

3. (a) Since the gas is ideal, its pressure p is given in terms of the number of moles n , the volume V , and the temperature T by $p = nRT/V$. The work done by the gas during the isothermal expansion is

$$W = \int_{V_1}^{V_2} p dV = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln \frac{V_2}{V_1} .$$

We substitute $V_2 = 2V_1$ to obtain

$$W = nRT \ln 2 = (4.00 \text{ mol}) \left(8.31 \frac{\text{J}}{\text{mol}\cdot\text{K}} \right) (400 \text{ K}) \ln 2 = 9.22 \times 10^3 \text{ J} .$$

- (b) Since the expansion is isothermal, the change in entropy is given by $\Delta S = \int (1/T) dQ = Q/T$, where Q is the heat absorbed. According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$. Now the internal energy of an ideal gas depends only on the temperature and not on the pressure and volume. Since the expansion is isothermal, $\Delta E_{\text{int}} = 0$ and $Q = W$. Thus,

$$\Delta S = \frac{W}{T} = \frac{9.22 \times 10^3 \text{ J}}{400 \text{ K}} = 23.1 \text{ J/K} .$$

- (c) $\Delta S = 0$ for all reversible adiabatic processes.

15. The ice warms to 0°C , then melts, and the resulting water warms to the temperature of the lake water, which is 15°C . As the ice warms, the energy it receives as heat when the temperature changes by dT is $dQ = mc_I dT$, where m is the mass of the ice and c_I is the specific heat of ice. If T_i ($= 263\text{K}$) is the initial temperature and T_f ($= 273\text{K}$) is the final temperature, then the change in its entropy is

$$\begin{aligned}\Delta S &= \int \frac{dQ}{T} = mc_I \int_{T_i}^{T_f} \frac{dT}{T} = mc_I \ln \frac{T_f}{T_i} \\ &= (0.010\text{ kg})(2220\text{ J/kg}\cdot\text{K}) \ln \left(\frac{273\text{ K}}{263\text{ K}} \right) = 0.828\text{ J/K} .\end{aligned}$$

Melting is an isothermal process. The energy leaving the ice as heat is mL_F , where L_F is the heat of fusion for ice. Thus, $\Delta S = Q/T = mL_F/T = (0.010\text{ kg})(333 \times 10^3\text{ J/kg})/(273\text{ K}) = 12.20\text{ J/K}$. For the warming of the water from the melted ice, the change in entropy is

$$\Delta S = mc_w \ln \frac{T_f}{T_i} ,$$

where c_w is the specific heat of water ($4190\text{ J/kg}\cdot\text{K}$). Thus,

$$\Delta S = (0.010\text{ kg})(4190\text{ J/kg}\cdot\text{K}) \ln \left(\frac{288\text{ K}}{273\text{ K}} \right) = 2.24\text{ J/K} .$$

The total change in entropy for the ice and the water it becomes is

$$\Delta S = 0.828\text{ J/K} + 12.20\text{ J/K} + 2.24\text{ J/K} = 15.27\text{ J/K} .$$

Since the temperature of the lake does not change significantly when the ice melts, the change in its entropy is $\Delta S = Q/T$, where Q is the energy it receives as heat (the negative of the energy it supplies the ice) and T is its temperature. When the ice warms to 0°C ,

$$Q = -mc_I(T_f - T_i) = -(0.010\text{ kg})(2220\text{ J/kg}\cdot\text{K})(10\text{ K}) = -222\text{ J} .$$

When the ice melts,

$$Q = -mL_F = -(0.010\text{ kg})(333 \times 10^3\text{ J/kg}) = -3.33 \times 10^3\text{ J} .$$

When the water from the ice warms,

$$Q = -mc_w(T_f - T_i) = -(0.010\text{ kg})(4190\text{ J/kg}\cdot\text{K})(15\text{ K}) = -629\text{ J} .$$

The total energy leaving the lake water is $Q = -222\text{ J} - 3.33 \times 10^3\text{ J} - 6.29 \times 10^2\text{ J} = -4.18 \times 10^3\text{ J}$. The change in entropy is

$$\Delta S = \frac{-4.18 \times 10^3\text{ J}}{288\text{ K}} = -14.51\text{ J/K} .$$

The change in the entropy of the ice-lake system is $\Delta S = (15.27 - 14.51)\text{ J/K} = 0.76\text{ J/K}$.

19. (a) Work is done only for the ab portion of the process. This portion is at constant pressure, so the work done by the gas is

$$W = \int_{V_0}^{4V_0} p_0 dV = p_0(4V_0 - V_0) = 3p_0V_0 .$$

- (b) We use the first law: $\Delta E_{\text{int}} = Q - W$. Since the process is at constant volume, the work done by the gas is zero and $E_{\text{int}} = Q$. The energy Q absorbed by the gas as heat is $Q = nC_V \Delta T$, where C_V is the molar specific heat at constant volume and ΔT is the change in temperature. Since the gas is a monatomic ideal gas, $C_V = \frac{3}{2}R$. Use the ideal gas law to find that the initial temperature is $T_b = p_b V_b / nR = 4p_0 V_0 / nR$ and that the final temperature is $T_c = p_c V_c / nR = (2p_0)(4V_0) / nR = 8p_0 V_0 / nR$. Thus,

$$Q = \frac{3}{2}nR \left(\frac{8p_0 V_0}{nR} - \frac{4p_0 V_0}{nR} \right) = 6p_0 V_0 .$$

The change in the internal energy is $\Delta E_{\text{int}} = 6p_0 V_0$. Since $n = 1$ mol, this can also be written $Q = 6RT_0$. Since the process is at constant volume, use $dQ = nC_V dT$ to obtain

$$\Delta S = \int \frac{dQ}{T} = nC_V \int_{T_b}^{T_c} \frac{dT}{T} = nC_V \ln \frac{T_c}{T_b} .$$

Substituting $C_V = \frac{3}{2}R$ and using the ideal gas law, we write

$$\frac{T_c}{T_b} = \frac{p_c V_c}{p_b V_b} = \frac{(2p_0)(4V_0)}{p_0(4V_0)} = 2 .$$

Thus, $\Delta S = \frac{3}{2}nR \ln 2$. Since $n = 1$, this is $\Delta S = \frac{3}{2}R \ln 2$.

- (c) For a complete cycle, $\Delta E_{\text{int}} = 0$ and $\Delta S = 0$.

29. (a) The net work done is the rectangular “area” enclosed in the pV diagram:

$$W = (V - V_0)(p - p_0) = (2V_0 - V_0)(2p_0 - p_0) = V_0p_0 .$$

Inserting the values stated in the problem, we obtain $W = 2.27$ kJ.

- (b) We compute the energy added as heat during the “heat-intake” portions of the cycle using Eq. 20-39, Eq. 20-43, and Eq. 20-46:

$$\begin{aligned} Q_{abc} &= nC_V(T_b - T_a) + nC_p(T_c - T_b) \\ &= n\left(\frac{3}{2}R\right)T_a\left(\frac{T_b}{T_a} - 1\right) + n\left(\frac{5}{2}R\right)T_a\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right) \\ &= nRT_a\left(\frac{3}{2}\left(\frac{T_b}{T_a} - 1\right) + \frac{5}{2}\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right)\right) \\ &= p_0V_0\left(\frac{3}{2}(2 - 1) + \frac{5}{2}(4 - 2)\right) = \frac{13}{2}p_0V_0 \end{aligned}$$

where, to obtain the last line, the gas law in ratio form has been used (see Sample Problem 20-1). Therefore, since $W = p_0V_0$, we have $Q_{abc} = 13W/2 = 14.8$ kJ.

- (c) The efficiency is given by Eq. 21-9:

$$\varepsilon = \frac{W}{|Q_H|} = \frac{2}{13} = 0.154 = 15.4\% .$$

- (d) A Carnot engine operating between T_c and T_a has efficiency equal to

$$\varepsilon = 1 - \frac{T_a}{T_c} = 1 - \frac{1}{4} = 0.750 = 75.0\%$$

where the gas law in ratio form has been used. This is greater than our result in part (c), as expected from the second law of thermodynamics.

34. (a) We use Eq. 21-12,

$$K = \frac{|Q_L|}{|W|} = \frac{600}{200} = 3 .$$

(b) Energy conservation for a refrigeration cycle requires $|Q_L| + |W| = |Q_H|$, so that the result is 800 J.