[illegible]

$$\vec{r} \equiv x\ \hat{i} + y\ \hat{j} + z\ \hat{k}$$

$$d\vec{r} \equiv dx\ \hat{i} + dy\ \hat{j} + dz\ \hat{k}$$

[illegible]

$$\vec{\nabla} \equiv \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \quad \Rightarrow \quad \vec{\nabla}T \equiv \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}$$

[illegible] $\vec{\nabla}\cdot\vec{T}$: measure how much the vector $\vec{T}$ spreads out (diverge) from the point in the questions.

$$\vec{\nabla}\cdot\vec{T} \equiv \frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} + \frac{\partial T_z}{\partial z} = \begin{cases} + & \vec{T} \text{ has a source, or "faucet"} \\ - & \vec{T} \text{ has a sink, or "drain"} \\ 0 & \vec{T} \text{ is said to be solenoidal} \end{cases}$$

[illegible] $\vec{\nabla}\times\vec{T}$: measure how much the vector $\vec{T}$ “curls around” the point in the questions.

$$\vec{\nabla}\times\vec{T} \equiv \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ T_x & T_y & T_z \end{vmatrix}$$

$$\vec{\nabla}\times\vec{T} = 0 \quad \vec{T} \text{ is classified as irrotational.}$$

[illegible]

$$\vec{\nabla}\cdot\left(\vec{\nabla}T\right) = \frac{\partial^2 T_x}{\partial x^2} + \frac{\partial^2 T_y}{\partial y^2} + \frac{\partial^2 T_z}{\partial z^2} \equiv \text{Laplacian}$$

$$\vec{\nabla}\times\left(\vec{\nabla}T\right) = 0$$

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{T}\right) = 0$$

The Fundamental theorem of Calculus

$$\int_a^b \frac{\partial T}{\partial x} dx = T(b) - T(a)$$

⇒ *"The integral of a derivative over an interval is given by the value of the function at the end points (boundaries)"*

a- for **Gradients**

$$\int_{a,P}^{b} \vec{\nabla} \cdot d\,\vec{l} = T(b) - T(a)$$

⇒ *"The line integral of the gradient is given by the value of the function at boundaries"*

Corollaries:

1- $\int_{a,P}^{b} \vec{\nabla} \cdot d\,\vec{l}$ is path independent

2- $\oint \vec{\nabla} \cdot d\,\vec{l} = 0$

Example 1.9, H.W. 1.31

b- For **Divergences** (Gauss' theorem, Green's theorem, Divergence theorem)

$$\int_V \left(\vec{\nabla} \cdot \vec{T}\right) d\tau = \oint_S \vec{T} \cdot d\,\vec{a}$$

⇒ *"The integral of the divergence over a volume is equal to the value of the surface, that bounds the volume, integral of the function"*

Example 1.10, H.W. 1.32

c- For **Curls** (Stokes' theorem)

$$\int_S \left(\vec{\nabla} \times \vec{T}\right) \cdot d\,\vec{a} = \oint_P \vec{T} \cdot d\,\vec{l}$$

⇒ *"The integral of the curl over surface is equal to the value of the closed line, that bounds the surface, integral of the function"*

Example 1.11, H.W. 1.33