<u>Example 3.4</u> Using the separation of variables to solve the Lap lace's equation:

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0 \qquad (1)$$

With the following boundary conditions:

$$V(x,0)=0 \qquad \text{(i)}$$
$$V(x,a)=0 \qquad \text{(ii)}$$
$$V(b,y)=V_o \qquad \text{(iii)}$$
$$V(-b,y)=V_o \qquad \text{(iv)}$$

The general solution of (1) is:

$$V(x,y)=X(x)Y(y)=N\cosh(kx)\sin(ky) \qquad (2)$$

$$\text{B.C. (ii)} \quad \Rightarrow \quad \sin(ka)=0 \quad \Rightarrow \quad k_n=\frac{n\pi}{a}, \qquad (n=1,2,3,\cdots)$$

$$V(x,y)=\sum_{n=1}^{\infty} N_n\cosh(k_n x)\sin(k_n y) \qquad (D)$$

Boundary condition (iii) implies that:

$$V(b,y)=\sum_{n=1}^{\infty} N_n\cosh(\frac{n\pi b}{a})\sin(\frac{n\pi y}{a})=V_o \qquad (E)$$

If V_o is constant, one finds

$$N_n\cosh(\frac{n\pi b}{a})=\begin{cases}0, & \text{if } n \text{ is even,}\\ \frac{4V_o}{n\pi}, & \text{if } n \text{ is odd}\end{cases}$$

Since *m* and *n* are dummy indices, finally

$$\boxed{V(x,y)=\frac{4V_o}{\pi}\sum_{n=1,3,5,\cdots}^{\infty}\frac{1}{n}\frac{\cosh(\frac{n\pi x}{a})}{\cosh(\frac{n\pi b}{a})}\sin(\frac{n\pi y}{a})}$$

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 (* Phys 305  Eq. 3.42   Page 134   *)
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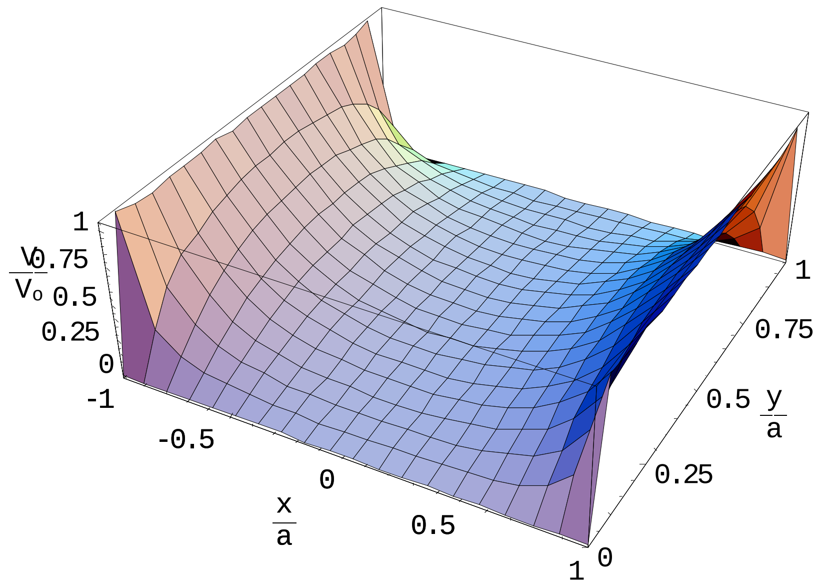
$$V[x\_, y\_, m\_, a\_] := \frac{4}{\pi}\,\mathrm{Sum}\left[\frac{1}{n}\,\frac{\mathrm{Cosh}\left[\frac{n\,\pi\,x}{a}\right]}{\mathrm{Cosh}\left[\frac{n\,\pi}{a}\right]}\,\mathrm{Sin}\left[\frac{n\,\pi\,y}{a}\right],\ \{n, 1, m, 2\}\right]$$

$$\mathrm{Plot3D}\left[V[x, y, 100, 1],\ \{x, -1, 1\},\ \{y, 0, 1\},\ \mathrm{AxesLabel} \to \left\{\text{"}\frac{x}{a}\text{"}, \text{"}\frac{y}{a}\text{"}, \text{"}\frac{V}{V_o}\text{"}\right\},\ \mathrm{PlotRange} \to \mathrm{All},\ \mathrm{PlotPoints} \to 20,\ \mathrm{PlotRange} \to \mathrm{All}\right]$$



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