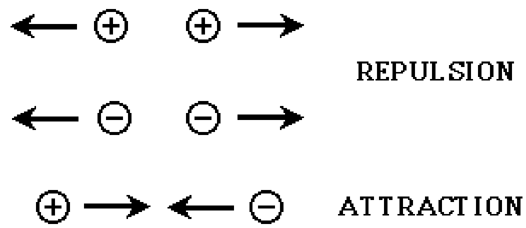
ELECTRIC CHARGE

Introduction: Ordinary matter consists of atoms. Each atom consists of a nucleus, consisting of protons and neutrons, surrounded by a number of electrons. In electricity, the electric charge (q, Q) plays the same rule as mass does in mechanics. The unit of electric charge q is the **Coulomb** (C). In nature there are two kinds of charges, **positive** and **negative**, with the properties that:

1. The force between the charged particles is called the **electric force**.
2. Unlike charges, opposite electrical sign, attract one another, and like charges, same electrical sign, repel one another, see the following figures.



3. The force between stationary charges is called electrostatic force. The electrostatic force obeys Coulomb's law.
4. The SI unit of charge is the Coulomb, C.
5. Electric charge is **quantized**, by mean that if the magnitude of the smallest charge is denoted by e (called the charge quantum), where e = 1.6×10^{-19} C, then all other charges are integer multiplies of e, i.e.

$$q = \pm n|e|, \qquad n = 1, 2, 3, \cdots$$

   Although there is a good reason to believe that charge magnitudes of e/3 and 2e/3 are possible, extensive experiments have not yet verified their existence.
6. An uncharged object becomes negatively charged if it gains electrons and becomes positively charged if it loses electrons. All electrons carry the same quantity of charge, so the more electrons an object gains or loses the greater is the overall negative or positive charge upon it.
7. Also, the algebraic sum of the charges in the universe is constant (**conservation of charge**). When a particle with charge +e is created, a particle with charge -e is created simultaneously. When a particle with charge +e disappears, a particle with -e also disappears. Hence the net charge of the universe remains constant.

The masses and charges of the electrons, protons and neutrons are listed in following table. Most of the mass of the atom is due to the mass of the nucleus.

particle	mass (kg)	charge (C)
electron	9.11×10^{-31}	-1.6×10^{-19}
proton	1.673×10^{-27}	1.6×10^{-19}
neutron	1.675×10^{-27}	0

Masses and charges of the building blocks of atoms.

The diameter of the nucleus is between 10^{-15} and 10^{-14} m. The electrons are contained in a roughly spherical region with a diameter of about 2×10^{-10} m. Measurements of the velocity of the orbital electrons in an atom have shown that the attractive force between the electrons and the nucleus is significantly stronger than the gravitational force between these two objects.

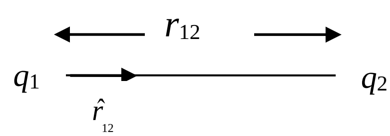
Coulomb's law: "The magnitude of the electric force, $\boldsymbol{F}$, that a charged particle, q, exerts on another particle, Q, is directly proportional to the product of their charges and inversely proportional to the square of the distance between them. The direction of the force is along the line joining the particles." Mathematically:

$$|\boldsymbol{F}| = k\,\frac{|q||Q|}{r^2}, \qquad k = \frac{1}{4\pi\varepsilon_o} \approx 9\times10^9\ \frac{\text{N.m}^2}{\text{C}^2},$$

where ε_o is the permittivity of free space, $\varepsilon_o \approx 9\times10^{-12}$ C²/(N.m²). Coulomb's law applies to elementary particles and small charged objects as long as their sizes are much less than the distance between them. It is also applies to uniform spherical shells or spheres of charge. In that case, r, the distance between the centers of the spheres, must be larger than the sum of the sphere radii; that is to say, the charge must be largely separated.

The electrostatic force between two point charges can be expressed in vector form. The force on q_1 due to q_2 is written as:

$$\boldsymbol{F}_{12} = k\,\frac{q_1 q_2}{r_{12}}\ \hat{r}_{12}$$

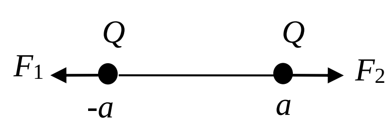


where $\hat{r}_{12}$ is a unit vector directed from q_1 to q_2 and r_{12} is the distance between them..
When more than one point charge is present, the total electrostatic force exerted on the i^{th} charge is the vector sum of the forces exerted on that charge by others individually, i.e.

$$\boldsymbol{F}_i = \sum_{\substack{j=1\\ i<j}} \boldsymbol{F}_{ij} = kq_i \sum_{i} \frac{q_j}{r_j^2}\boldsymbol{r}_j$$

where $\boldsymbol{r}_j$ is a vector directed from the charge, q_i, to the point in question.

❖ Two identical positive charges Q are on the x-axis at $-a$ and $+a$. Find the magnitude and direction of the net force on each charge.



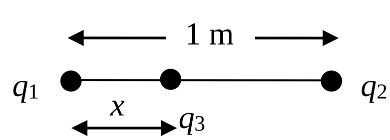
Answer:

(i) The net force acting on the charge at $-a$ is:

$$\Rightarrow \qquad |F_1| = k\,\frac{QQ}{(2a)^2} = k\,\frac{Q^2}{4a^2},\ \text{to the negative x-axis.}$$

The net force acting on the charge at a is:

$$\Rightarrow \quad |F_2| = k\,\frac{QQ}{(2a)^2} = k\,\frac{Q^2}{4a^2},\ \text{to the positive x-axis.}$$



❖ Two charges one of $q_1 = 2\ \mu\text{C}$ and one of $q_2 = 8\ \mu\text{C}$ are one meter apart. Find the location of a third charge of $q_3 = 3\ \mu\text{C}$ which feels no net force on it.

$\Rightarrow$ If the third charge, q_3, is located between the two charges, the effective repulsive forces it feels will cancel each other. Suppose *x* is the distance of q_3 from q_1, then:

$$\because |F_{13}| = |F_{23}|$$

$$\therefore k\,\frac{q_1 q_3}{x^2} = k\,\frac{q_2 q_3}{(1-x)^2} \qquad \Rightarrow \qquad \frac{q_2}{q_1} = \frac{(1-x)^2}{x^2} = \frac{8}{2}$$

Taking the square root, one can have:

$$(1-x) = \pm 2x \qquad \Rightarrow \qquad x = \underset{\text{accepted}}{1/3} \quad , \quad \underset{\text{rejected}}{-1}$$

So, the answer is 0.33 m from q_1.

Notice that the point of equilibrium does not depend on the charge q_3, but on the other charges.

H. W. Two charges one of $q_1 = 9\ \mu\text{C}$ and one of $q_2 = -4\ \mu\text{C}$ are two meters apart. Find the location (from q_1) of a third charge q_3 which feels no net force on it. [Ans: 6 m]

❖ Three point charges of 2 μC, 7 μC, and - 4 μC are located at the corners of an equilateral triangle of side length 0.5 m. Calculate the net force on the 7 μC charge.

$$|F_{7,2}| = (9\times10^9)\frac{(2\times10^{-6})(7\times10^{-6})}{(0.5)^2} = 0.504\ \text{N} \qquad \text{(repulsive)}$$

$$|F_{7,-4}| = (9\times10^9)\frac{(4\times10^{-6})(7\times10^{-6})}{(0.5)^2} = 1.008\ \text{N} \qquad \text{(attractive)}$$

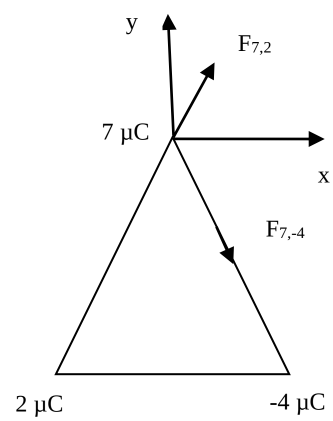
$$|F_x| = \left(|F_{7,2}| + |F_{7,-4}|\right)\cos 60^o = 0.756\ \text{N},$$

$$|F_y| = \left(|F_{7,2}| - |F_{7,-4}|\right)\sin 60^o = -0.436\ \text{N},$$

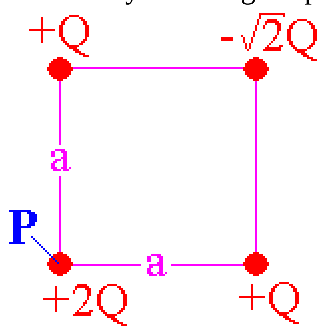
As a vector

$$\mathbf{F} = (0766\ \mathbf{i} - 0.436\ \mathbf{j})\ \text{N}$$

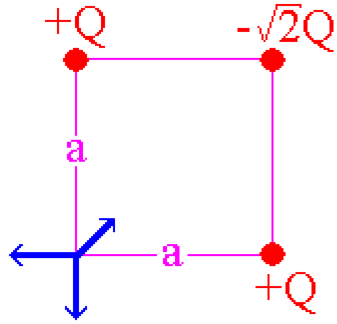
$|\mathbf{F}| = 0.873$ N, at an angle of 330° with the positive x-axis.



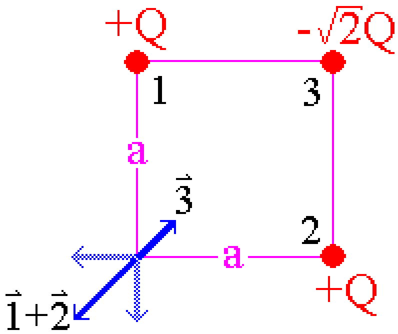
Example: What is the **net force** experienced by the charge at point P?



Solution:
The force experienced by the +2Q charge from the other three charges are indicated below:



Note that the magnitudes of the contributions from 1 and 2 are the same since charges 1 and 2 are the same. A direction has been assigned to the contribution from charge 3 [diagonally inward] but its magnitude relative to the others is not known until we evaluate it.
Recognizing that the resultant of the contributions from 1 and 2 points diagonally outward, as shown below. The solution to this question reduces to a determination of the relative magnitude of the contribution from charge 3 to the resultant of the contributions from 1 and 2.



The relationship for the force between two charges:

$$|\boldsymbol{F}| = k\,\frac{q_i q_j}{r_{ij}^2}$$

Thus the magnitude of the contribution from 1 (and 2) is

$$|\boldsymbol{F_1}| = |\boldsymbol{F_2}| = k\,\frac{(2Q)Q}{a^2}$$

and the resultant is

$$|\boldsymbol{F_1} + \boldsymbol{F_2}| = k\,2\sqrt{2}\,\frac{Q^2}{a^2}$$

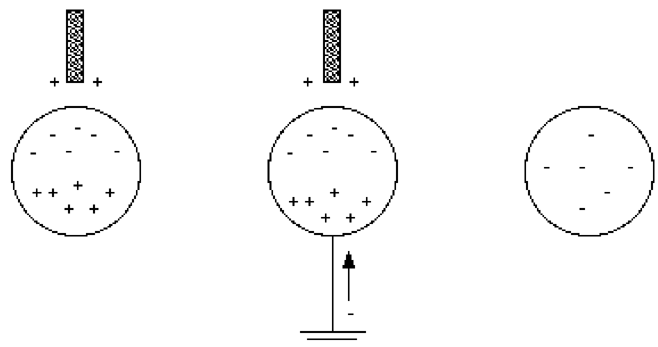
The magnitude of the contribution from 3 is

$$|\boldsymbol{F}_3| = k\,\frac{(2Q)\left(\sqrt{2}Q\right)}{\left(\sqrt{2}a\right)^2} = k\,\sqrt{2}\,\frac{Q^2}{a^2}$$

which is one-half that from 1 and 2 combined, so the relative contributions shown in the previous figure are correct and the direction of ***F*** from all three contributions points diagonally outward (225 degrees or southwest).

Conductors and Insulators

A **conductor** is a material that permits the motion of electric charge through its volume. Examples of conductors are copper, aluminum and iron. An electric charge placed on the end of a conductor will spread out over the entire conductor until an equilibrium distribution is established. In contrast, electric charge placed on an **insulator** stays in place: an insulator (like glass, rubber and Mylar) does not permit the motion of electric charge.



Induction of Charge on Metal Sphere

The properties of a conductor are a result of the presence of free electrons in the material. These electrons are free to move through the entire volume of the conductor. Because of the free electrons, the charge distribution of a conductor can be changed by the presence of external charges. For example, the metal sphere shown in the figure is initially uncharged. This implies that the free electrons (and positive ions) are distributed uniformly over its surface. If a rod with a positive charge is placed in the vicinity of the sphere, it will produce an attractive force on the free electrons. As a consequence of this attractive force the free electrons will be redistributed, and the top of the conductor will get a negative charge (excess of electrons). Since the number of free electrons on the sphere is unchanged, the bottom of the sphere will have a deficit of free electrons (and will have a positive charge). The positive ions are bound to the lattice of the material, and their distribution is not affected by the presence of the charged rod. If we connect the bottom of the sphere to ground (a source or drain of electrons) electrons will be attracted by the positive charge. The number of electrons on the sphere will increase, and the sphere will have a net negative charge. If we break the connection to the ground before removing the charged rod, we are left with a negative charge on the sphere. If we first remove the charged rod, the excess of electrons will drain to the ground, and the sphere will become uncharged.

Suppose that isolated charges Q and q attract each other with a force F. If the separation between these charges were made half as great, each charge would then experience a force

(a) 2F/3.
(b) F.
(c) Can not be determined unless we know the magnitude of Q and q.
(d)@ 4 F.
(e) F/2.

Three charges are located as shown in Figure 1. If a = 3.0 m, Q_1 = 2.0 μC, and Q_2 = Q_3 = 8.0 μC, what is the magnitude of the electric force on charge Q_1?

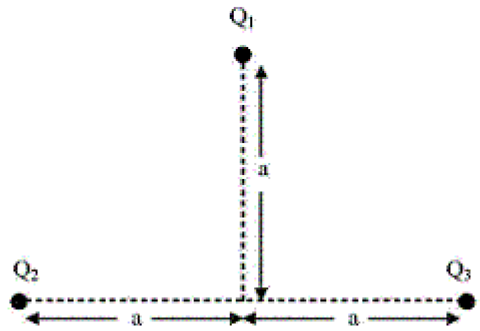


Figure 1

(a) 0.023 N
(b) 0.090 N
(c) 0.046 N
(d)@ 0.011 N
(e) 0.055 N

Three points charges are located on the x-y plane as follows: Q_1 = -10 μC at (4 m, 0), Q_2 = 20 μC at (0, 10 m), and Q_3 at (4 m, 10 m). If the net force on Q_1 points in the negative x-direction, find the charge Q_3.

(a) -24 μC
(b) +16 μC
(c) 0 μC
(d)@ -16 μC
(e) +24 μC

A 0.2 g metallic ball hangs from an insulating string in a vertical electric field of 3000 N/C and directed upward as shown in Figure 1. If the tension in the string is 0.004 N, then the charge on the ball is:

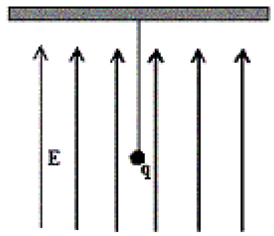


Figure 1

(a) -1.0 μC
(b)@ -0.7 μC
(c) 1.0 μC
(d) -2.0 μC
(e) 0.7 μC

A charge +2q is placed at the origin and a charge -q is placed at x= 0.200 m on the x-axis. Where, on the x-axis, can a third charge +q be placed so that the force on it is zero?
(a) 0.327 m
(b) -0.740 m
(c) -0.440 m
(d) 0.112 m
(e)@ 0.683 m

Consider three point charges, Q_1=Q_2= 2 μC and Q3= 4 μC, located as shown in Figure 1. Find the magnitude of the resultant force on Q_3.

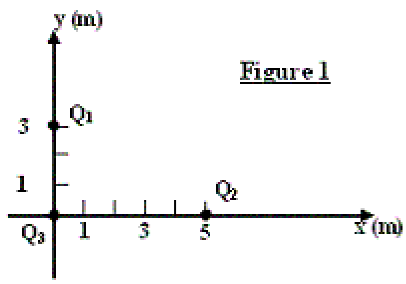


Figure 1

(a) 10*10**(-3) N
(b) 2.9*10**(-3) N
(c) 6.0*10**(-3) N
(d)@ 8.5*10**(-3) N
(e) zero

A negative charge is placed at the center of a square. Each corner of the square has a fixed charge of 1.00 μC. If the resulting force acting on each charge is zero, the magnitude of the negative charge is:
(a) 9.60 μC.
(b) 0.77 μC.
(c) 0.69 μC.

(d)@ 0.96 μC.
(e) 6.92 μC.

Two neutral metal sphere are separated by 0.3 km. How much electric charge must be transferred from one sphere to the other so that their electrical attraction is 10**3 N?
(a) 0.2 C.
(b) 0.9 C.
(c) 0.4 C.
(d) 0.6 C.
(e)@ 0.1 C.

A charge of + 3.2 μC is placed at the origin. A second charge (q_2) is placed at x = 3.0 m. If a charge of 1.0 μC experiences no force if placed at x = 4.0 m, then q2 is:
(a) + 0.2 μC
(b) - 3.3 μC
(c)@ - 0.2 μC
(d) + 2.1 μC
(e) - 2.1 μC

Two small charged objects repel each other with a force F when separated by a distance d. If the charge on each object is reduced to one-fourth of its original value and the distance between them is reduced to d/2 the force becomes:
(a) F.
(b)@ F/4.
(c) F/16.
(d) F/8.
(e) F/2.

Two fixed particles, of charges q1 = + 1.0 μC and q2= - 9.0 μC, are 10 cm apart. How far from each should a third charge be located so that no net electrostatic force acts on it?
(a) 1.1 cm from q1 and 11.1 cm form q2.
(b)@ 5.0 cm from q1 and 15.0 cm from q2.
(c) 3.0 cm from q1 and 7.0 cm from q2.
(d) 1.0 cm from q1 and 11.0 cm from q2.
(e) 1.0 cm from q1 and 9.0 cm from q2.

A mass with a charge "Q" is suspended in equilibrium from a beam balance. A point charge q = + 10 μC is then fixed at a distance d = 5.0 cm below "Q" and an extra mass m = 4.0 g has to be placed on the pan to obtain equilibrium, see figure (3). Find the value of the charge "Q".

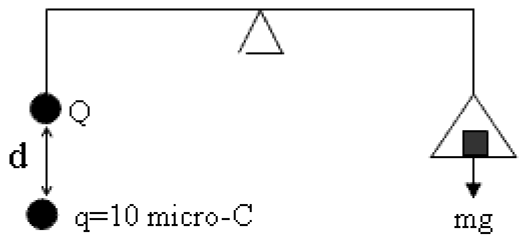


Figure 3

(a) + 6.2*10**(-9) C.
(b)@ - 1.1*10**(-9) C.
(c) + 3.3*10**(-9) C.
(d) + 1.1*10**(-9) C.
(e) - 3.3*10**(-9) C.

Charges q1 and q2 are on the x-axis. q1 is at x = a and q2 is at x = 2a. The net force on a third charge at the origin is zero. Which of the following is TRUE ?
(a)@ q2 = -4*q1
(b) q2 = 2*q1
(c) q2 = 4*q1
(d) q2 = -q1
(e) q2 = -2*q1

Two point charges q1 and q2 lie along the x-axis. q1 = + 16.0 μC is at x = 2.00 m and q2 = + 9.00 μC is at the origin. Where must a negative charge q3 be placed on the x-axis such that the net electrostatic force on it is zero?
(a)@ x = + 0.857 m
(b) x = - 0.857 m
(c) x = + 1.14 m
(d) x = + 2.86 m
(e) x = - 1.14 m

As in figure (1), a charge Q is fixed at each of two opposite corners of a square. A charge q is fixed at each of the other two corners. If the resultant electrical force on Q is zero, then Q and q are related as:

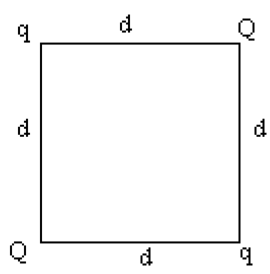


Figure (1)

(a) Q = - 2 Sqrt(2) q**2
(b)@ Q = - 2 Sqrt(2) q
(c) Q = q**2
(d) Q = q

(e) Q = - 4 q

Consider two identical conductor spheres, A and B. Initially, sphere A has a charge of -80 Q and Sphere B has a charge of +20 Q. If the spheres touched and then are separated by a distance of 0.3 m, what is the resultant force between them? [Take Q = 5.7*10**(-8) C]

(a)@ 0.3 N, repulsive.
(b) 0.4 N, attractive.
(c) 0.2 N, attractive.
(d) 0.2 N, repulsive.
(e) 0.3 N, attractive.

In figure 3, Q = 60 micro-C, q = 20 micro-C, a = 3.0 m, and b = 4.0 m. Calculate the total electric force on q. [i and j are the unit vectors in the positive direction of x-axis and y-axis, respectively].

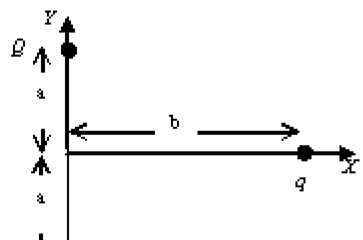


Figure 3

(a) 1.12 j (N).
(b) -0.34 i (N).
(c) -0.69 i (N).
(d) 0.34 i (N).
(e)@ 0.69 i (N).

In figure (1), if Q = 30 μC, q = 5.0 μC and d = 0.3 m, find the net force on q. [i and j are the unit vectors in the positive direction of x-axis and y-axis, respectively].

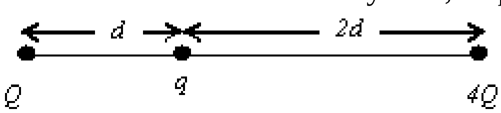


Figure 1

(a) -7.5 i (N).
(b) 7.5 i (N).
(c) 3.8 i (N).
(d) -3.8 j (N).
(e)@ zero.

What is the electric force between two protons which are separated by 1.6*10**(-15) m.

(a) zero.
(b) 90 N, attractive.
(c) 2.2 N, repulsive.
(d) 2.2 N, attractive.

(e)@ 90 N, repulsive.

Two positive charges (+8.0 C and +2.0 C) are separated by 300 m. A third charge is placed a distance r from the +8.0 C charge so that the resultant electric force on the third charge due to the other two charges is zero. The distance r is:

(a) 500 m.
(b) 100 m.
(c) 300 m.
(d)@ 200 m.
(e) 400 m.

Two positively charged particles q1 and q2 (with q2>q1) are fixed in place on the x-axis at the positions shown in figure 1. A third charge q3 is to be placed somewhere on the x-axis such that the net electrostatic force on q3 is zero. Which one of the following statements is TRUE?

Figure 1

(a) q3 should be placed at a point between q1 and q2 but closer to q2.
(b) q3 should be placed to the left of q1.
(c) q3 should be placed to the right of q2.
(d) q3 should be placed at the mid point between q1 and q2.
(e)@ q3 should be placed at a point between q1 and q2 but closer to q1

Consider the arrangement in Fig. 1. If the charged rod is approaching the neutral metal rod without touching it, which of the following is correct?

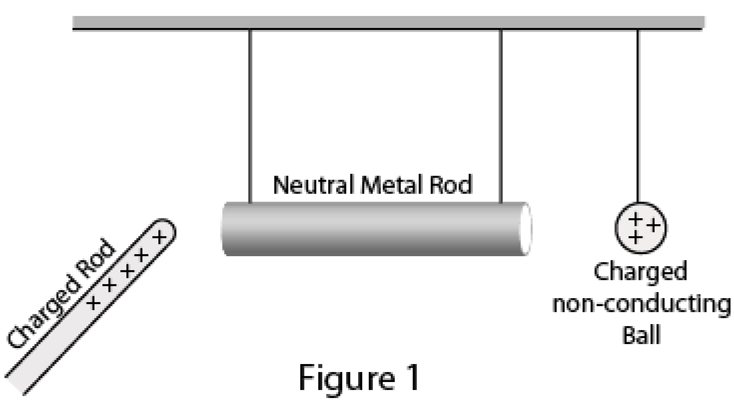


Figure 1

(a)@ The ball will be repelled to the right
(b) The ball will be attracted to the left
(c) The ball will stay hanging vertically
(d) The neutral metal rod will be pushed up
(e) The neutral metal rod will be pushed down.

Supplementary Problems

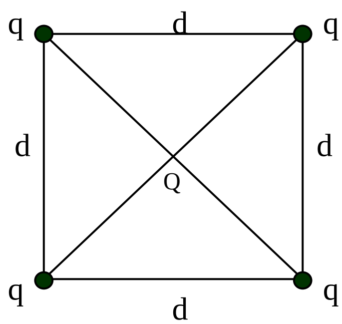


FIGURE 1

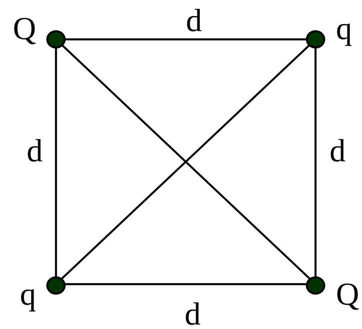


FIGURE 2

1- A negative charge "Q" is placed at the center of a square, see figure (1). Each corner of the square has a fixed charge of q = 1.00 x10^{-6} C. If the resulting force acting on each charge is zero, the magnitude of the negative charge is: Ans: 0.96x10^{-6} C.

2- Two neutral metal sphere are separated by 0.3 km. How much electric charge must be transferred from one sphere to the other so that their electrical attraction is 10^3 N? Ans: 0.1 C.

3- As in figure (2), a charge Q is fixed at each of two opposite corners of a square. A charge q is fixed at each of the other two corners. If the resultant electrical force on Q is zero, then Q and q are related as: Ans: Q = - 2 Sqrt(2) q

4- Consider two identical conductor spheres, A and B. Initially, sphere A has a charge of -80 Q and Sphere B has a charge of +20 Q. If the spheres touched and then are separated by a distance of 0.3 m, what is the resultant force between them? [Take Q = 5.7 x10^{-8} C] Ans:0.3 N.

5- A 2.0 micro-C charge is placed at the origin. An identical charge is placed 2.0 m from the origin on the x-axis, and a third identical charge is placed 2 m from the origin on the y-axis. The magnitude of the force on the charge at the origin is: [1.3*10**(-2) N]

Question 1
Two point charges q1 and q2 lie along the x-axis.
q1 = + 16.0 micro-Coulombs is at x = 2.00 m and
q2 = + 9.00 micro-Coulombs is at the origin.
Where must a negative charge q3 be placed on
the x-axis such that the net electrostatic force
on it is zero ?

a x = + 1.14 m
b x = - 1.14 m
c x = + 0.857 m
d x = - 0.857 m
e x = + 2.86 m

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Question 2
As in figure (1), a charge Q is fixed at each of two opposite corners of a square. A charge q is fixed at each of the other two corners. If the resultant electrical force on Q is zero, then Q and q are related as:`

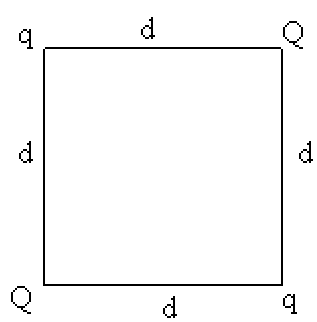

Figure (1)

a Q = - 2 Sqrt(2) q**2
b Q = q
c Q = - 2 Sqrt(2) q
d Q = q**2
e Q = - 4 q

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Question 3
Two fixed particles, of charges q1 = + 1.0*10**(-6) C and q2= - 9.0*10**(-6) C, are 10 cm apart. How far from each should a third charge be located so that no net electrostatic force acts on it?

a 1 cm from q1 and 11 cm from q2.
b 5 cm from q1 and 15 cm from q2.
c 3 cm from q1 and 7 cm from q2.
d 1 cm from q1 and 9 cm from q2.
e 1.1 cm from q1 and 11.1 cm form q2.

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Question 4
Consider two identical conductor spheres, A and B. Initially, sphere A has a charge of -80 Q and Sphere B has a charge of +20 Q. If the spheres touched and then are separated by a distance of 0.3 m, what is the resultant force between them? [Take Q = 5.7*10**(-8) C]

a 0.2 N.
b 0.3 N.
c 0.4 N.
d 0.6 N.
e 0.9 N.

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Question 5
Charges q1 and q2 are on the x-axis. q1 is at x = a and q2 is at x = 2a. The net force on a third charge at the origin is zero.
Which of the following is TRUE ?

a q2 = -4*q1
b q2 = 2*q1
c q2 = -q1
d q2 = 4*q1
e q2 = -2*q1

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Answers
1 c
2 c
3 b
4 b
5 a