3. (a)

$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}(6.0t^2 + 7.0t) = 12t + 7.0 = 12(2.0) + 7.0 = 31 \text{ mV} .$$

(b) Appealing to Lenz's law (especially Fig. 31-5(a)) we see that the current flow in the loop is clockwise. Thus, the current is from right to left through R.

12. (a) Eq. 30-12 gives the field at the center of the large loop with $R = 1.00$ m and current $i(t)$. This is approximately the field throughout the area ($A = 2.00 \times 10^{-4}$ m^2) enclosed by the small loop. Thus, with $B = \mu_0 i/2R$ and $i(t) = i_0 + kt$ (where $i_0 = 200$ A and $k = (-200\,\text{A} - 200\,\text{A})/1.00\,\text{s} = -400\,\text{A/s}$), we find

$$\begin{aligned} B\big|_{t=0} &= \frac{\mu_0 i_0}{2R} = \frac{(4\pi \times 10^{-7}\,\text{H/m})(200\,\text{A})}{2(1.00\,\text{m})} = 1.26 \times 10^{-4}\,\text{T} , \\ B\big|_{t=0.500\,\text{s}} &= \frac{(4\pi \times 10^{-7}\,\text{H/m})[200\,\text{A} - (400\,\text{A/s})(0.500\,\text{s})]}{2(1.00\,\text{m})} = 0 , \\ B|_{t=1.00\,\text{s}} &= \frac{(4\pi \times 10^{-7}\,\text{H/m})[200\,\text{A} - (400\,\text{A/s})(1.00\,\text{s})]}{2(1.00\,\text{m})} = -1.26 \times 10^{-4}\ \text{T} . \end{aligned}$$

(b) Let the area of the small loop be a. Then $\Phi_B = Ba$, and Faraday's law yields

$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d(Ba)}{dt} = -a\frac{dB}{dt} = -a\left(\frac{\Delta B}{\Delta t}\right) \\ &= -(2.00 \times 10^{-4}\,\text{m}^2)\left(\frac{-1.26 \times 10^{-4}\,\text{T} - 1.26 \times 10^{-4}\,\text{T}}{1.00\,\text{s}}\right) = 5.04 \times 10^{-8}\ \text{V} . \end{aligned}$$

15. (a) Let L be the length of a side of the square circuit. Then the magnetic flux through the circuit is $\Phi_B = L^2B/2$, and the induced emf is

$$\mathcal{E}_i = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt} .$$

Now $B = 0.042 - 0.870t$ and $dB/dt = -0.870\,\text{T/s}$. Thus,

$$\mathcal{E}_i = \frac{(2.00\,\text{m})^2}{2}(0.870\,\text{T/s}) = 1.74\ \text{V} .$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is $\mathcal{E}+\mathcal{E}_i = 20.0\,\text{V}+1.74\,\text{V} = 21.7\,\text{V}$.

(b) The current is in the sense of the total emf (counterclockwise).

27. (a) Eq. 31-10 leads to

$$\mathcal{E} = BLv = (0.350\,\mathrm{T})(0.250\,\mathrm{m})(0.550\,\mathrm{m/s}) = 0.0481\ \mathrm{V}\ .$$

(b) By Ohm's law, the induced current is $i = 0.0481\,\mathrm{V}/18.0\,\Omega = 0.00267\,\mathrm{A}$. By Lenz's law, the current is clockwise in Fig. 31-46.

(c) Eq. 27-22 leads to $P = i^2R = 0.000129$ W.