4. The points must be along a line parallel to the wire and a distance r from it, where r satisfies

$$B_{\text{wire}} = \frac{\mu_0 i}{2\pi r} = B_{\text{ext}} ,$$

 $\operatorname{or}$ 

$$r = \frac{\mu_0 i}{2\pi B_{\text{ext}}} = \frac{(1.26 \times 10^{-6} \,\text{T} \cdot \text{m/A})(100 \,\text{A})}{2\pi (5.0 \times 10^{-3} \,\text{T})} = 4.0 \times 10^{-3} \,\text{m} \,.$$

9. Recalling the *straight sections* discussion in Sample Problem 30-1, we see that the current in the straight segments colinear with P do not contribute to the field at that point. Using Eq. 30-11 (with  $\phi = \theta$ ) and the right-hand rule, we find that the current in the semicircular arc of radius b contributes  $\mu_0 i\theta/4\pi b$  (out of the page) to the field at P. Also, the current in the large radius arc contributes  $\mu_0 i\theta/4\pi a$  (into the page) to the field there. Thus, the net field at P is

$$\vec{B} = \frac{\mu_0 i \theta}{4\pi} \left( \frac{1}{b} - \frac{1}{a} \right)$$
 out of the page .

27. We use Eq. 30-15 and the superposition of forces:  $\vec{F}_4 = \vec{F}_{14} + \vec{F}_{24} + \vec{F}_{34}$ . With  $\theta = 45^\circ$ , the situation is as shown below:



The components of  $\vec{F}_4$  are given by

$$F_{4x} = -F_{43} - F_{42} \cos \theta$$
  
=  $-\frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \cos 45^\circ}{2\sqrt{2\pi a}}$   
=  $-\frac{3\mu_0 i^2}{4\pi a}$ 

 $\quad \text{and} \quad$ 

$$F_{4y} = F_{41} - F_{42} \sin \theta$$
  
=  $\frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \sin 45^\circ}{2\sqrt{2\pi a}}$   
=  $\frac{\mu_0 i^2}{4\pi a}$ .

Thus,

$$F_4 = (F_{4x}^2 + F_{4y}^2)^{1/2} = \left[ \left( -\frac{3\mu_0 i^2}{4\pi a} \right)^2 + \left( \frac{\mu_0 i^2}{4\pi a} \right)^2 \right]^{1/2} = \frac{\sqrt{10}\mu_0 i^2}{4\pi a} ,$$

and  $\vec{F_4}$  makes an angle  $\phi$  with the positive x axis, where

$$\phi = \tan^{-1}\left(\frac{F_{4y}}{F_{4x}}\right) = \tan^{-1}\left(-\frac{1}{3}\right) = 162^{\circ}$$

30. A close look at the path reveals that only currents 1, 3, 6 and 7 are enclosed. Thus, noting the different current directions described in the problem, we obtain

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (7i - 6i + 3i + i) = 5\mu_0 i \; .$$

40. It is possible (though tedious) to use Eq. 30-28 and evaluate the contributions (with the intent to sum them) of all 1200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 30-25 for the ideal solenoid (which does not make use of the coil radius) is the preferred method:

$$B = \mu_0 in = \mu_0 i\left(\frac{N}{\ell}\right)$$

where i = 3.60 A,  $\ell = 0.950$  m and N = 1200. This yields B = 0.00571 T.

56. (a) By the right-hand rule,  $\vec{B}$  points into the paper at P (see Fig. 30-6(c)). To find the magnitude of the field, we use Eq. 30-11 for each semicircle ( $\phi = \pi$  rad), and use superposition to obtain the result:

$$B = \frac{\mu_0 i\pi}{4\pi a} + \frac{\mu_0 i\pi}{4\pi b} = \frac{\mu_0 i}{4} \left(\frac{1}{a} + \frac{1}{b}\right) \; .$$

(b) The direction of  $\vec{\mu}$  is the same as the  $\vec{B}$  found in part (a): into the paper. The enclosed area is  $A = (\pi a^2 + \pi b^2)/2$  which means the magnetic dipole moment has magnitude

$$|\vec{\mu}| = \frac{\pi i}{2} \left( a^2 + b^2 \right) \; .$$