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Tutorial on the Use of Significant Figures

All measurements are approximations--no measuring device can give perfect measurements without experimental uncertainty. By convention, a mass measured to 13.2 g is said to have an absolute uncertainty of 0.1 g and is said to have been measured to the nearest 0.1 g. In other words, we are somewhat uncertain about that last digit—it could be a "2"; then again, it could be a "1" or a "3". A mass of 13.20 g indicates an absolute uncertainty of 0.01 g.

The objectives of this tutorial are:

- Explain the concept of significant figures.
- Define rules for deciding the number of significant figures in a measured quantity.
- Explain the concept of an exact number.
- Define rules for determining the number of significant figures in a number calculated as a result of a mathematical operation.
- Explain rules for rounding numbers.
- Present guidelines for using a calculator.
- Provide some exercises to test your skill at significant figures.

What is a "significant figure"?

The number of significant figures in a result is simply the number of figures that are known with some degree of reliability. The number 13.2 is said to have 3 significant figures. The number 13.20 is said to have 4 significant figures

Rules for deciding the number of significant figures in a measured quantity:

(1) All nonzero digits are significant:

1.234 g has 4 significant figures,
1.2 g has 2 significant figures.

(2) Zeroes between nonzero digits are significant:

1002 kg has 4 significant figures,
3.07 mL has 3 significant figures.

(3) Leading zeros to the left of the first nonzero digits are not significant; such zeroes merely indicate the position of the decimal point:

0.001 °C has only 1 significant figure,
0.012 g has 2 significant figures.

(4) Trailing zeroes that are also to the right of a decimal point in a number are significant:

0.0230 mL has 3 significant figures,
0.20 g has 2 significant figures.

(5) When a number ends in zeroes that are not to the right of a decimal point, the zeroes are not necessarily significant:

190 miles may be 2 or 3 significant figures,
50,600 calories may be 3, 4, or 5 significant figures.

The potential ambiguity in the last rule can be avoided by the use of standard exponential, or "scientific," notation. For example, depending on whether the number of significant figures is 3, 4, or 5, we would write 50,600 calories as:

5.06×10^4 calories (3 significant figures)
 5.060×10^4 calories (4 significant figures), or
 5.0600×10^4 calories (5 significant figures).

What is a "exact number"?

Some numbers are exact because they are known with complete certainty.

Most exact numbers are integers: exactly 12 inches are in a foot, there might be exactly 23 students in a class. Exact numbers are often found as conversion factors or as counts of objects.

Exact numbers can be considered to have an infinite number of significant figures. Thus, the number of apparent significant figures in any exact number can be ignored as a limiting factor in determining the number of significant figures in the result of a calculation.

Rules for mathematical operations

In carrying out calculations, the general rule is that the accuracy of a calculated result is limited by the least accurate measurement involved in the calculation.

(1) In addition and subtraction, the result is rounded off to the last common digit occurring furthest to the right in all components. Another way to state this rule, it

that, in addition and subtraction, the result is rounded off so that it has the same number of decimal places as the measurement having the fewest decimal places. For example,

$$100 \text{ (assume 3 significant figures)} + 23.643 \text{ (5 significant figures)} = 123.643,$$

which should be rounded to 124 (3 significant figures).

(2) In multiplication and division, the result should be rounded off so as to have the same number of significant figures as in the component with the least number of significant figures. For example,

$$3.0 \text{ (2 significant figures)} \times 12.60 \text{ (4 significant figures)} = 37.8000$$

which should be rounded off to 38 (2 significant figures).

General guidelines for using calculators

When using a calculator, if you work the entirety of a long calculation without writing down any intermediate results, you may not be able to tell if an error is made. Further, even if you realize that one has occurred, you may not be able to tell where the error is.

In a long calculation involving mixed operations, carry as many digits as possible through the entire set of calculations and then round the final result appropriately. For example,

$$(5.00 / 1.235) + 3.000 + (6.35 / 4.0) = 4.04858... + 3.000 + 1.5875 = 8.630829...$$

The first division should result in 3 significant figures. The last division should result in 2 significant figures. The three numbers added together should result in a number that is rounded off to the last common significant digit occurring furthest to the right; in this case, the final result should be rounded with 1 digit after the decimal. Thus, the correct rounded final result should be 8.6. This final result has been limited by the accuracy in the last division.

Warning: carrying all digits through to the final result before rounding is critical for many mathematical operations in statistics. Rounding intermediate results when calculating sums of squares can seriously compromise the accuracy of the result.

Sample problems on significant figures

Instructions: print a copy of this page and work the problems. When you are ready to check your answers, go to the next page.

1. $37.76 + 3.907 + 226.4 =$

2. $319.15 - 32.614 =$
3. $104.630 + 27.08362 + 0.61 =$
4. $125 - 0.23 + 4.109 =$
5. $2.02 \times 2.5 =$
6. $600.0 / 5.2302 =$
7. $0.0032 \times 273 =$
8. $(5.5)^3 =$
9. $0.556 \times (40 - 32.5) =$
10. $45 \times 3.00 =$
11. $3.00 \times 10^5 - 1.5 \times 10^2 =$ (Give the exact numerical result, then express it the correct number of significant figures).
12. What is the average of 0.1707, 0.1713, 0.1720, 0.1704, and 0.1715?

Answer key to sample problems on significant figures

1. $37.76 + 3.907 + 226.4 = 268.1$
2. $319.15 - 32.614 = 286.54$
3. $104.630 + 27.08362 + 0.61 = 132.32$
4. $125 - 0.23 + 4.109 = 129$ (assuming that 125 has 3 significant figures).
5. $2.02 \times 2.5 = 5.0$
6. $600.0 / 5.2302 = 114.7$
7. $0.0032 \times 273 = 0.87$
8. $(5.5)^3 = 1.7 \times 10^2$
9. $0.556 \times (40 - 32.5) = 4$
10. $45 \times 3.00 = 1.4 \times 10^2$ (assuming that 45 has two significant figures)
11. $3.00 \times 10^5 - 1.5 \times 10^2 = 299850 = 3.00 \times 10^5$

12. What is the average of 0.1707, 0.1713, 0.1720, 0.1704, and 0.1715?
Answer = 0.1712

Links to other resources on the use of significant figures

[Uncertainty in Measurements](http://antoine.frostburg.edu/cgi-bin/senese/tutorials/sigfig/index.cgi): A tutorial by Professor Frederick A. Senese at Frostburg State University that shows how uncertainty arises from length, temperature, and volume measurements. How to count significant figures for a single measurement and for a series of measurements. How to round measurements to the correct number of significant figures.
<http://antoine.frostburg.edu/cgi-bin/senese/tutorials/sigfig/index.cgi>

[Significant figures quiz](http://antoine.frostburg.edu/chem/senese/101/measurement/sigfig-quiz.shtml): A JavaScript tutorial by Professor Frederick A. Senese at Frostburg State University that lets you test your knowledge of the use of significant figures.
<http://antoine.frostburg.edu/chem/senese/101/measurement/sigfig-quiz.shtml>

[Frequently asked questions](http://antoine.frostburg.edu/chem/senese/101/measurement/faq.shtml) about measurements, including FAQs on significant figures (such as:
"Why should the rules for propagating significant digits not be applied to averages?"
"Why does $1101 \text{ cm} - 1091 \text{ cm} = 10 \text{ cm}$ with 2 significant figures?"
"Are there simpler rules for counting significant digits?"
<http://antoine.frostburg.edu/chem/senese/101/measurement/faq.shtml>

[Determining the number of significant figures](http://science.widener.edu/svb/tutorial/sigfigures.html): a drill by Scott Van Bramer of Widener University involving significant figures that presents you with successive number displays and grades you on your answer to the question, "How many significant figures are there?"
<http://science.widener.edu/svb/tutorial/sigfigures.html>

[Significant Figures and Rounding Rules](http://www.angelfire.com/oh/cmulliss): a discussion of issues relating to the proper teaching of significant figures and rounding rules. This site, authored by Christopher Mulliss, includes links to other pages and to interactive tutorials.
<http://www.angelfire.com/oh/cmulliss>