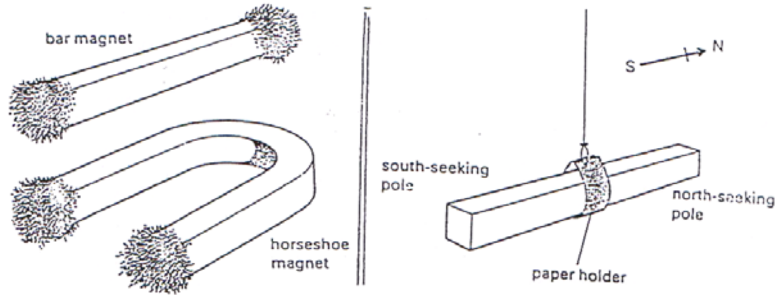
MAGNETIC FIELDS

Properties of Magnet



1- A magnet attracts iron pieces or iron fillings. The filings cling near the ends, at the (poles}) of the magnet.
2- The magnetization is zero at the middle of the magnet and maximum at the poles.
3- When a magnet is freely suspended so that it can swing in a horizontal plane, it comes to rest in N-S direction. N-pole points towards the north, and S-pole points towards the south.
4- First law of magnetism: " like poles repel, unlike poles attract”.
5- When a magnet is broken into pieces, each piece is found to be a magnet with two poles, i.e. no monopole magnet.

What causes magnetism? It is the spinning of the electrons in the atoms of materials, and the behavior of these electrons, which give a material its magnetic properties.

Magnetic field: "It is the region around a magnet in which a magnetic force is exerted". The magnetic flux lines or lines of force represent the magnitude and direction of the magnetic field, i.e. it is vector quantities. Not that:

1- The density of lines is proportional to the strength of the field (i.e. the lines are close together where the field is strong and vice versa).
2- The lines go from **north** pole to the **south** pole.
3- The lines never cross each other.

Classifying the Materials: Regarding the magnetic properties, materials could be classified as:

A) **Non-magnetic materials**: (materials that can not be magnetized by an external magnetic field). For example brass, copper, aluminum, and non-metals.

B) **Magnetic materials** (*Ferromagnetic*): (materials that are magnetized strongly by an external magnetic field). For example Iron, nickel, cobalt, and many alloys, e.g. steel. Ferromagnetic materials are classified as:

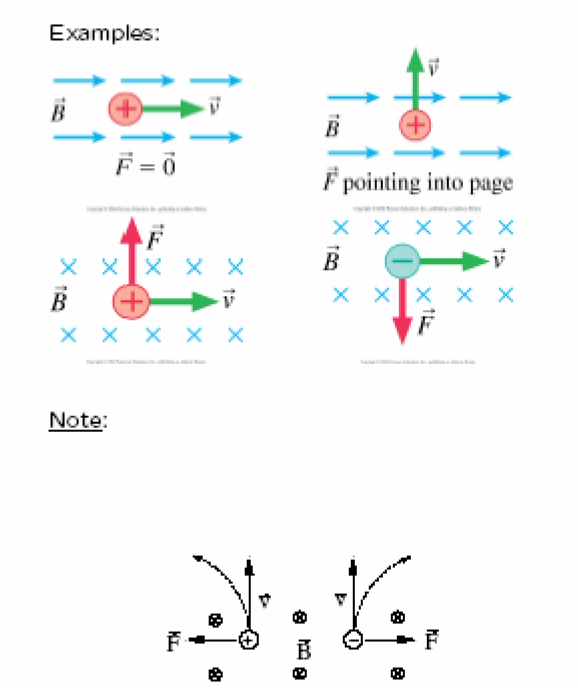
i- **Hard magnetic materials**: such as steel and alcomax (steel-like alloy) are difficult to magnetize and difficult to demagnetize. They are used as a permanent magnet, also in cassette magnetic taps.

ii- **Soft magnetic materials**: such as iron and mumetal (a nickel-based alloy) are easy to magnetize temporary. They are used as electromagnets because in this case they remain magnetized as long as a current is passing through a surrounding coil, i.e. it can be switched on or off.

The Magnetic Force on a Moving Charge: The magnetic force, $\boldsymbol{F}_B$, on a test charge, q_o , moving with velocity v in a magnetic field, is

$$\boxed{\boldsymbol{F}_B = q_o \mathrm{v} \times \boldsymbol{B}}$$

The magnitude of the magnetic force is $|F_B| = q_o \mathrm{v} B \sin\theta$, where θ is the angle between v and $\boldsymbol{B}$. The force is perpendicular both to the velocity and to the magnetic field. Since the force is proportional to the velocity, charges at rest do not experience magnetic forces. [B] = T (Tesla) = $\frac{\text{N}}{\text{A.m}}$. Also, 1T $=10^4$ Gauss.



Example: an electron is projected into a uniform magnetic field given by $\boldsymbol{B}=\left(1.4\,\hat{\mathrm{i}}+2.1\,\hat{\mathrm{j}}\right)\mathrm{T}$.. Find the vector expression for the force on the electron when its velocity is $\mathbf{v}=3.7\times10^5\,\hat{\mathrm{j}}\frac{\mathrm{m}}{\mathrm{s}}$.

Ans:

$$\boldsymbol{F}_B=q_e\mathrm{v}\times\boldsymbol{B}=\begin{vmatrix}\hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}}\\ \mathrm{v}_x & \mathrm{v}_y & \mathrm{v}_z\\ B_x & B_y & B_z\end{vmatrix}=\begin{vmatrix}\hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}}\\ 0 & 3.7\times10^5 & 0\\ 1.4 & 2.1 & 0\end{vmatrix}$$

$$=q_e(1.4)(3.7\times10^5)(-\hat{\mathrm{k}})=\underline{(8.3\times10^{-14}\hat{\mathrm{k}})\ \mathrm{N}}$$

Example: A proton is moving with a velocity of $\mathbf{v}=6.0\times10^6\,\hat{\mathrm{i}}\frac{\mathrm{m}}{\mathrm{s}}$ at a point where the magnetic field is given by $\boldsymbol{B}=\left(3.0\,\hat{\mathrm{i}}-1.5\,\hat{\mathrm{j}}+2.0\,\hat{\mathrm{k}}\right)\mathrm{T}$. What is the magnitude of the acceleration of the proton at this point?

Ans:

$$\because \boldsymbol{F}_B=m\boldsymbol{a}=q_e\mathrm{v}\times\boldsymbol{B}$$

$$\therefore \boldsymbol{a}=\frac{q_e}{m}\mathrm{v}\times\boldsymbol{B}=\frac{1.6\times10^{-19}}{1.67\times10^{-27}}\begin{vmatrix}\hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}}\\ 6.0\times10^6 & 0 & 0\\ 3.0 & -1.5 & 2.0\end{vmatrix}=-1.2\times10^7\,\hat{\mathrm{j}}-9.0\times10^6\,\hat{\mathrm{k}}$$

$$\therefore |\boldsymbol{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} = \underline{1.44\times10^{15}\ \text{m/s}^2}$$

EXTRA NOTICES ABOUT THE FORCE

1- The charge should be in motion (i.e. current).
2- Positive and negative charges experience forces in opposite directions.
3- The force is greatest when the charge moves perpendicular to the magnetic field and zero when the charges move parallel to the field.
4- The size of the force also depends on the magnitudes of the magnetic field and the electric charge (current) and on the speed of the moving charge.
5- The magnetic force does not accelerate the charge, but deflect it.

A Beam of Charged Particles in a Magnetic Field: Consider a beam of positively charged particles with velocity v .If this beam enters a magnetic field at right angle to its direction of motion, it will experience a force perpendicular to both velocity and magnetic field, i.e. it will be deflected.

- When a particle is accelerated through a potential difference, ΔV , energy conservation requires that

$$\boxed{q\Delta V = \frac{1}{2}m\text{v}^2 \Rightarrow \frac{q}{m} = \frac{\text{v}^2}{2\Delta V}}$$

Example: A singly charged positive ion, of mass 3.2×10^{-26} kg, has been accelerated through a potential difference of 833 V. Find its final speed.
Answer:

$$\because q\Delta V = \frac{1}{2}m\text{v}^2$$

$$\therefore \text{v} = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times833}{3.2\times10^{-26}}} = \underline{9.1\times10^{4}\ \text{m/s}}$$

- **Velocity selector:** In a region of crossed or perpendicular magnetic field $\boldsymbol{B}$ and electric field $\boldsymbol{E}$ perpendicular to v, the forces cancel when

$$\boxed{q\text{v}B = eE \Rightarrow \text{v} = \frac{E}{B}}$$

Example: An electric field and a magnetic field normal to each other. The electric field is 4.0 kV/m and the magnetic field strength is 2.0 mT. They are act on a moving electron to produce no force, calculate the electron speed.

Answer: $$\text{v} = \frac{|E|}{|B|} = \frac{4.0\times10^{3}}{2.0\times10^{-3}} = \underline{2.0\times10^{6}\ \text{m/s}}$$

H.W. A proton with velocity $\mathbf{v} = 2.0\times10^{6}\ \hat{\text{i}}$ (m/s) moves horizontally into a region of space in which there is an electric field $\boldsymbol{E} = -5.0\times10^{3}\ \hat{\text{j}}$ N/C and a magnetic field B. Find the smallest magnetic field such that the proton will continue to move horizontally undeflected. [Ans: $\boldsymbol{B} = -2.5\times10^{-3}\ \hat{\text{k}}$ T]

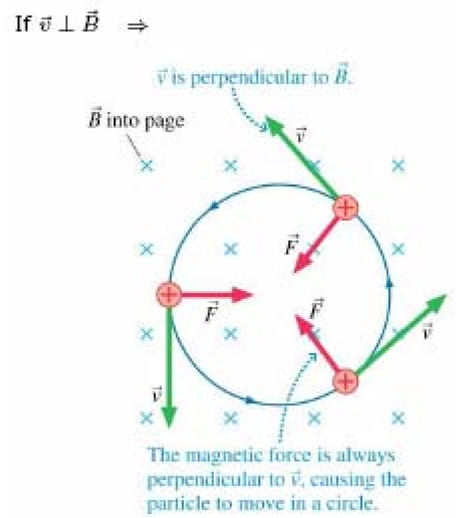
✸ **Circular motion:** The beam will move in circular path if the velocity is small and the magnetic field is strong. In that case:

Magnetic force = Centripetal force

$$qvB = m\frac{v^2}{R} \quad \Rightarrow \quad B = \frac{mv}{qR}$$

where R is the radius of the circle.
Also, the angular frequency (ω) and the periodic time (T) of a rotating charged particle are:

$$\omega = \frac{v}{R} = \frac{qB}{m},$$
$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{qvB}$$



☞ The beam will move in helical (spiral) path if the velocity vector has an angel with the magnetic field.
☞ The beam of negative charges will be deflected in the reverse direction.

Example: Alpha particles (m = 3.3×10^{-27} kg, $q = 2|e|$) are accelerated from rest through a potential difference of 1.0 kV. They then enter a region of magnetic field *B* = 0.2 T perpendicular to their direction of motion. What is the radius of the bath?
Answer:

$$\because q\Delta V = \frac{1}{2}mv^2 \quad \therefore \quad v = \sqrt{\frac{2q\Delta V}{m}}$$

$$\Rightarrow R = \frac{mv}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}} = \frac{1}{0.2\text{ T}}\sqrt{\frac{2\left(3.3\times10^{-27}\text{ kg}\right)\times1000\text{ V}}{2\left(1.6\times10^{-19}\text{C}\right)}}$$

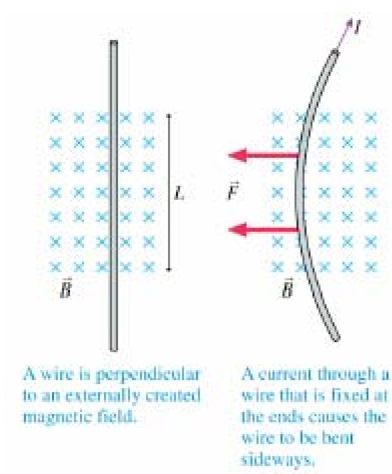
$$= \underline{2.2\times10^{-2}\text{ m}}$$

H.W. A proton with a velocity of 6.0×10^6 m/s Travels at right angels to magnetic field of 0.5 Tesla. What is the frequency of the proton's orbit? [**Ans:** 7.6×10^6 Hz]

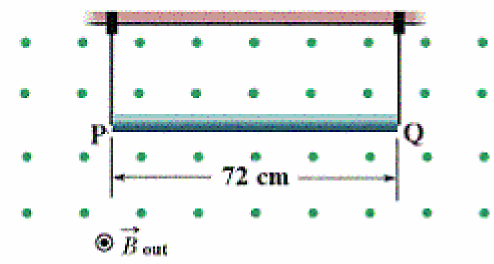
The Magnetic Force on a Current Carrying Wire: The magnetic force $\boldsymbol{F}_B$ on a segment of wire, $\boldsymbol{L}$, carrying current, *I*, in a magnetic field, $\boldsymbol{B}$, is given by

$$\boldsymbol{F}_B = q_o v\times\boldsymbol{B} = q_o\frac{\boldsymbol{L}}{t}\times\boldsymbol{B} = \frac{q_o}{t}\boldsymbol{L}\times\boldsymbol{B} = I\left(\boldsymbol{L}\times\boldsymbol{B}\right)$$

The total force on the wire is the vector sum of the forces on the segments.



Example: A wire 72 cm in length has a mass of 15 g. It is suspended by a pair of flexible leads in a magnetic field $B = 0.54$ T pointing out of the page, as shown in the following figure. What current must exist in the wire for the tension in the supporting leads to be zero?

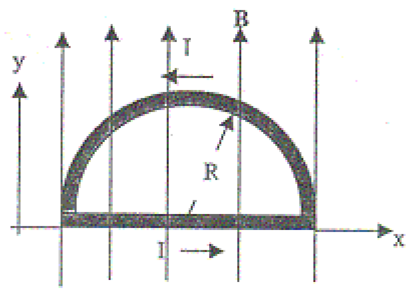


Ans: At the balance, tension Magnetic force = Gravitational force (i.e. $\boldsymbol{F}_B = \boldsymbol{F}_G$), then we have:

$$\frac{|\boldsymbol{F}_B|}{L} = I\,\frac{L \times B}{L} = \frac{mg}{L}$$

$$\Rightarrow I = \frac{mg}{LB_{in}} = \frac{0.015 \times 9.8}{0.72 \times 0.54} = 0.38 \text{ A from Q to P}$$

Example: A wire bent into a semicircle of radius R = 2.0 m forms a closed circuit and carries a current of 1.5 A. The circuit lies in the xy-plane, and a uniform magnetic field B = 3.0 T is present along the y axis, as shown in the figure. Find the magnitude of the magnetic force on the curved portion of the wire.



Answer:

$$F_{circle} + F_{straight} = 0 \quad \Rightarrow \quad |F_{circle}| = |F_{straight}|$$

$$F_{straight} = i\,\vec{l} \times \vec{B} = i\,(2R)3 = \underline{18 \text{ N}}$$

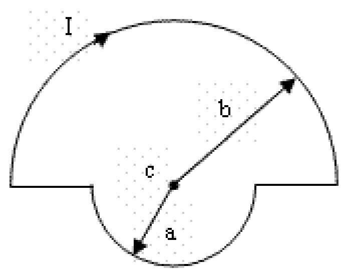
Magnetic Dipoles: A current loop, of area *A*, behaves like a magnet and has a magnetic dipole moment

$$\boldsymbol{M} = IA\hat{\boldsymbol{n}}$$

Where $\hat{\boldsymbol{n}}$ is a unit vector normal to the plane of the loop. Its direction is found by placing the **fingers** of the right hand along the **current**; the **right thumb** then indicates the direction of $\hat{\boldsymbol{n}}$ and $\boldsymbol{M}$. for N - current loops

$$\boldsymbol{M} = NIA\hat{\boldsymbol{n}}\,.$$

Example: The current loop, in the following figure, consists of one loop with two semicircles of different radii. If the current in the circuit is 2 A, a = 3.0 cm and b = 5.0 cm, then the magnetic dipole moment of the current loop is:



Answer:

$$\boldsymbol{M} = NI\boldsymbol{A} = (1)(2)\frac{\pi}{2}\left[(1.05)^2 + (0.3)^2\right]$$

$$= 0.01 \text{ A.m}^2, \text{ into the page.}$$

In a **uniform magnetic field**, a magnetic dipole behaves much like an electric dipole in uniform electric field, and the net force on the dipole will be zero. The torque, τ , on the dipole and the potential energy, *U*, are given by:

$$\tau = \boldsymbol{M} \times \boldsymbol{B}, \qquad U = MB\cos\theta$$

The magnetic torque on a flat current-carrying loop of wire by a uniform magnetic field $\boldsymbol{B}$ is maximum when the plane of the loop is parallel to $\boldsymbol{B}$.

Example: A current of 16 mA is maintained in a single circular loop of radius 0.32 m. An external magnetic field of 0.8 T is directed parallel to the plane of the loop.

a- Calculate the magnetic moment of the current loop.

Ans:

$$|\boldsymbol{M}| = IA = 16\times10^{-3}\times\pi(0.32)^2$$
$$= \underline{5.1\times10^{-3}\ \mathrm{C\cdot m^2}}$$

b- What is the magnitude of the torque exerted on the loop by the magnetic field?

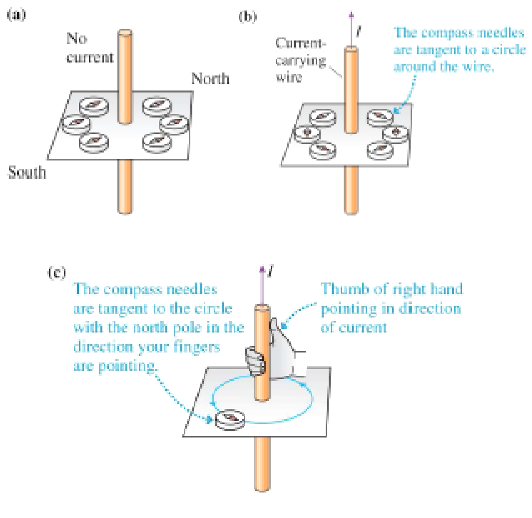
Ans:

$$|\tau| = |\boldsymbol{M}\times\boldsymbol{B}| = |\boldsymbol{M}||\boldsymbol{B}|\sin 90^\circ = 5.1\times10^{-3}\times0.8$$
$$= \underline{4.1\times10^{-3}\mathrm{C\cdot m^2 T}}$$

Magnetic Field Due to Currents

Electric current and magnetism

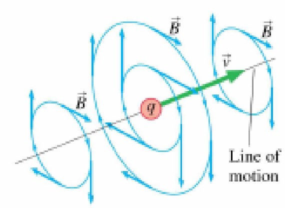
H. C. Oersted discovered in 1819:



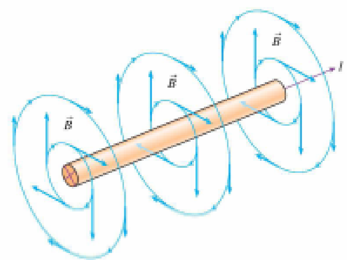
Electric current (i.e. moving charges) is an origin of magnetism

$\vec{B}$-field of a straight current-carrying wire

Field due to single moving charge



Field to due current:



Typical magnetic field strengths

Source of Field	B (T)
Surface of earth	5×10^{-5}
Fridge magnet	5×10^{-3}
MRI magnet	1.5
Superconducting lab magnet	$\lesssim 30$

Biot –Savart law

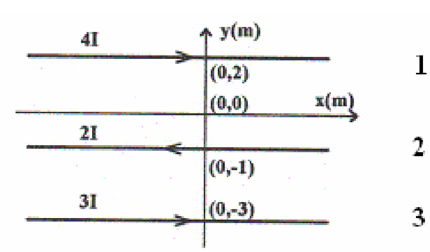
$$B = \frac{\mu_o}{2\pi} \frac{I\, d\vec{s} \times \hat{r}}{r}$$

Magnetic field due to a current in a long straight wire

$$B = \mu_o \frac{Ni}{2\pi R}$$

Example: Three long wires parallel to the x-axis carry currents as shown in the following figure. If $I = 20$ A, what is the magnitude of the magnetic field at the origin?

Ans:



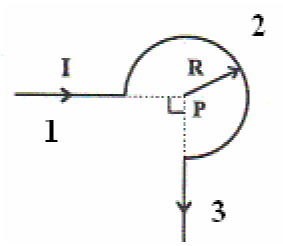
$$B = B_1 + B_2 + B_3 = \frac{\mu_o}{2\pi}\left(\frac{I_1}{|a_1|} + \frac{I_2}{|a_2|} + \frac{I_3}{|a_3|}\right),$$

$$= \frac{\mu_o}{2\pi}\left(\frac{4I}{2} + \frac{2I}{1} - \frac{3I}{3}\right) = \frac{3\mu_o}{2\pi} I = \underline{1.2\times10^{-5} \text{ T}}.$$

Magnetic field due to a current in a circular arc wire

$$B = \mu_o \frac{N}{4\pi R}\varphi$$

Example: A segment of wire is formed into the shape shown in the following figure and carries a current I. What is the resulting magnetic field at the point P?



Ans:

$$B = B_1 + B_2 + B_3 = 0 + B_2 + 0,$$

$$B = B_2 = \mu_o \frac{I}{4\pi R}\theta = \mu_o \frac{I}{4\pi R}\left(\frac{3\pi}{2}\right) = \underline{\mu_o \frac{3I}{8R} \text{ into the page}}$$

Example: Figure 10 shows two concentric, circular wire loops, of radii r_1 = 15 cm and r_2 = 30 cm, are located in the xy plane. The inner loop carries a current of 8.0 A in the clockwise direction, and the outer loop carries a current of 10.0 A in the counter clockwise direction. Find the net magnetic field at the center.

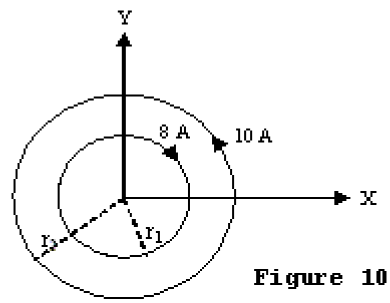


Figure 10

Ans:

$$B_{r_1} = \frac{\mu_o I}{2r_1} = \frac{4\pi\times10^{-7}\times8}{2(0.15)} = 33.5 \ \mu\text{T into the page.}$$

$$.\ B_{r_2} = \frac{\mu_o I}{2r_2} = \frac{4\pi\times10^{-7}\times10}{2(0.3)} = 20.9 \ \mu\text{T out of the page.}$$

$$B = B_1 - B_2 = 33.5 - 20.9 = \underline{12.6 \ \mu\text{T into the page}}.$$

Example: A circular loop of radius 0.1 m has a resistance of 6 Ohms. If it is attached to a 12 V battery, how large a magnetic field is produced at the center of the loop?

Ans:

$$\because V = \frac{I}{R} = \frac{12}{6} = 2 \text{ A},$$

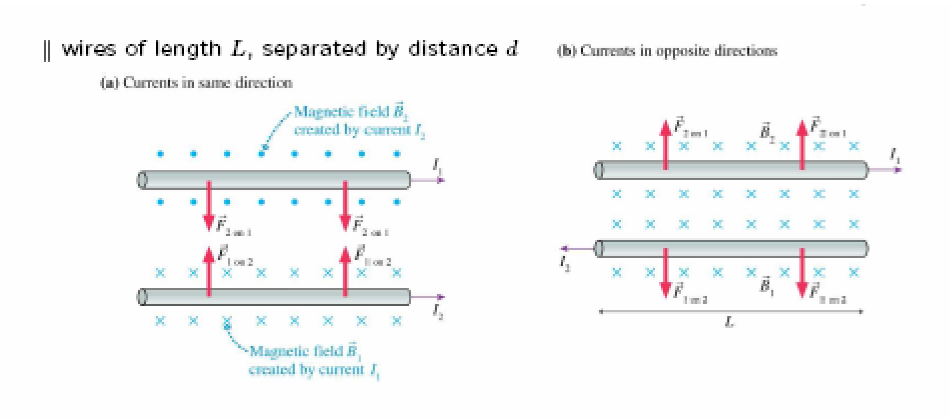
$$\therefore B = \mu_o \frac{I}{4\pi R}\theta = 4\pi\times10^{-7}\frac{2}{4\pi\times0.1}\times2\pi = \underline{1.3\times10^{-5} \text{ T}}$$

Example: How many turns should be in a flat circular coil of radius 0.1 m in order for a current of 10 A to produce a magnetic field of 3.0×10^{-3} T at its center?

Ans:

$$B = \mu_o \frac{NI}{2\pi R} \Rightarrow N = \frac{2\pi BR}{\mu_o I} = \frac{2\times\pi\times 3.0\times 10^{-3}\times 0.1}{4\pi\times 10^{-7}} = \underline{48 \text{ Turns}}$$

Force between two parallel currents



$$\frac{F}{L} = \mu_o \frac{i_1 i_2}{2\pi d}$$

Example: Two long parallel wires, a distance *d* apart, carry currents of *I* and 5*I* in the same direction. Locate the point r, from I, at which their magnetic fields cancel each other.
Ans:

at equilibrium $|F_1| = |F_2| \Rightarrow \frac{\mu_o I}{2\pi r} = \frac{\mu_o I}{2\pi(d-r)}$

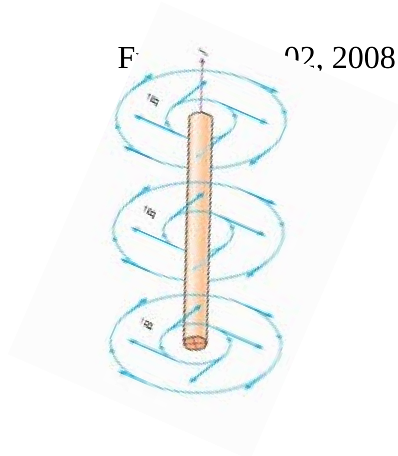
.

Solve for r, one can find $r = \underline{\frac{d}{6}}$

Example: Two long parallel conductors, separated by a distance $a = 10$ cm, carry currents in the same direction. If $I_1 = 5.0$ A and $I_2 = 8.0$ A, what is the force per unit length exerted on each conductor by the other?
Ans:

$$\frac{F}{l} = \mu_o \frac{I_1 I_2}{2\pi a}$$

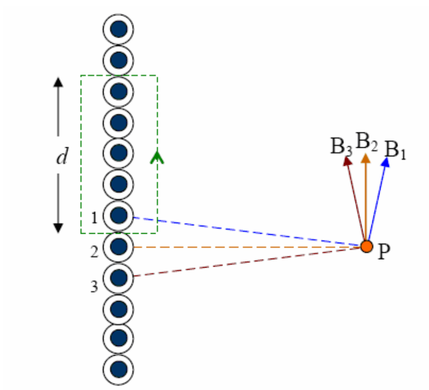
$$= 4\pi\times 10^{-7} \frac{5.0\times 8.0}{2\pi\times 0.1} = 8.0\times 10^{-5} \text{ N/m} \quad \text{attractive}$$



Ampere's Law:

$$\boxed{\oint \vec{B}.d\vec{s} = \mu_o i_{enc}}$$

The symbol $\oint$ denotes integration around the entire Amperian loop. Here it is called *Amperian loop*. This type of integral is called a "line integral" or "path integral."



The magnetic field of a planar array of wires.
The current is out of the screen.

Example: Magnetic field outside a long straight wire with current:

$$\overbrace{B\oint ds = B\left(2\pi r\right) \qquad \mu_o i_{enc}}^{\oint \vec{B}.d\vec{s}}$$

$$\Rightarrow \boxed{B = \mu_o \frac{i_{enc}}{2\pi r}}$$

Example: Magnetic field inside a long straight wire with current:

$$\overbrace{B\oint ds = B\left(2\pi r\right) \qquad \mu_o i_{enc} = i\,\frac{\pi r^2}{\pi R^2}}^{\oint \vec{B}.d\vec{s}}$$

$$\Rightarrow \boxed{B = \frac{\mu_o i}{2\pi R^2} r}$$

Example: A cylindrical conductor of radius $R = 2.5$ cm carries a current $I = 2.5$ A along its length; this current is uniformly distributed through the cross section of the conductor. Calculate the magnetic field midway along the radius of the wire (that is, at $r = R/2$).

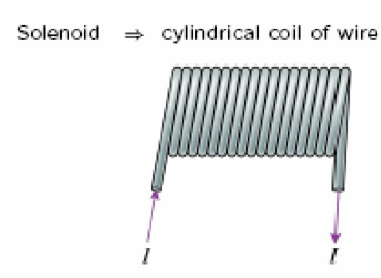
Ans:

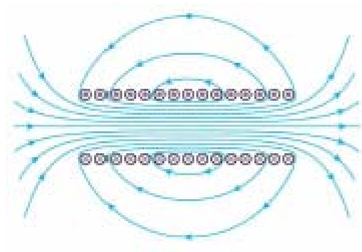
$$\because B = \mu_o \frac{I}{2\pi R^2} r, \qquad r < R$$

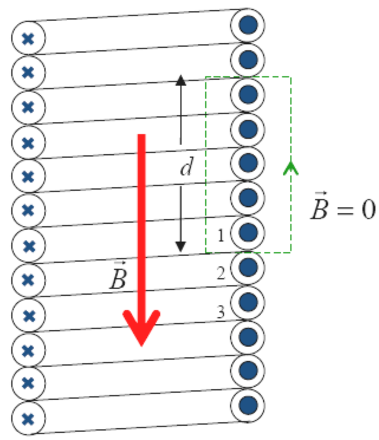
$$\therefore B = \mu_o \frac{I}{2\pi R^2}(R/2) = 4\pi \times 10^{-7} \frac{2.5}{4 \times \pi \times (0.05)} = \underline{1.0 \times 10^{-5}\ \text{T}}$$

Magnetic field of a solenoid:

a coil of wire that acts as a magnet when an electric current flows through it







A solenoid

$$B_s = \mu_o n I = \mu_o \left(\frac{N}{L}\right) I$$

Where:
B is the magnetic field anywhere within the solenoid. The units are tesla (T).
μ_o is the permeability of free space, $\mu_o = 4\pi \times 10^{-7}$ T.m/A .
n is the number of coils per meter in the solenoid.
L = Length of the solenoid,
I is the current passing through the solenoid.
.

Example: What current in a solenoid 15-cm long wound with 100 turns would produce a magnetic field equal to that of the earth, which is 5.1×10^{-5} T?
Ans:

$$\because B_s = \mu_o n I = \mu_o \left(\frac{N}{L}\right) I$$

$$\therefore I = \frac{B_s N}{\mu_o L} = \frac{5.1\times10^{-5}\times0.15}{4\pi\times10^{-7}\times100} = \underline{6.1\times10^{-2}\ \text{A}}$$

Example: A solenoid is formed by tightly winding a single layer of wire. The wire is 1.0 mm in diameter. What is the magnitude of the magnetic field inside the solenoid when there is a current of 0.081 A in the windings?
Ans:

$$B_s = \mu_o n I = \mu_o \left(\frac{N}{L}\right) I$$

$$= 4\pi\times10^{-7}\left(\frac{1}{10^{-3}}\right)\times0.081 = \underline{1.02\times10^{-4}\ \text{T}}$$