A pendulum consists of a sphere of mass m attached to a light cord of length L, as shown in Figure 8.7. The sphere is released from rest at point Ⓐ when the cord makes an angle θ_A with the vertical, and the pivot at P is frictionless.

(A) Find the speed of the sphere when it is at the lowest point Ⓑ.

Solution The only force that does work on the sphere is the gravitational force. (The force applied by the cord is always perpendicular to each element of the displacement and hence does no work.) Because the pendulum–Earth system is isolated, the energy of the system is conserved. As the pendulum swings, continuous transformation between potential and kinetic energy occurs. At the instant the pendulum is released, the energy of the system is entirely potential energy. At point Ⓑ the pendulum has kinetic energy, but the system has lost some potential energy. At Ⓒ the system has regained its initial potential energy, and the kinetic energy of the pendulum is again zero.

If we measure the y coordinates of the sphere from the center of rotation, then $y_A = -L\cos\theta_A$ and $y_B = -L$. Therefore, $U_A = -mgL\cos\theta_A$ and $U_B = -mgL$.

Applying the principle of conservation of mechanical energy to the system gives

$$K_B + U_B = K_A + U_A$$

$$\frac{1}{2}mv_B^2 - mgL = 0 - mgL\cos\theta_A$$

$$(1) \qquad v_B = \sqrt{2gL(1-\cos\theta_A)}$$

(B) What is the tension T_B in the cord at Ⓑ?

Solution Because the tension force does no work, it does not enter into an energy equation, and we cannot determine the tension using the energy method. To find T_B, we can apply Newton's second law to the radial direction. First, recall that the centripetal acceleration of a particle moving in a circle is equal to v^2/r directed toward the center of rotation. Because $r = L$ in this example, Newton's second law gives

$$(2) \qquad \sum F_r = mg - T_B = ma_r = -m\frac{v_B^2}{L}$$

Substituting Equation (1) into Equation (2) gives the tension at point Ⓑ as a function of θ_A:

$$(3) \qquad T_B = mg + 2mg(1-\cos\theta_A) = mg(3-2\cos\theta_A)$$

From Equation (2) we see that the tension at Ⓑ is greater than the weight of the sphere. Furthermore, Equation (3) gives the expected result that $T_B = mg$ when the initial angle $\theta_A = 0$. Note also that part (A) of this example is categorized as an energy problem while part (B) is categorized as a Newton's second law problem.

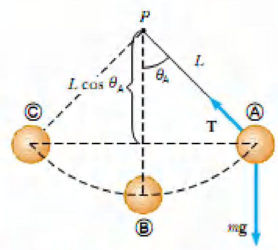

Figure 8.7 (Example 8.3) If the sphere is released from rest at the angle θ_A, it will never swing above this position during its motion. At the start of the motion, when the sphere is at position Ⓐ, the energy of the sphere–Earth system is entirely potential. This initial potential energy is transformed into kinetic energy when the sphere is at the lowest elevation Ⓑ. As the sphere continues to move along the arc, the energy again becomes entirely potential energy when the sphere is at Ⓒ.

A bead slides without friction around a loop-the-loop (Fig. P8.5). The bead is released from a height $h = 3.50R$. (a) What is its speed at point Ⓐ? (b) How large is the normal force on it if its mass is 5.00 g?

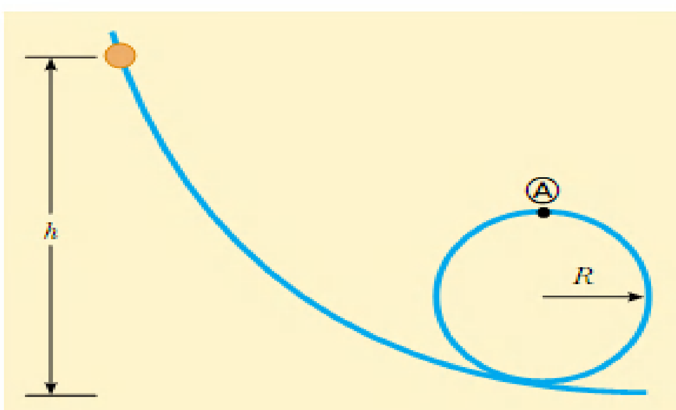

Figure P8.5

$U_i + K_i = U_f + K_f$: $mgh + 0 = mg(2R) + \frac{1}{2}mv^2$

$$g(3.50R) = 2g(R) + \frac{1}{2}v^2$$

$$\boxed{v = \sqrt{3.00gR}}$$

$\sum F = m\frac{v^2}{R}$: $n + mg = m\frac{v^2}{R}$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00gR}{R} - g\right] = 2.00mg$$

$$n = 2.00\left(5.00\times10^{-3}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)$$

$$= \boxed{0.098\,0\text{ N downward}}$$

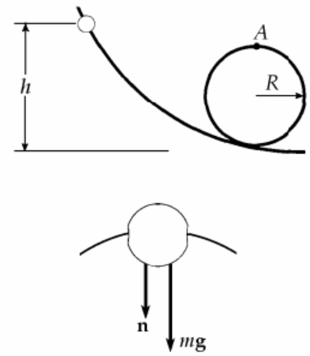

FIG. P8.5

A particle is subject to a force F_x that varies with position as in Figure P7.13. Find the work done by the force on the particle as it moves (a) from $x = 0$ to $x = 5.00$ m, (b) from $x = 5.00$ m to $x = 10.0$ m, and (c) from $x = 10.0$ m to $x = 15.0$ m. (d) What is the total work done by the force over the distance $x = 0$ to $x = 15.0$ m?

P7.13 $W = \int F_x dx$

and W equals the area under the Force-Displacement curve

(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$,

$$W = (3.00\text{ N})(5.00\text{ m}) = \boxed{15.0\text{ J}}$$

(c) For the region $10.0 \le x \le 15.0$,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(d) For the region $0 \le x \le 15.0$

$$W = (7.50 + 7.50 + 15.0)\text{ J} = \boxed{30.0\text{ J}}$$

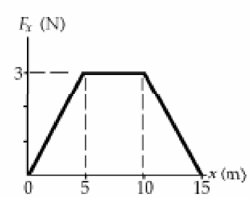

FIG. P7.13

]If it takes 4.00 J of work to stretch a Hooke's-law spring 10.0 cm from its unstressed length, determine the extra work required to stretch it an additional 10.0 cm.

$$4.00\ \text{J} = \frac{1}{2}k(0.100\ \text{m})^2$$

$\therefore k = 800\ \text{N/m}$ and to stretch the spring to 0.200 m requires

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\ \text{J} = \boxed{12.0\ \text{J}}$$

]A 40.0-kg box initially at rest is pushed 5.00 m along a rough, horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between box and floor is 0.300, find (a) the work done by the applied force, (b) the increase in internal energy in the box-floor system due to friction, (c) the work done by the normal force, (d) the work done by the gravitational force, (e) the change in kinetic energy of the box, and (f) the final speed of the box.

$$\sum F_y = ma_y: \quad n - 392\ \text{N} = 0$$
$$n = 392\ \text{N}$$
$$f_k = \mu_k n = (0.300)(392\ \text{N}) = 118\ \text{N}$$

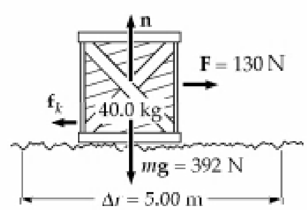

FIG. P7.31

(a) $W_F = F\Delta r\cos\theta = (130)(5.00)\cos 0° = \boxed{650\ \text{J}}$

(b) $\Delta E_{\text{int}} = f_k \Delta x = (118)(5.00) = \boxed{588\ \text{J}}$

(c) $W_n = n\Delta r\cos\theta = (392)(5.00)\cos 90° = \boxed{0}$

(d) $W_g = mg\Delta r\cos\theta = (392)(5.00)\cos(-90°) = \boxed{0}$

(e) $\Delta K = K_f - K_i = \sum W_{\text{other}} - \Delta E_{\text{int}}$

$\frac{1}{2}mv_f^2 - 0 = 650\ \text{J} - 588\ \text{J} + 0 + 0 = \boxed{62.0\ \text{J}}$

(f) $v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(62.0\ \text{J})}{40.0\ \text{kg}}} = \boxed{1.76\ \text{m/s}}$

A 2.00-kg block is attached to a spring of force constant 500 N/m as in Figure 7.10. The block is pulled 5.00 cm to the right of equilibrium and released from rest. Find the speed of the block as it passes through equilibrium if (a) the horizontal surface is frictionless and (b) the coefficient of friction between block and surface is 0.350.

(a) $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}(500)(5.00\times10^{-2})^2 - 0 = 0.625\text{ J}$

$W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - 0$

so $v_f = \sqrt{\frac{2(\sum W)}{m}} = \sqrt{\frac{2(0.625)}{2.00}}\text{ m/s} = \boxed{0.791\text{ m/s}}$

(b) $\frac{1}{2}mv_i^2 - f_k\Delta x + W_s = \frac{1}{2}mv_f^2$

$0 - (0.350)(2.00)(9.80)(0.0500)\text{ J} + 0.625\text{ J} = \frac{1}{2}mv_f^2$

$0.282\text{ J} = \frac{1}{2}(2.00\text{ kg})v_f^2$

$v_f = \sqrt{\frac{2(0.282)}{2.00}}\text{ m/s} = \boxed{0.531\text{ m/s}}$

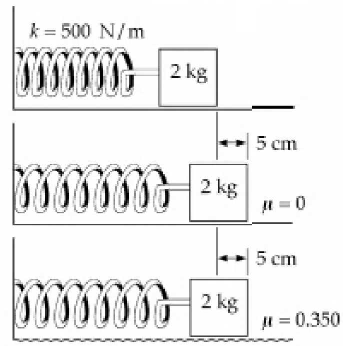

FIG. P7.32

A crate of mass 10.0 kg is pulled up a rough incline with an initial speed of 1.50 m/s. The pulling force is 100 N parallel to the incline, which makes an angle of 20.0° with the horizontal. The coefficient of kinetic friction is 0.400, and the crate is pulled 5.00 m. (a) How much work is done by the gravitational force on the crate? (b) Determine the increase in internal energy of the crate–incline system due to friction. (c) How much work is done by the 100-N force on the crate? (d) What is the change in kinetic energy of the crate? (e) What is the speed of the crate after being pulled 5.00 m?

(a) $W_g = mg\ell\cos(90.0° + \theta)$

$W_g = (10.0\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m})\cos 110° = \boxed{-168\text{ J}}$

(b) $f_k = \mu_k n = \mu_k mg\cos\theta$

$\Delta E_{\text{int}} = \ell f_k = \ell\mu_k mg\cos\theta$

$\Delta E_{\text{int}} = (5.00\text{ m})(0.400)(10.0)(9.80)\cos 20.0° = \boxed{184\text{ J}}$

(c) $W_F = F\ell = (100)(5.00) = \boxed{500\text{ J}}$

(d) $\Delta K = \sum W_{\text{other}} - \Delta E_{\text{int}} = W_F + W_g - \Delta E_{\text{int}} = \boxed{148\text{ J}}$

(e) $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$v_f = \sqrt{\frac{2(\Delta K)}{m} + v_i^2} = \sqrt{\frac{2(148)}{10.0} + (1.50)^2} = \boxed{5.65\text{ m/s}}$

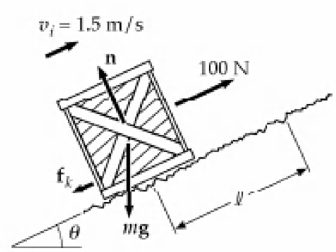

FIG. P7.33

A 15.0-kg block is dragged over a rough, horizontal surface by a 70.0-N force acting at 20.0° above the horizontal. The block is displaced 5.00 m, and the coefficient of kinetic friction is 0.300. Find the work done on the block by (a) the 70-N force, (b) the normal force, and (c) the gravitational force. (d) What is the increase in internal energy of the block-surface system due to friction? (e) Find the total change in the block's kinetic energy.

$\sum F_y = ma_y$: $n + (70.0\text{ N})\sin 20.0° - 147\text{ N} = 0$

$n = 123\text{ N}$

$f_k = \mu_k n = 0.300(123\text{ N}) = 36.9\text{ N}$

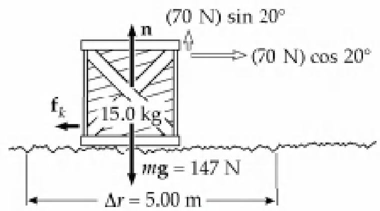

FIG. P7.34

(a) $W = F\Delta r\cos\theta = (70.0\text{ N})(5.00\text{ m})\cos 20.0° = \boxed{329\text{ J}}$

(b) $W = F\Delta r\cos\theta = (123\text{ N})(5.00\text{ m})\cos 90.0° = \boxed{0\text{ J}}$

(c) $W = F\Delta r\cos\theta = (147\text{ N})(5.00\text{ m})\cos 90.0° = \boxed{0}$

(d) $\Delta E_{\text{int}} = F\Delta x = (36.9\text{ N})(5.00\text{ m}) = \boxed{185\text{ J}}$

(e) $\Delta K = K_f - K_i = \sum W - \Delta E_{\text{int}} = 329\text{ J} - 185\text{ J} = \boxed{+144\text{ J}}$

The electric motor of a model train accelerates the train from rest to 0.620 m/s in 21.0 ms. The total mass of the train is 875 g. Find the average power delivered to the train during the acceleration.

$$\mathcal{P}_{\text{av}} = \frac{W}{\Delta t} = \frac{K_f}{\Delta t} = \frac{mv^2}{2\Delta t} = \frac{0.875\text{ kg}(0.620\text{ m/s})^2}{2(21\times 10^{-3}\text{ s})} = \boxed{8.01\text{ W}}$$

A 700-N Marine in basic training climbs a 10.0-m vertical rope at a constant speed in 8.00 s. What is his power output?

$$\text{Power} = \frac{W}{t} \qquad \mathcal{P} = \frac{mgh}{t} = \frac{(700\text{ N})(10.0\text{ m})}{8.00\text{ s}} = \boxed{875\text{ W}}$$

]Two objects are connected by a light string passing over a light frictionless pulley as shown in Figure P8.13. The object of mass 5.00 kg is released from rest. Using the principle of conservation of energy, (a) determine the speed of the 3.00-kg object just as the 5.00-kg object hits the ground. (b) Find the maximum height to which the 3.00-kg object rises.

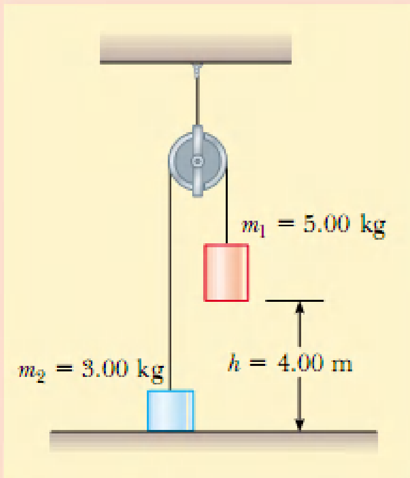

Figure P8.13 Problems 13 and 14.

Using conservation of energy for the system of the Earth and the two objects

(a) $$(5.00\text{ kg})g(4.00\text{ m}) = (3.00\text{ kg})g(4.00\text{ m}) + \frac{1}{2}(5.00+3.00)v^2$$

$$v = \sqrt{19.6} = \boxed{4.43\text{ m/s}}$$

(b) Now we apply conservation of energy for the system of the 3.00 kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00 kg object reaches its highest position in its free fall.

$$\frac{1}{2}(3.00)v^2 = mg\,\Delta y = 3.00g\Delta y$$

$$\Delta y = 1.00\text{ m}$$

$$y_{max} = 4.00\text{ m} + \Delta y = \boxed{5.00\text{ m}}$$

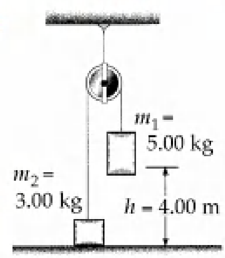

FIG. P8.13

] A single conservative force acts on a 5.00-kg particle. The equation $F_x = (2x + 4)$ N describes the force, where x is in meters. As the particle moves along the x axis from $x = 1.00$ m to $x = 5.00$ m, calculate (a) the work done by this force, (b) the change in the potential energy of the system, and (c) the kinetic energy of the particle at $x = 5.00$ m if its speed is 3.00 m/s at $x = 1.00$ m.

(a) $W = \int F_x dx = \int_1^{5.00\text{ m}} (2x+4)dx = \left(\frac{2x^2}{2} + 4x\right)_1^{5.00\text{ m}} = 25.0 + 20.0 - 1.00 - 4.00 = \boxed{40.0\text{ J}}$

(b) $\Delta K + \Delta U = 0$ $\qquad \Delta U = -\Delta K = -W = \boxed{-40.0\text{ J}}$

(c) $\Delta K = K_f - \frac{mv_1^2}{2}$ $\qquad K_f = \Delta K + \frac{mv_1^2}{2} = \boxed{62.5\text{ J}}$