Prof. Dr. I. M. Nasser | Friday, June 26, 2009

Multiple Choice Questions: Circle the correct answer.

Correct answer	**1 point**
Wrong answer	**-0.25 point**
No answer	**0 point**

1- A particle of mass m has the wave function $\psi(x,t)=e^{i\omega t}\left[\alpha\cos(kx)+\beta\sin(kx)\right]$, where α and β are complex constants and ω and k are real constants. The probability current density is equal to which of the following? (Note: α^* denotes the complex conjugate of α, and $|\alpha|^2=\alpha^*\alpha$.)

(A) $\hbar k/m$

(B) $\frac{\hbar k}{m}\left(|\alpha|^2+|\beta|^2\right)$

(C) $\frac{\hbar k}{m}\left(|\alpha|^2-|\beta|^2\right)$

(D) $\frac{\hbar k}{2mi}\left(\alpha^*\beta-\beta^*\alpha\right)$

(E) 0

2- The solution to the Schrödinger equation for a particle bound in a one-dimensional, infinitely deep potential well, indexed by quantum number *n*, indicates that in the middle of the well the probability density vanishes for:

(A) the ground state $(n=1)$ only

(B) states of odd n $(n=1,3,\cdots)$

(C) states of even n $(n=2,4,\cdots)$

(D) all states $(n=1,2,3,\cdots)$

(E) all states except the ground state

3- The lowering operator $\hat{a}\equiv\frac{1}{\sqrt{2m\hbar\omega}}\left(m\omega\hat{x}+i\hat{p}\right)$ has the flowing property (or properties).

I. $\hat{a}$ commutes with the Hamiltonian $\hat{H}=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}kx^2$

II. $\hat{a}$ is a Hermitian operator and therefore an observable.

III. The adjoint operator $\hat{a}^\dagger\neq\hat{a}$.

(A) II only

(B) I only

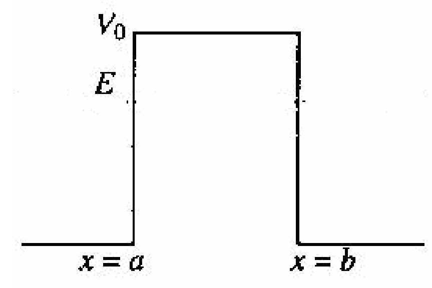
(C) III only

(D) III and II only

(E) I and III only

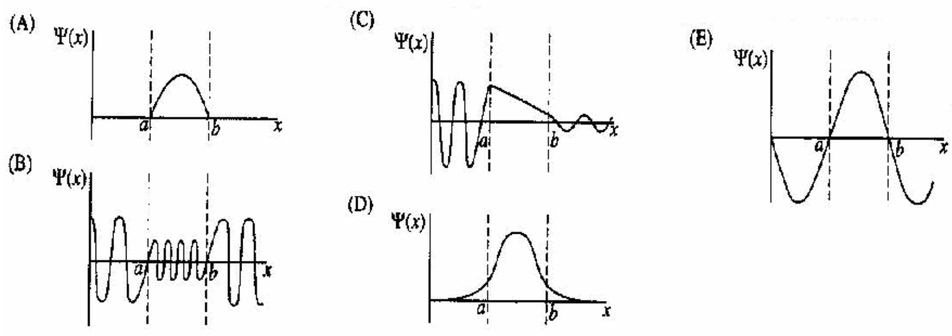
4- Consider a potential of the form

$$V(x)=\begin{cases}0 & x\le a\\ V_0 & a<x<b\\ 0 & x\ge b\end{cases}$$



as shown in the figure.

Which of the corresponding wave functions is possible for a particle incident from left with energy $E < V_o$? C



5- The degree of degenracy of the energy-level $14\frac{\pi^2\hbar^2}{2mL^2}$ of a particle in a cubiccal potential box is

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

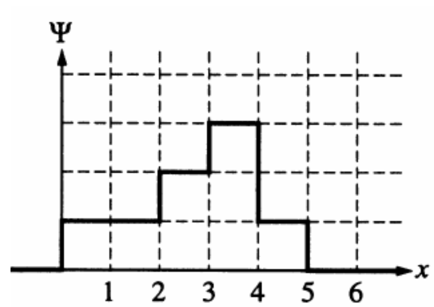
$14 \equiv (1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), (3,2,1)$

6- Which of the following functions could represent the radial wave function for an electron in an atom? (r is the distance of the electron from the nucleus; A and b are constants).

I. Ae^{-br}, II. $A\sin(br)$, III. A/r

(A) II only
(B) I only
(C) III only
(D) III and II only
(E) I, II and III

7- The wave function for a particle constrained to move in one dimension is shown in the graph ($\Psi = 0$ for $x \leq 0$ and $x \geq 5$). What is the probability that the particle would be found between $x = 2$ and $x = 4$?

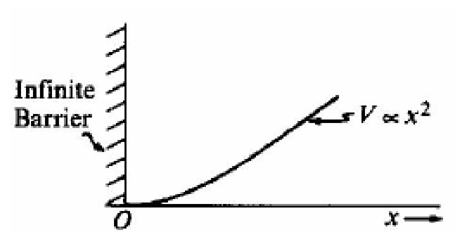


(A) $17/64$
(B) $13/16$
(C) $\sqrt{5/8}$
(D) $5/8$
(D) $25/64$

Normalization condition $\Rightarrow A^2(1+1+4+9+1) = 1 \Rightarrow A^2 = 1/16;$

$$P(2 \rightarrow 4) = A^2(4+9) = \frac{13}{16}$$

8- A particle of mass m is acted on by a harmonic force with potential energy function $V(x) = \frac{1}{2} m\omega^2 x^2$ (a one-dimensional simple harmonic oscillator). If there is a wall at $x = 0$ so that $V = \infty$ for $x < 0$ then the energy levels are equal to:



(A) $0, \hbar\omega, 2\hbar\omega, \cdots$
(B) $\frac{1}{2}\hbar\omega, \frac{3}{2}\hbar\omega, \frac{5}{2}\hbar\omega, \cdots$
(C) $0, \frac{1}{2}\hbar\omega, \hbar\omega, \cdots$
(D) $\frac{3}{2}\hbar\omega, \frac{7}{2}\hbar\omega, \frac{11}{2}\hbar\omega, \cdots$
(E) $0, \frac{3}{2}\hbar\omega, \frac{5}{2}\hbar\omega, \cdots$

9- A particle of mass m moving in the harmonic oscillator potential $V(x)=\frac{1}{2}m\omega^2x^2$ is in the normalized state

$$\psi(x,0)=\frac{1}{\sqrt{14}}\left[|1\rangle-2|2\rangle+3|3\rangle\right]$$

where $\psi_n(x)$ are the eigenfunctions of the harmonic oscillator corresponding to the energies $E_n=(n+\frac{1}{2})\hbar\omega$. What is the average value of the energy in this state?

(A) $\frac{102}{14}\hbar\omega$

(B) $\frac{43}{14}\hbar\omega$

$$\langle E\rangle=\frac{1}{14}\left[\frac{3}{2}\times1+\frac{5}{2}\times4+\frac{7}{2}\times9\right]\hbar\omega=\frac{43}{14}\hbar\omega$$

(C) $\frac{23}{14}\hbar\omega$

(D) $\frac{17}{\sqrt{14}}\hbar\omega$

(E) $\frac{7}{\sqrt{14}}\hbar\omega$

10- For the eigen value equation $\left[ar^2,p_x\right]\psi=E\psi$, where a is a constant, r is magnitude of the displacement in 3D, and p_x is the x-component of the momentum operator, the E will be equal to:

(A) $2i\,a\hbar x$

(B) $i\,a\hbar x$

$$\left[ar^2,p_x\right]=a\left[x^2+y^2+z^2,p_x\right]=a\left[x^2,p_x\right]=a(2i\hbar x)$$

(C) $2\,a\hbar$

(D) $2\,a\hbar x$

(E) $3i\,a\,x$

11- The raising and lowering operators for the quantum harmonic oscillator satisfy: $\hat{a}^\dagger|n\rangle=\sqrt{n+1}|n+1\rangle$; $\hat{a}|n\rangle=\sqrt{n}|n-1\rangle$ for energy eigenstates $|n\rangle$ with energy E_n. For $n=2$. Calculate is the expectation value of the operator $\hat{C}=V\left(\hat{a}+\hat{a}^\dagger\right)^2$, where V is a constant.

(A) V

$(B)\quad 2\sqrt{2}\,V$

$(C)\quad \sqrt{2}\,V$

$(D)\quad 5V$

$(E)\quad 7V$

Answer:

$$V\ \langle n|\left(\hat{a}+\hat{a}^{\dagger}\right)^2|n\rangle = V\ \langle n|\,\hat{a}^2 + \hat{a}^{\dagger}\hat{a} + \hat{a}\hat{a}^{\dagger} + \hat{a}^{\dagger 2}\,|n\rangle;$$

$$\hat{a}\hat{a}^{\dagger}|n\rangle = \sqrt{(n+1)}\sqrt{(n+1)}\,|n\rangle = 3|n\rangle;$$

$$\hat{a}^{\dagger}\hat{a}|n\rangle = \sqrt{n}\sqrt{n}\,|n\rangle = 2|n\rangle;$$

$$V\ \langle n|\left(\hat{a}+\hat{a}^{\dagger}\right)^2|n\rangle = 5V$$

12-In the eigen value equation $\hat{A}|\psi\rangle = a|\psi\rangle$, calculate the eigen function $|\psi\rangle$ for the operator $\hat{A} = \frac{d}{dx} - 2x$.

$(A)\quad |\psi\rangle = e^{x+x^2/2}$

$(B)\quad |\psi\rangle = e^{ax-x^2/2}$

$(C)\quad |\psi\rangle = e^{ax+x^2}$

$(D)\quad |\psi\rangle = e^{ax-x^2}$

$(E)\quad |\psi\rangle = e^{ax+x^2/2}$

Answer:

$$\hat{A}|\psi>=a|\psi>, \qquad \hat{A} = \frac{d}{dx} - 2x$$

$$\left(\frac{d}{dx} - 2x\right)|\psi> = a|\psi> \Rightarrow \frac{d}{dx}|\psi> = (a+2x)|\psi>$$

$$\int \frac{d|\psi>}{|\psi>} = \int (a+2x)dx \Rightarrow |\psi> = e^{ax+x^2}$$

13-For the wave function:

$$\psi(x)=\begin{cases} c\,x, & 0\le x\le 1, \quad c=\text{ constant} \\ 0, & \text{otherwise} \end{cases}$$

Calculate the probability to find the particle in the range from 0.5 to 1.0.

$(A)\quad P(0.5,1.0)=2/9$

$(B)\quad P(0.5,1.0)=13/15$

$(C)\quad P(0.5,1.0)=1/4$

$(D)\quad P(0.5,1.0)=1/2$

$(E)\quad P(0.5,1.0)=7/8$

Answer

$$c^2\int_0^1 x^2 dx=1 \Rightarrow c^2\frac{x^3}{3}\Big|_o^1=c^2(\frac{1}{3})=1$$

$$\Rightarrow c=\sqrt{3}$$

$$P(0.5,1.0)=1-P(0.0,0.5)$$

$$=1-3\int_0^{0.5} x^2 dx=1-3\frac{x^3}{3}\Big|_o^{0.5}=\frac{7}{8}$$

14- If you define the operators $\hat{A}=\frac{1}{(\,)}$, $\hat{B}=\frac{d}{dx}$, $\hat{C}=x$, and the wave function $\psi=\frac{1}{4}x^3-\frac{1}{x}$, calculate the result of the expression $\hat{A}\hat{B}\hat{C}\psi$

$(A)\quad x^3$

$(B)\quad x^{-3}$

$(C)\quad x^4$

$(D)\quad x^{-2}$

$(E)\quad 0$

Answer:

$$\hat{B}\hat{C}\psi=\frac{d}{dx}\left[x(\frac{1}{4}x^3-\frac{1}{x})\right]=\frac{d}{dx}\left[\frac{1}{4}x^4-1\right]$$

$$=x^3,$$

$$\Rightarrow \hat{A}\hat{B}\hat{C}\psi=\frac{1}{x^3}.$$

15- If the relation $\left[\hat{A},\hat{B}\right]^{\dagger}=-\left[\hat{A},\hat{B}\right]$ is satisfied, then the two operators $\hat{B}$ and $\hat{A}$ are:

(A) Hermitian
(B) anti – Hermitian
(C) Orthogonal
(D) Normalized
(E) Equal

Answer:

$$\left[\hat{A},\hat{B}\right]^{\dagger}=\left[\hat{A}\hat{B}-\hat{B}\hat{A}\right]^{\dagger}=\left[\hat{B}^{\dagger}\hat{A}^{\dagger}-\hat{A}^{\dagger}\hat{B}^{\dagger}\right]=-\left[\hat{A}^{\dagger},\hat{B}^{\dagger}\right]$$
$$\Rightarrow \hat{B}=\hat{B}^{\dagger}, \quad \hat{A}=\hat{A}^{\dagger}$$

16- The degeneracy of the state 4*f* in the hydrogen atom will be (neglect the spin):

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

$$f \Rightarrow \ell=3 \Rightarrow d=2\ell+1=7$$

17- If you define the operator $\hat{D}=\frac{d}{dx}$, then the operator $(\hat{D}+\hat{x})^2$ will be:

(A) $\hat{D}^2+2\hat{x}\,\hat{D}+\hat{x}^2+1$
(B) $\hat{D}^2+\hat{x}^2+1$
(C) $\hat{D}^2+2\hat{x}\,\hat{D}+\hat{x}^2$
(D) $\hat{D}^2+2\hat{x}\,\hat{D}-\hat{x}^2$
(E) $\hat{D}^2+\hat{x}\,\hat{D}+\hat{x}^2+1$

Answer:

$$\begin{aligned}(\hat{D}+\hat{x})^2 &= (\hat{D}+\hat{x})(\hat{D}+\hat{x})\\ &= \hat{D}^2+\hat{x}\hat{D}+\hat{D}\hat{x}+\hat{x}^2\\ &= \hat{D}^2+\hat{x}\hat{D}+\hat{x}\hat{D}+1+\hat{x}^2\\ &= \hat{D}^2+2\hat{x}\hat{D}+\hat{x}^2+1\end{aligned}$$

18- A particle in an infinite square well (of width *a and bounded at* $x=\pm 1$) has as its initial wave function an equal mixture of the first two stationary states:

$$\psi(x) = A\left[\psi_1(x)+\psi_2(x)\right]$$

Find the expectation value of the Hamiltonian operator, *H*, in terms of E_1 and E_2.

[Hint: Use the symmetric wave functions: $\psi_1 = \left(\frac{2}{a}\right)^{\frac{1}{2}}\cos\left(\frac{\pi}{a}x\right)$,

$\psi_2 = \left(\frac{2}{a}\right)^{\frac{1}{2}}\sin\left(\frac{2\pi}{a}x\right)$ and $E_n = n^2\left(\frac{\hbar^2\pi^2}{2ma^2}\right)$, $n=1,2,\cdots$]

(A) $(E_1+E_2)/\sqrt{2}$
(B) $(E_1-E_2)/2$
(C) $(E_1+E_2)/2$
(D) $2(E_1+E_2)$
(E) $(E_1+E_2)/4$

$$\begin{aligned}\langle H\rangle &= \int_{-a/2}^{a/2}\psi(x,t)^*(H)\psi(x,t)x\,dx = A^2\int_{-a/2}^{a/2}\left(\psi_1^*+\psi_2^*\right)\left(H\psi_1+H\psi_2\right)dx\\ &= A^2\int_{-a/2}^{a/2}\left(\psi_1^*+\psi_2^*\right)\left(E_1\psi_1+E_2\psi_2\right)dx = A^2\int_{-a/2}^{a/2}\left(E_1\left|\psi_1\right|^2+E_2\left|\psi_2\right|^2\right)dx\\ &= A^2\left(E_1+E_2\right) = \left(E_1+E_2\right)/2\end{aligned}$$

19- A particle moves in the gravitational field given by the Hamiltonian:

$$H(t) = \frac{p^2(t)}{2m}+mgx\,.$$

The equation of motion $x(t)$ will be:

(A) $x(t) = x(0)+\frac{1}{m}\left[p(0)-\frac{1}{2}mgt\right]$

(B) $x(t) = x(0)+\frac{1}{m}\left[p(0)t-\frac{1}{2}mgt^2\right]$

$(C)\quad x(t)=x(0)+\frac{1}{m}\left[p(0)t-mgt^2\right]$

$(D)\quad x(t)=x(0)+\left[p(0)t-\frac{1}{4}mgt^2\right]$

$(E)\quad x(t)=x(0)+\frac{1}{2}mgt^2$

Answer: Using the expressions:

$$\frac{dx(t)}{dt}=\frac{i}{\hbar}\left[H(t),x(t)\right]=\frac{i}{\hbar}\left[\frac{p^2(t)}{2m}+x(t)\right]=\frac{p(t)}{m};$$

and

$$\frac{dp(t)}{dt}=\frac{i}{\hbar}\left[H(t),p(t)\right]=\frac{i}{\hbar}\left[mgx(t)+p(t)\right]=-mg$$

$$\Rightarrow \underline{p(t)=p(0)-mgt}$$

then

$$\frac{dx(t)}{dt}=\frac{p(t)}{m}=\frac{1}{m}\left[p(0)-mgt\right]$$

$$\Rightarrow \underline{x(t)=x(0)+\frac{1}{m}\left[p(0)t-\frac{1}{2}mgt^2\right]}$$

20- The total energy of an electron in the ground state, **1s**, of the hydrogen atom is -13.6 eV. What is the expectation value of the potential energy of the electron in **3p** state:

$(A)\quad -13.6$ eV

$(B)\quad -1.51$ eV

$(C)\quad -3.02$ eV

$(D)\quad 1.51$ eV

$(E)\quad 3.02$ eV

$$E_{3p}=\frac{E_{1s}}{n^2}=\frac{-13.6}{9}=-1.51 \text{ eV}$$

$$\langle V\rangle=\frac{2E_{3p}}{m+2}=\frac{2\times(-1.51)}{-1+2}=-3.02 \text{ eV}$$

In case of Coulmbic potential, $m=-1$.

21- Consider the operators whose action is defined by the equations below

$$O_1\psi(x)=x^3\psi(x);$$

$$O_2\psi(x)=x\frac{d}{dx}\psi(x)$$

Find the commutator $\left[O_1,O_2\right]$.

$(A)\quad -3O_1$

$(B)\quad -3O_2$

$(C)\quad -3O_2\,O_1$

$(D)\quad -2O_1$

$(E)\quad -2O_2$

Answer:

$[O_1,O_2]\psi(x) = (O_1O_2 - O_2O_1)\psi(x)$

$= O_1O_2\psi(x) - O_2O_1\psi(x)$

$= O_1 x d\psi(x)/dx - O_2 x^3\psi(x) = x^4 d\psi(x)/dx - xd(x^3\psi(x))/dx$

$= x^4 d\psi(x)/dx - 3x^3 d\psi(x)/dx) - x^4 d\psi(x)/dx$

$= -3x^3 d\psi(x)/dx) = -3O_1\psi(x)$ for any $\psi(x)$.

Therefore $[O_1, O_2] = -3O_1$.

22-A beam of monoenergetic particles, of mass m, energy E and wave number $k = \sqrt{\frac{2mE}{\hbar^2}}$, is incident from left (negative x.) on a potential expressed in terms of the Dirac delta function in the form:

$$V(x) = \frac{\hbar^2}{2m}\alpha\,\delta(x); \qquad \alpha = \text{ positive constant}$$

Calculate the transmission coefficient.

$(A)\quad \dfrac{\alpha^2}{\alpha^2 + k^2}$

$(B)\quad \dfrac{k^2}{\alpha^2 + k^2}$

$(C)\quad \dfrac{\alpha^2}{\alpha^2 - k^2}$

$(D)\quad \dfrac{k^2}{\alpha^2 - k^2}$

$(E)\quad \dfrac{2k^2}{\alpha^2 + i\,k^2}$

Answer: The Schrödinger equation in the regions other than the infinitesimal neighborhood of x = 0 is:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0 \quad \text{for} \quad x < 0_- \text{ and } x > 0_+$$

With the solution $k^2 = \dfrac{2m}{\hbar^2}E$.

$$\psi(x) = Ae^{ikx} + Be^{-ikx} \qquad x < 0_-$$
$$\psi(x) = Ce^{ikx} \qquad x > 0_+ \text{ (there is no reflection beam in this region)}$$

We have

$$\psi(0_+) = \psi(0_-)$$

And the integration of the Schrödinger equation gives:

$\psi'(0_+) = \psi'(0_-) = 2\alpha\psi(0)$

This leads to

$\frac{A}{B} = \frac{ik - \alpha}{\alpha}, \quad \frac{C}{B} = \frac{ik}{\alpha}$

Hence the coefficients are:

$R = \left|\frac{B}{A}\right|^2 = \frac{\alpha^2}{\alpha^2 + k^2}, \quad T = \left|\frac{C}{A}\right|^2 = \frac{k^2}{\alpha^2 + k^2}$

It is easily seen that $R + T = 1$

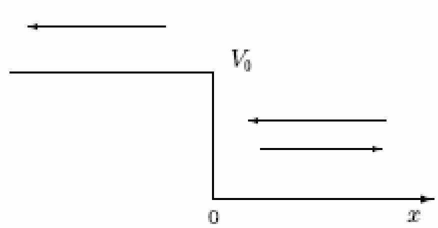
If the potential barrier is highly opaque, i.e. $\alpha \to \infty$, we have

$R = 1, \qquad T = 0$

23- The energy eigenvalues of the Hermitian operators:

(A) are always real numbers
(B) are always complex numbers
(C) are always unbounded
(D) are always zeros
(E) are always singulars

24-A particle of mass m and energy $E < V_o$ approaches the "step" at $x = 0$ from $x = \infty$. At $x = 0$ it can be reflected or transmitted as indicated by the arrows.



$$V(x) = \begin{cases} V_0 & \text{for } -\infty < x < 0 \\ 0 & \text{for } 0 < x < \infty \end{cases}$$

If the relation between the reflection amplitude "B" and the incident amplitude "A" is expressed as $B = Ae^{-2i\delta}$, then δ will be:

$(A)\quad \tan^{-1}\sqrt{\dfrac{E}{(V_o+E)}}$

$(B)\quad \tan^{-1}\sqrt{\dfrac{(V_o+E)}{E}}$

$(C)\quad \tan^{-1}\sqrt{\dfrac{E}{(V_o-E)}}$

$(D)\quad \tan^{-1}\sqrt{\dfrac{(V_o-E)}{E}}$

$(E)\quad \tan^{-1}\sqrt{\dfrac{iE}{(V_o+iE)}}$

Answer:

Define $\alpha=\sqrt{\dfrac{2mE}{\hbar^2}}$; $\quad \beta=\sqrt{\dfrac{2m\,(E-V_o)}{\hbar^2}}$, then by solving Schrodinger equations one can find:

$$\psi_I(x)=Ae^{i\alpha x}+Be^{-i\alpha x} \qquad x>0$$

$$\psi_{II}(x)=Ce^{-\beta x} \qquad x<0$$

B.C. implies $A+B=C$, $\quad A-B=i\dfrac{\beta}{\alpha}C$

Solve for B and C, $\quad B=\dfrac{\alpha-i\beta}{\alpha-i\beta}$, $\quad C=\dfrac{2\alpha}{\alpha+i\beta}$

$$B=\frac{\alpha-i\beta}{\alpha+i\beta}A=Ae^{-2i\delta}, \tan\delta=\frac{\beta}{\alpha}=\sqrt{\frac{(V_o-E)}{E}}$$

25- Let $\{|1\rangle,|2\rangle\}$ be an orthonormal basis for a two dimensional Hilbert space. Let $|A\rangle$ and $|B\rangle$ be defined as: $|A\rangle=\dfrac{1}{\sqrt{2}}\{|1\rangle+e^{i\theta}|2\rangle\}$, $\quad |B\rangle=\dfrac{1}{\sqrt{2}}\{|1\rangle-e^{i\theta}|2\rangle\}$.

The operator $\hat{N}=|A\rangle\langle A|+|B\rangle\langle B|$ will be:

$(A)\quad |1\rangle\langle 1|+|2\rangle\langle 2|$

$(B)\quad |1\rangle\langle 1|-|2\rangle\langle 2|$

$(C)\quad |1\rangle\langle 1|+e^{2i\theta}|2\rangle\langle 2|$

$(D)\quad |1\rangle\langle 1|-e^{2i\theta}|2\rangle\langle 2|$

(E) $|2\rangle\langle 2| + e^{2i\theta}|1\rangle\langle 1|$

Answer:

$$|A\rangle\langle A| = \frac{1}{\sqrt{2}}\{|1\rangle + e^{i\theta}|2\rangle\}\frac{1}{\sqrt{2}}\{|1\rangle + e^{-i\theta}|2\rangle\} = \frac{1}{2}\{|1\rangle\langle 1| + |2\rangle\langle 2| + e^{i\theta}|2\rangle\langle 1| + e^{-i\theta}|1\rangle\langle 2|\},$$

$$|B\rangle\langle B| = \frac{1}{\sqrt{2}}\{|1\rangle - e^{i\theta}|2\rangle\}\frac{1}{\sqrt{2}}\{|1\rangle - e^{-i\theta}|2\rangle\} = \frac{1}{2}\{|1\rangle\langle 1| + |2\rangle\langle 2| - e^{i\theta}|2\rangle\langle 1| - e^{-i\theta}|1\rangle\langle 2|\};$$

$$|A\rangle\langle A| + |B\rangle\langle B| = |1\rangle\langle 1| + |2\rangle\langle 2|$$

26-Suppose that $\{|1\rangle, |2\rangle, |3\rangle\}$ is an orthonormal basis for a three dimensional Hilbert space an operator $\hat{B}$ has the following action on the base kets:

$$\hat{B}|1\rangle = i|1\rangle + (1+i)|2\rangle, \qquad \hat{B}|2\rangle = |1\rangle - 3i|3\rangle, \qquad \hat{B}|3\rangle = 0.$$

What is $\hat{B}^{\dagger}|3\rangle$

(A) 0
(B) $3i|2\rangle$
(C) $3i|1\rangle$
(D) $|1\rangle$
(E) $|2\rangle$

Answer:

$$\hat{B}^{\dagger}|3\rangle = |1\rangle\langle 1|\hat{B}^{\dagger}|3\rangle + |2\rangle\langle 2|\hat{B}^{\dagger}|3\rangle + |3\rangle\langle 3|\hat{B}^{\dagger}|3\rangle = |1\rangle 0 + |2\rangle(3i) + |3\rangle 0 = (3i)|2\rangle$$

27- Find the average value of the distance of the electron from the nucleus in the ground state of a hydrogen atom.

(A) $1/2$
(B) 3
(C) 1
(D) 2
(E) $3/2$

Answer

$$\langle r\rangle = \int_{r=0}^{\infty}\int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi}\psi_{100}^{*} r \psi_{100} r^{2} dr d\Omega = \frac{4\pi}{\pi}\int_{r=0}^{\infty} r^{3} e^{-2r} dr = \frac{3}{2}$$

28- For the function $\psi(\varphi) = 2\sqrt{\frac{2}{3}}\sin^{2}\varphi$, the probability to find a particle in the state of $m = 0$ will be?

(A) $1/2$
(B) $1/5$
(C) $1/3$
(D) $1/6$
(E) $2/3$

$$\sin^{2}\varphi = \left(\frac{e^{i\varphi} - e^{-i\varphi}}{2i}\right)^{2} = \left(\frac{2}{4} - \frac{1}{4}e^{\overset{m=2}{\overbrace{2}}\, i\varphi} - \frac{1}{4}e^{\overset{m=-2}{\overbrace{-2}}\, i\varphi}\right)$$

$$P(m = 0) = 4 \times \frac{2}{3} \times \left(\frac{2}{4}\right)^{2} = \frac{2}{3}$$

29- The function $\sqrt{\pi} - \sqrt{3\pi}\cos^{2}\theta$ is an eigenfunction of the operator (s)

(A) $\hat{L}_z$ only
(B) $\hat{L}^2$ only
(C) $\hat{a}$
(D) $\hat{a}^{\dagger}$
(E) $\hat{L}^2$ and $\hat{L}_z$

$$\sqrt{\pi} - \sqrt{3\pi}\cos^{2}\theta = -\pi\sqrt{\frac{16}{5}}Y_{2,0}$$

30- The commutator $\left[\hat{L}_x \hat{L}_y, \hat{L}_z\right]$ reduces to

(A) $2i\hbar \hat{L}_x \hat{L}_y$
(B) $-i\hbar\left(\hat{L}_x^2 + \hat{L}_y^2\right)$
(C) $i\hbar\left(\hat{L}_x^2 - \hat{L}_y^2\right)$

$(D)\quad i\hbar\left(\hat{L}_x^2+\hat{L}_y^2\right)$

$(E)\quad -i\hbar\left(\hat{L}_x^2-\hat{L}_y^2\right)$

Answer

$$\left[\hat{L}_x\hat{L}_y,\hat{L}_z\right]=\hat{L}_x\left[\hat{L}_y,\hat{L}_z\right]+\left[\hat{L}_x,\hat{L}_z\right]\hat{L}_y=$$

$$=i\hbar\left\{\hat{L}_x\hat{L}_x+\left(-\hat{L}_y\right)\hat{L}_y\right\}=i\hbar\left(\hat{L}_x^2-\hat{L}_y^2\right)$$