CHAPTER 21.

6E. Four moles of an ideal gas are expanded from volume V_1 to volume $V_2 = 2V_1$. If the expansion is isothermal at temperature $T = 400$ K, find (a) the work done by the expanding gas and (b) the change in its entropy. (c) If the expansion is reversibly adiabatic instead of isothermal, what is the entropy change of the gas?

<u>21.6</u> $n = 4$ moles, $V_i = V_1$, $V_f = 2V_1$, $T = 400$ K

(a) for Isothermal process

$$W = \int_{V_1}^{V_2} p\,dV = nRT\int_{V_i}^{V_f}\frac{dV}{V} = nRT\ln\left(\frac{V_f}{V_i}\right)$$

$$= 4(8.31)(400)\ln(2) = \boxed{9.22\times10^{3}\ \mathrm{J}}$$

(b) In Isothermal process $dE_{in} = 0 \Rightarrow dQ = W$

$$\Delta S = \frac{\Delta Q}{T} = \frac{W}{T} = nR\ln\left(\frac{V_f}{V_i}\right) = \boxed{23.0\ \frac{\mathrm{J}}{\mathrm{K}}}$$

(c) $\boxed{\Delta S = 0}$ because $\Delta Q = 0$ in adiabatic process.

15P. A 2.0 mol sample of an ideal monatomic gas undergoes the reversible process shown in Fig. 21-19. (a) How much heat is absorbed by the gas? (b) What is the change in the internal energy of the gas? (c) How much work is done by the gas?

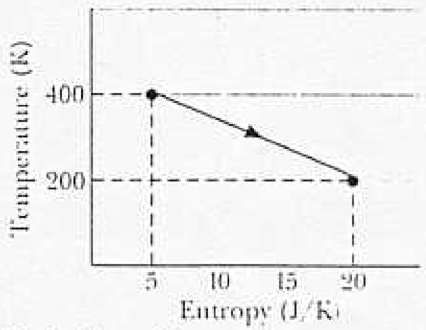


FIGURE 21-19 Problem 15.

$n = 2.0$ moles , $C_V = \frac{3}{2} R$ (monatomic gas)

<u>21-15</u>

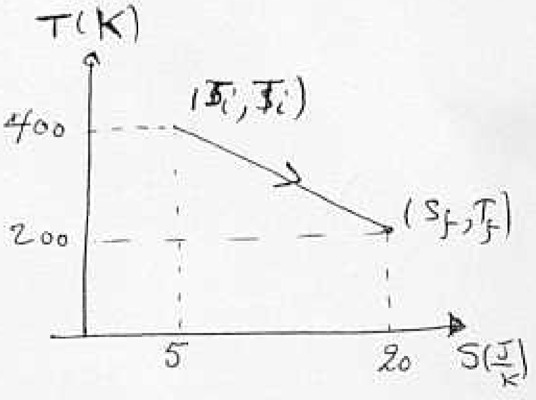
$\Delta Q = T \Delta S$.

(a) $\therefore Q = \int T\, dS \equiv$ area under the curve

$\equiv$ Area of trapezoidal area

$= \frac{1}{2}(200 + 400)(20 - 5)$

$= \boxed{4.5 \times 10^3 \text{ J.}}$



(b) $\Delta E_{int} = n C_V \Delta T = \frac{3}{2} n R (T_f - T_i)$

$= \frac{3}{2}(2)(8.31)(200 - 400)$

$= \boxed{-4.99 \times 10^3 \text{ J}}$

(c) $Q = \Delta E_{int} + W \Rightarrow W = Q - \Delta E_{int}$

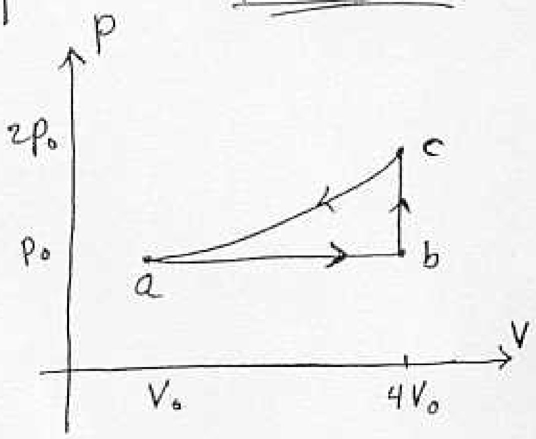
$= 4.5 \times 10^3 - (-4.99 \times 10^3)$

$= \boxed{9.5 \times 10^3 \text{ J}}$

29P. One mole of an ideal monatomic gas is taken through the cycle in Fig. 21-22. (a) How much work is done by the gas in going from state a to state c along path abc? What are the changes in internal energy and entropy in going (b) from b to c and (c) through one complete cycle? Express all answers in terms of the pressure p_0, volume V_0, and temperature T_0 of state a.

$PV = RT$

21-29 $n = 1$ mole $C_V = \frac{3}{2}R$ (monatomic)



a) $W_{a\to c} = W_{a\to b} + W_{bc}$

$= P_0(4V_0 - V_0) + 0 = \boxed{3P_0V_0}$

b) Along bc, since

$\Delta E_{int} = n C_V \Delta T = Q$ (since $\Delta V = 0$, $W = 0$)

$Q = nC_V(T_c - T_b) = nC_V\left(\frac{P_cV_c}{nR} - \frac{P_bV_b}{nR}\right) = nC_V\left(\frac{8P_0V_0}{nR} - \frac{4P_0V_0}{nR}\right)$

$= \frac{3}{2}(4P_0V_0) = \boxed{6P_0V_0} = 6RT_0$

then

$\boxed{\Delta E_{int} = Q = 6P_0V_0}$

. for the entropy

$\Delta S = \int \frac{\Delta Q}{T} = nC_V \int_{T_b}^{T_c} \frac{dT}{T} = nC_V \ln\left(\frac{T_c}{T_b}\right)$

$= nC_V \ln\left(\frac{P_cV_c}{P_bV_b}\right) = nC_V \ln\left(\frac{2\times 4P_0V_0}{1\times 4P_0V_0}\right)$

$= nC_V \ln 2 = \boxed{\frac{3}{2}R\ln 2}$

(c) Through one complete reversible cycle $\Delta T = 0$, $\Delta Q = 0$

$\Rightarrow \Delta E_{int} = 0$, $\Delta S = 0$

37E. An ideal engine whose low-temperature reservoir is at 17°C has an efficiency of 40%. By how much should the temperature of the high-temperature reservoir be increased to increase the efficiency to 50%?

21-37 $T_c = 17 + 273 = 290\,K \quad , \quad \varepsilon = 40\%$

$$\varepsilon = \frac{T_H - T_c}{T_H} = 1 - \frac{T_c}{T_H} \Rightarrow \frac{T_c}{T_H} = 1 - \varepsilon$$

$$T_H = \frac{T_c}{1-\varepsilon}$$

$$T_H' - T_H = \frac{T_c}{1-\varepsilon'} - \frac{T_c}{1-\varepsilon} = T_c\left(\frac{1}{1-\varepsilon'} - \frac{1}{1-\varepsilon}\right)$$

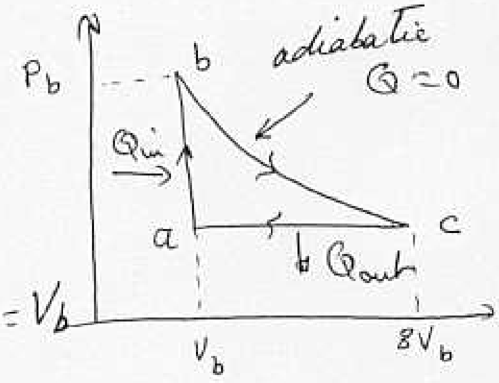
$$= 290\left(\frac{1}{1-.5} - \frac{1}{1-.4}\right)$$

$$= 290\,(0.333) = 96.7\,K$$

42P. One mole of a monatomic ideal gas is taken through the reversible cycle shown in Fig. 21-23. Process bc is an adiabatic expansion, with $p_b = 10.0$ atm and $V_b = 1.00 \times 10^{-3}$ m³. Find (a) the heat added to the gas, (b) the heat leaving the gas, (c) the net work done by the gas, and (d) the efficiency of the cycle.

$n = 1$, $C_V = \frac{3}{2}R$ (monatomic gas), $P_b = 10$ atm $= 1.01 \times 10^6$ Pa

$V_b = 1 \times 10^{-3}$ m^3, $\gamma = C_P / C_V = 5/3$



21-42

(a) Heat added to the gas only at ab

$$Q_{in} = n C_V \Delta T = n C_V (T_b - T_a)$$

$$= n C_V \left(\frac{P_b V_b - P_a V_a}{nR} \right) , \quad V_a = V_b$$

$$= (1) \frac{3}{2} R (P_b - P_a) V_a$$

note that $P_a = P_c$, and $P_c V_c^\gamma = P_b V_b^\gamma$

$$P_c = P_b \left(\frac{V_b}{V_c} \right)^\gamma = 1.01 \times 10^6 \left(\frac{1}{8} \right)^{5/3}$$

$$= 3.16 \times 10^4 \text{ pa}$$

$$\therefore Q_{in} = \frac{3}{2} \cdot (1.01 \times 10^6 - 3.16 \times 10^4) \times (1 \times 10^{-3})$$

$$= \underline{1.47 \times 10^3 \text{ J}}$$

(b) $Q_{out} = n C_p \Delta T = (1) \frac{5}{2} (V_a - V_c) P_a$

$$= \frac{5}{2} (V_b - 8 V_b) P_a = \frac{5}{2} (-7 \times 10^{-3})(3.16 \times 10^4)$$

$$= \underline{-5.53 \times 10^2 \text{ J}}$$

$\therefore |Q_{out}| = \underline{5.53 \times 10^2 \text{ J}}$

(c) $W = |Q_{in}| - |Q_{out}|$

$$= 9.17 \times 10^2 \text{ J}$$

(d) $\varepsilon = \frac{W}{Q_{in}} = \frac{9.17 \times 10^2}{1.47 \times 10^3} = 0.624 \Rightarrow \underline{62.4\%}$