

$$\theta = \tan^{-1}\left(\frac{30.0}{150}\right) = \boxed{11.3^\circ \text{ north of west}}$$

. To start an avalanche on a mountain slope, an artillery shell is fired with an initial velocity of 300 m/s at 55.0° above the horizontal. It explodes on the mountainside 42.0 s after firing. What are the *x* and *y* coordinates of the shell where it explodes, relative to its firing point?

$$x = v_{xi}t = v_i \cos\theta_i t$$

$$x = (300\ \text{m/s})(\cos 55.0°)(42.0\ \text{s})$$

$$x = \boxed{7.23 \times 10^3\ \text{m}}$$

$$y = v_{yi}t - \frac{1}{2}gt^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

$$y = (300\ \text{m/s})(\sin 55.0°)(42.0\ \text{s}) - \frac{1}{2}\left(9.80\ \text{m/s}^2\right)(42.0\ \text{s})^2 = \boxed{1.68 \times 10^3\ \text{m}}$$

A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of 8.00 m/s at an angle of 20.0° below the horizontal. It strikes the ground 3.00 s later. (a) How far horizontally from the base of the building does the ball strike the ground? (b) Find the height from which the ball was thrown. (c) How long does it take the ball to reach a point 10.0 m below the level of launching?

(a) $x_f = v_{xi}\, t = 8.00 \cos 20.0° (3.00) = \boxed{22.6 \text{ m}}$

(b) Taking y positive downwards,

$$y_f = v_{yi} t + \frac{1}{2} g t^2$$

$$y_f = 8.00 \sin 20.0° (3.00) + \frac{1}{2}(9.80)(3.00)^2 = \boxed{52.3 \text{ m}}.$$

(c) $10.0 = 8.00(\sin 20.0°)t + \frac{1}{2}(9.80)t^2$

$$4.90t^2 + 2.74t - 10.0 = 0$$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18 \text{ s}}$$