

## Chapter 20

5. We use the following relation derived in Sample Problem 20-2:

$$\Delta S = mc \ln \left( \frac{T_f}{T_i} \right).$$

(a) The energy absorbed as heat is given by Eq. 19-14. Using Table 19-3, we find

$$Q = cm\Delta T = \left( 386 \frac{\text{J}}{\text{kg} \cdot \text{K}} \right) (2.00 \text{ kg})(75 \text{ K}) = 5.79 \times 10^4 \text{ J}$$

where we have used the fact that a change in Kelvin temperature is equivalent to a change in Celsius degrees.

(b) With  $T_f = 373.15 \text{ K}$  and  $T_i = 298.15 \text{ K}$ , we obtain

$$\Delta S = (2.00 \text{ kg}) \left( 386 \frac{\text{J}}{\text{kg} \cdot \text{K}} \right) \ln \left( \frac{373.15}{298.15} \right) = 173 \text{ J/K}.$$

11. The connection between molar heat capacity and the degrees of freedom of a diatomic gas is given by setting  $f = 5$  in Eq. 19-51. Thus,  $C_v = 5R/2$ ,  $C_p = 7R/2$ , and  $\gamma = 7/5$ . In addition to various equations from Chapter 19, we also make use of Eq. 20-4 of this chapter. We note that we are asked to use the ideal gas constant as  $R$  and not plug in its numerical value. We also recall that isothermal means constant-temperature, so  $T_2 = T_1$  for the  $1 \rightarrow 2$  process. The statement (at the end of the problem) regarding “per mole” may be taken to mean that  $n$  may be set identically equal to 1 wherever it appears.

(a) The gas law in ratio form (see Sample Problem 19-1) is used to obtain

$$p_2 = p_1 \left( \frac{V_1}{V_2} \right) = \frac{p_1}{3} \Rightarrow \frac{p_2}{p_1} = \frac{1}{3} = 0.333.$$

(b) The adiabatic relations Eq. 19-54 and Eq. 19-56 lead to

$$p_3 = p_1 \left( \frac{V_1}{V_3} \right)^\gamma = \frac{p_1}{3^{1.4}} \Rightarrow \frac{p_3}{p_1} = \frac{1}{3^{1.4}} = 0.215.$$

(c) and

$$T_3 = T_1 \left( \frac{V_1}{V_3} \right)^{\gamma-1} = \frac{T_1}{3^{0.4}} \Rightarrow \frac{T_3}{T_1} = \frac{1}{3^{0.4}} = 0.644.$$

• process 1  $\rightarrow$  2

(d) The work is given by Eq. 19-14:  $W = nRT_1 \ln(V_2/V_1) = RT_1 \ln 3$  which is approximately  $1.10RT_1$ . Thus,  $W/nRT_1 = \ln 3 = 1.10$ .

(e) The internal energy change is  $\Delta E_{\text{int}} = 0$  since this is an ideal gas process without a temperature change (see Eq. 19-45). Thus, the energy absorbed as heat is given by the first law of thermodynamics:  $Q = \Delta E_{\text{int}} + W \approx 1.10RT_1$ , or  $Q/nRT_1 = \ln 3 = 1.10$ .

(f)  $\Delta E_{\text{int}} = 0$  or  $\Delta E_{\text{int}}/nRT_1 = 0$

(g) The entropy change is  $\Delta S = Q/T_1 = 1.10R$ , or  $\Delta S/R = 1.10$ .

• process 2  $\rightarrow$  3

(h) The work is zero since there is no volume change. Therefore,  $W/nRT_1 = 0$

(i) The internal energy change is

$$\Delta E_{\text{int}} = nC_V(T_3 - T_2) = (1) \left( \frac{5}{2} R \right) \left( \frac{T_1}{3^{0.4}} - T_1 \right) \approx -0.889 RT_1 \Rightarrow \frac{\Delta E_{\text{int}}}{nRT_1} \approx -0.889.$$

This ratio ( $-0.889$ ) is also the value for  $Q/nRT_1$  (by either the first law of thermodynamics or by the definition of  $C_V$ ).

(j)  $\Delta E_{\text{int}}/nRT_1 = -0.889$ .

(k) For the entropy change, we obtain

$$\frac{\Delta S}{R} = n \ln \left( \frac{V_3}{V_1} \right) + n \frac{C_V}{R} \ln \left( \frac{T_3}{T_1} \right) = (1) \ln(1) + (1) \left( \frac{5}{2} \right) \ln \left( \frac{T_1/3^{0.4}}{T_1} \right) = 0 + \frac{5}{2} \ln(3^{-0.4}) \approx -1.10 .$$

• process 3  $\rightarrow$  1

(l) By definition,  $Q = 0$  in an adiabatic process, which also implies an absence of entropy change (taking this to be a reversible process). The internal change must be the negative of the value obtained for it in the previous process (since all the internal energy changes must add up to zero, for an entire cycle, and its change is zero for process 1  $\rightarrow$  2), so  $\Delta E_{\text{int}} = +0.889RT_1$ . By the first law of thermodynamics, then,

$$W = Q - \Delta E_{\text{int}} = -0.889RT_1,$$

or  $W/nRT_1 = -0.889$ .

(m)  $Q = 0$  in an adiabatic process.

(n)  $\Delta E_{\text{int}}/nRT_1 = +0.889$ .

(o)  $\Delta S/nR = 0$

15. The ice warms to  $0^\circ\text{C}$ , then melts, and the resulting water warms to the temperature of the lake water, which is  $15^\circ\text{C}$ . As the ice warms, the energy it receives as heat when the temperature changes by  $dT$  is  $dQ = mc_I dT$ , where  $m$  is the mass of the ice and  $c_I$  is the specific heat of ice. If  $T_i (= 263 \text{ K})$  is the initial temperature and  $T_f (= 273 \text{ K})$  is the final temperature, then the change in its entropy is

$$\Delta S = \int \frac{dQ}{T} = mc_I \int_{T_i}^{T_f} \frac{dT}{T} = mc_I \ln \frac{T_f}{T_i} = (0.010 \text{ kg})(2220 \text{ J/kg} \cdot \text{K}) \ln \left( \frac{273 \text{ K}}{263 \text{ K}} \right) = 0.828 \text{ J/K}.$$

Melting is an isothermal process. The energy leaving the ice as heat is  $mL_F$ , where  $L_F$  is the heat of fusion for ice. Thus,

$$\Delta S = Q/T = mL_F/T = (0.010 \text{ kg})(333 \times 10^3 \text{ J/kg})/(273 \text{ K}) = 12.20 \text{ J/K}.$$

For the warming of the water from the melted ice, the change in entropy is

$$\Delta S = mc_w \ln \frac{T_f}{T_i},$$

where  $c_w$  is the specific heat of water ( $4190 \text{ J/kg} \cdot \text{K}$ ). Thus,

$$\Delta S = (0.010 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln \left( \frac{288 \text{ K}}{273 \text{ K}} \right) = 2.24 \text{ J/K}.$$

The total change in entropy for the ice and the water it becomes is

$$\Delta S = 0.828 \text{ J/K} + 12.20 \text{ J/K} + 2.24 \text{ J/K} = 15.27 \text{ J/K}.$$

Since the temperature of the lake does not change significantly when the ice melts, the change in its entropy is  $\Delta S = Q/T$ , where  $Q$  is the energy it receives as heat (the negative of the energy it supplies the ice) and  $T$  is its temperature. When the ice warms to  $0^\circ\text{C}$ ,

$$Q = -mc_i(T_f - T_i) = -(0.010 \text{ kg})(2220 \text{ J/kg} \cdot \text{K})(10 \text{ K}) = -222 \text{ J}.$$

When the ice melts,

$$Q = -mL_F = -(0.010 \text{ kg})(333 \times 10^3 \text{ J/kg}) = -3.33 \times 10^3 \text{ J}.$$

When the water from the ice warms,

$$Q = -mc_w(T_f - T_i) = -(0.010 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(15 \text{ K}) = -629 \text{ J}.$$

The total energy leaving the lake water is

$$Q = -222 \text{ J} - 3.33 \times 10^3 \text{ J} - 6.29 \times 10^2 \text{ J} = -4.18 \times 10^3 \text{ J}.$$

The change in entropy is

$$\Delta S = -\frac{4.18 \times 10^3 \text{ J}}{288 \text{ K}} = -14.51 \text{ J/K}.$$

The change in the entropy of the ice-lake system is  $\Delta S = (15.27 - 14.51) \text{ J/K} = 0.76 \text{ J/K}$ .

23. With  $T_L = 290 \text{ K}$ , we find

$$\varepsilon = 1 - \frac{T_L}{T_H} \Rightarrow T_H = \frac{T_L}{1 - \varepsilon} = \frac{290 \text{ K}}{1 - 0.40}$$

which yields the (initial) temperature of the high-temperature reservoir:  $T_H = 483 \text{ K}$ . If we replace  $\varepsilon = 0.40$  in the above calculation with  $\varepsilon = 0.50$ , we obtain a (final) high temperature equal to  $T'_H = 580 \text{ K}$ . The difference is

$$T'_H - T_H = 580 \text{ K} - 483 \text{ K} = 97 \text{ K}.$$

40. (a) Eq. 20-11 gives the Carnot efficiency as  $1 - T_L/T_H$ . This gives 0.222 in this case. Using this value with Eq. 20-9 leads to  $W = (0.222)(750 \text{ J}) = 167 \text{ J}$ .

(b) Now, Eq. 20-14 gives  $K_C = 3.5$ . Then, Eq. 20-12 yields  $|W| = 1200/3.5 = 343 \text{ J}$ .