5. (a) Let i be the current in the circuit and take it to be positive if it is to the left in R_1. We use Kirchhoff's loop rule: $\mathcal{E}_1 - iR_2 - iR_1 - \mathcal{E}_2 = 0$. We solve for i:

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{12\,\text{V} - 6.0\,\text{V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\text{ A} .$$

A positive value is obtained, so the current is counterclockwise around the circuit.

(b) If i is the current in a resistor R, then the power dissipated by that resistor is given by $P = i^2R$. For R_1, $P_1 = (0.50\,\text{A})^2(4.0\,\Omega) = 1.0\,\text{W}$ and for R_2, $P_2 = (0.50\,\text{A})^2(8.0\,\Omega) = 2.0\,\text{W}$.

(c) If i is the current in a battery with emf $\mathcal{E}$, then the battery supplies energy at the rate $P = i\mathcal{E}$ provided the current and emf are in the same direction. The battery absorbs energy at the rate $P = i\mathcal{E}$ if the current and emf are in opposite directions. For $\mathcal{E}_1$, $P_1 = (0.50\,\text{A})(12\,\text{V}) = 6.0\,\text{W}$ and for $\mathcal{E}_2$, $P_2 = (0.50\,\text{A})(6.0\,\text{V}) = 3.0\,\text{W}$. In battery 1 the current is in the same direction as the emf. Therefore, this battery supplies energy to the circuit; the battery is discharging. The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.

21. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is upward. When the loop rule is applied to the lower loop, the result is

$$\mathcal{E}_2 - i_1 R_1 = 0 \ .$$

and when it is applied to the upper loop, the result is

$$\mathcal{E}_1 - \mathcal{E}_2 - \mathcal{E}_3 - i_2 R_2 = 0 \ .$$

The first equation yields

$$i_1 = \frac{\mathcal{E}_2}{R_1} = \frac{5.0\,\mathrm{V}}{100\,\Omega} = 0.050\,\mathrm{A} \ .$$

The second yields

$$i_2 = \frac{\mathcal{E}_1 - \mathcal{E}_2 - \mathcal{E}_3}{R_2} = \frac{6.0\,\mathrm{V} - 5.0\,\mathrm{V} - 4.0\,\mathrm{V}}{50\,\Omega} = -0.060\,\mathrm{A} \ .$$

The negative sign indicates that the current in R_2 is actually downward. If V_b is the potential at point b, then the potential at point a is $V_a = V_b + \mathcal{E}_3 + \mathcal{E}_2$, so $V_a - V_b = \mathcal{E}_3 + \mathcal{E}_2 = 4.0\,\mathrm{V} + 5.0\,\mathrm{V} = 9.0\,\mathrm{V}$.

31. (a) We first find the currents. Let i_1 be the current in R_1 and take it to be positive if it is upward. Let i_2 be the current in R_2 and take it to be positive if it is to the left. Let i_3 be the current in R_3 and take it to be positive if it is to the right. The junction rule produces

$$i_1 + i_2 + i_3 = 0 .$$

The loop rule applied to the left-hand loop produces

$$\mathcal{E}_1 - i_3 R_3 + i_1 R_1 = 0$$

and applied to the right-hand loop produces

$$\mathcal{E}_2 - i_2 R_2 + i_1 R_1 = 0 .$$

We substitute $i_1 = -i_2 - i_3$, from the first equation, into the other two to obtain

$$\mathcal{E}_1 - i_3 R_3 - i_2 R_1 - i_3 R_1 = 0$$

and

$$\mathcal{E}_2 - i_2 R_2 - i_2 R_1 - i_3 R_1 = 0 .$$

The first of these yields

$$i_3 = \frac{\mathcal{E}_1 - i_2 R_1}{R_1 + R_3} .$$

Substituting this into the second equation and solving for i_2, we obtain

$$\begin{aligned} i_2 &= \frac{\mathcal{E}_2(R_1 + R_3) - \mathcal{E}_1 R_1}{R_1 R_2 + R_1 R_3 + R_2 R_3} \\ &= \frac{(1.00\,\text{V})(5.00\,\Omega + 4.00\,\Omega) - (3.00\,\text{V})(5.00\,\Omega)}{(5.00\,\Omega)(2.00\,\Omega) + (5.00\,\Omega)(4.00\,\Omega) + (2.00\,\Omega)(4.00\,\Omega)} = -0.158\,\text{A} . \end{aligned}$$

We substitute into the expression for i_3 to obtain

$$i_3 = \frac{\mathcal{E}_1 - i_2 R_1}{R_1 + R_3} = \frac{3.00\,\text{V} - (-0.158\,\text{A})(5.00\,\Omega)}{5.00\,\Omega + 4.00\,\Omega} = 0.421\,\text{A} .$$

Finally,

$$i_1 = -i_2 - i_3 = -(-0.158\,\text{A}) - (0.421\,\text{A}) = -0.263\,\text{A} .$$

Note that the current in R_1 is actually downward and the current in R_2 is to the right. The current in R_3 is also to the right. The power dissipated in R_1 is $P_1 = i_1^2 R_1 = (-0.263\,\text{A})^2(5.00\,\Omega) = 0.346\,\text{W}$.

(b) The power dissipated in R_2 is $P_2 = i_2^2 R_2 = (-0.158\,\text{A})^2(2.00\,\Omega) = 0.0499\,\text{W}$.

(c) The power dissipated in R_3 is $P_3 = i_3^2 R_3 = (0.421\,\text{A})^2(4.00\,\Omega) = 0.709\,\text{W}$.

(d) The power supplied by $\mathcal{E}_1$ is $i_3 \mathcal{E}_1 = (0.421\,\text{A})(3.00\,\text{V}) = 1.26\,\text{W}$.

(e) The power "supplied" by $\mathcal{E}_2$ is $i_2 \mathcal{E}_2 = (-0.158\,\text{A})(1.00\,\text{V}) = -0.158\,\text{W}$. The negative sign indicates that $\mathcal{E}_2$ is actually absorbing energy from the circuit.

47. (a) The voltage difference V across the capacitor varies with time as $V(t) = \mathcal{E}(1 - e^{-t/RC})$. At $t = 1.30\,\mu\text{s}$ we have $V(t) = 5.00\,\text{V}$, so $5.00\,\text{V} = (12.0\,\text{V})(1 - e^{-1.30\,\mu\text{s}/RC})$, which gives $\tau = (1.30\,\mu\text{s})/\ln(12/7) = 2.41\,\mu\text{s}$.

(b) $C = \tau/R = 2.41\,\mu\text{s}/15.0\,\text{k}\Omega = 161\,\text{pF}$.