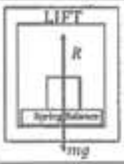
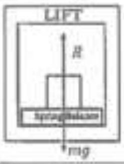
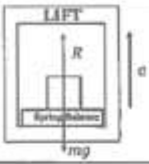
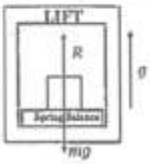
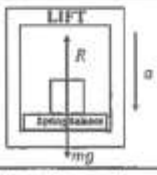
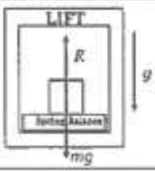
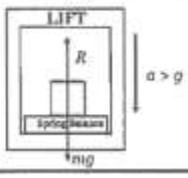
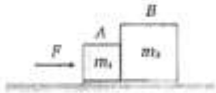
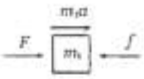
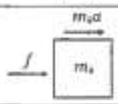
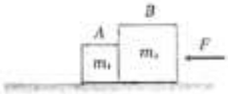
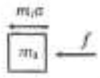
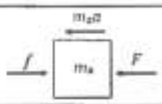
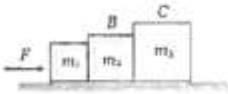
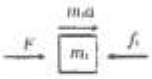
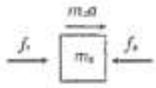
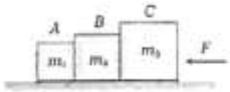
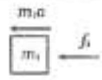
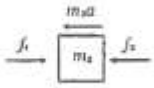
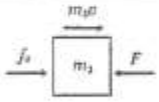
Chapter 5

4.13 Apparent Weight of a Body in a Lift

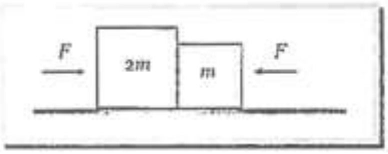
When a body of mass m is placed on a weighing machine which is placed in a lift, then actual weight of the body is mg. This acts on a weighing machine which offers a reaction R given by the reading of weighing machine. This reaction exerted by the surface of contact on the body is the *apparent weight* of the body.

Condition	Figure	Velocity	Acceleration	Reaction	Conclusion
Lift is at rest		$v = 0$	$a = 0$	$R - mg = 0$ $\therefore R = mg$	Apparent weight = Actual weight
Lift moving upward or downward with constant velocity		v = constant	$a = 0$	$R - mg = 0$ $\therefore R = mg$	Apparent weight = Actual weight
Lift accelerating upward at the rate of 'a'		v = variable	$a < g$	$R - mg = ma$ $\therefore R = m(g + a)$	Apparent weight > Actual weight
Lift accelerating upward at the rate of 'g'		v = variable	$a = g$	$R - mg = mg$ $R = 2mg$	Apparent weight = 2 Actual weight
Lift accelerating downward at the rate of 'a'		v = variable	$a < g$	$mg - R = ma$ $\therefore R = m(g - a)$	Apparent weight < Actual weight
Lift accelerating downward at the rate of 'g'		v = variable	$a = g$	$mg - R = mg$ $R = 0$	Apparent weight = Zero (weightlessness)
Lift accelerating downward at the rate of $a(>g)$		v = variable	$a > g$	$mg - R = ma$ $R = mg - ma$ $R = -ve$	Apparent weight negative means the body will rise from the floor of the lift and stick to the ceiling of the lift.

4.16 Motion of Blocks in Contact

Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$f = m_2 a$	$f = \frac{m_2 F}{m_1 + m_2}$
		$f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - f = m_2 a$	$f = \frac{m_1 F}{m_1 + m_2}$
		$F - f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2 a$	$f_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$f_2 = m_3 a$	$f_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$
		$f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_2 - f_1 = m_2 a$	$f_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$
		$F - f_2 = m_3 a$	$f_2 = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$

Problem Two blocks are in contact on a frictionless table one has a mass m and the other $2m$ as shown in figure. Force F is applied on mass $2m$ then system moves towards right. Now the same force F is applied on m. The ratio of force of contact between the two blocks will be in the two cases respectively.

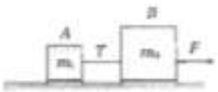
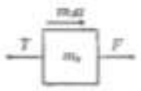
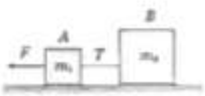
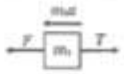
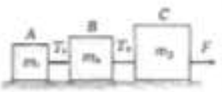
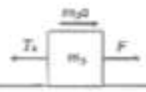
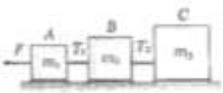
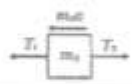
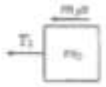


Solution : When the force is applied on mass $2m$ contact force $f_1 = \frac{m}{m + 2m} g = \frac{g}{3}$

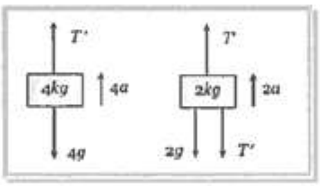
When the force is applied on mass m contact force $f_2 = \frac{2m}{m + 2m} g = \frac{2}{3} g$

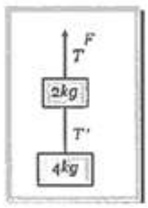
Ratio of contact forces $\frac{f_1}{f_2} = \frac{1}{2}$

4.17 Motion of Blocks Connected by Mass Less String

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - T = m_2 a$	$T = \frac{m_1 F}{m_1 + m_2}$
		$F - T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$T = m_2 a$	$T = \frac{m_2 F}{m_1 + m_2}$
		$T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	$T_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$
		$F - T_2 = m_3 a$	$T_2 = \frac{(m_1 + m_2) F}{m_1 + m_2 + m_3}$
		$F - T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_1 - T_2 = m_2 a$	$T_1 = \frac{(m_2 + m_3) F}{m_1 + m_2 + m_3}$
		$T_2 = m_3 a$	$T_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$

Problem Two blocks are connected by a string as shown in the diagram. The upper block is hung by another string. A force F applied on the upper string produces an acceleration of $2m/s^2$ in the upward direction in both the blocks. If T and T' be the tensions in the two parts of the string, then





Solution : From F.B.D. of mass 4 *kg* $4a = T' - 4g$ .....(i)

From F.B.D. of mass 2 *kg* $2a = T - T' - 2g$ .....(ii)

For total system upward force

$F = T = (2+4)(g+a) = 6\ (9.8+2)\ \text{N} = 70.8\ \text{N}$

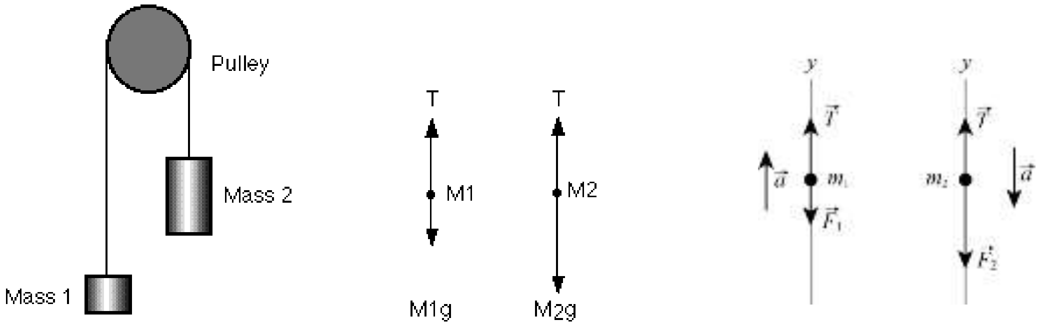
by substituting the value of T in equation (i) and (ii)

and solving we get $T' = 47.2N$

It is important to draw the free body diagram in each problem, if needed.

Example: The figure shows two blocks connected by a cord (of negligible mass) that passes over a frictionless pulley (also negligible mass). The arrangement is known as Atwood's machine. One block has mass $m_1 = 1.3$ kg; the other has mass $m_2 = 2.8$ kg . What are:

a- the magnitude of the block's acceleration and
b- the tension in the cord.



The free-body diagrams for m_1 and m_2 are shown in the figures above. The only forces on the blocks are the upward tension T and the downward gravitational forces $\boldsymbol{F_1} = m_1\boldsymbol{g}$ and $\boldsymbol{F_2} = m_2\boldsymbol{g}$. Applying Newton's second law, we obtain:

For mass m_1: $T - m_1 g = m_1 a$

For mass m_2: $m_2 g - T = m_2 a$

which can be solved to yield

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

Substituting the result back, we have

$$T = \left(\frac{2 m_2 m_1}{m_2 + m_1}\right) g$$

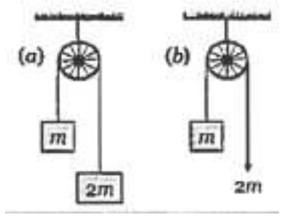
a- With $m_1 = 1.3$ kg and $m_2 = 2.8$ kg, the acceleration becomes

$$a = \left(\frac{2.8 - 1.3}{2.8 + 1.3}\right) g = 3.6 \ \frac{\text{m}}{\text{s}^2}$$

b- Similarly, the tension in the cord is

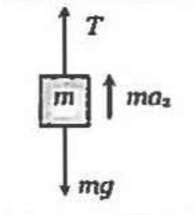
$$T = \left(\frac{2 \times 2.8 \times 1.3}{2.8 + 1.3}\right) g = 17 \text{ N}$$

Problem: The two pulley arrangements shown in the figure are identical. The mass of the rope is negligible. In (a) the mass m is lifted up by attaching a mass 2 m to the other end of the rope. In (b). m is lifted up by pulling the other end of the rope with a constant downward force of *2mg*. The ratio of accelerations in two cases will be



Answer: Part (a) is the Atwood machine and has the common acceleration

$$a_1 = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g = \left(\frac{2m - m}{2m + m}\right) g = \frac{g}{3} \qquad \text{(i)}$$



For part (b), the free body diagram gives:

$$ma_2 = T - mg = 2mg - mg \quad \Rightarrow \quad a_2 = g \qquad \text{(ii)}$$

Note that: we used $T = 2mg$

From (i) and (ii), one finds:

$$\frac{a_1}{a_2} = \frac{1}{3}$$

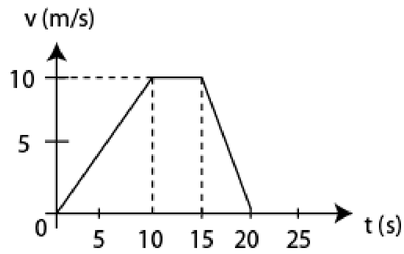
Q15. An elevator of mass 480 kg is designed to carry a maximum load of 3000 N. What is the tension in the elevator cable at maximum load when the elevator moves down accelerating at 9.8 m/s^2?

Answer: The equation of motion for the mass when the elevator is going down is:

$$ma = -T + mg;$$

$$\Rightarrow \quad T = ma - mg = m(9.8 - 9.8) = \underline{0 \text{ N}}$$

Q16.: A car of mass 1000 kg is initially at rest. It moves along a straight road for 20 s and then comes to rest again. The velocity – time graph for the movement is given in the figure. The magnitude of the net force that acts on the car while it is slowing down to stop from t = 15 s to t = 20 s is:

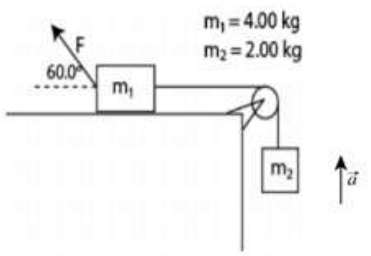


Answer: The acceleration when car slowing down to stop is:

$$a = \frac{dv}{dt} = \frac{0 - 10}{20 - 15} = -2 \text{ m/s}^2;$$

$$\Rightarrow \quad |F| = m|a| = 1000 \times 2 = \underline{2000 \text{ N}}$$

Q15.: Two blocks of masses $m_1 = 4.00$ kg and $m_2 = 2.00$ kg are connected by a string passing over a massless and frictionless pulley and placed on a frictionless horizontal table as shown in the figure. A force of F = 10.0 N at an angle of 60.0° with the horizontal is applied to m_1. The magnitude of acceleration of the system is:

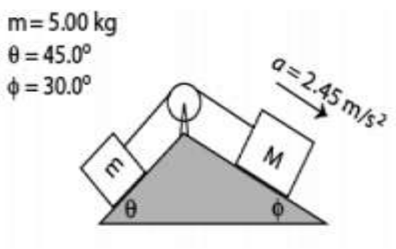


Answer: The equations of motion for the two masses are:

$$m_2 a = T - m_2 g; \quad m_1 a = F\cos 60^\circ - T$$

Adding the above two equations, we have

$$\Rightarrow \quad a = \frac{F\cos 60^\circ - m_2 \times g}{m_1 + m_2} = \frac{F\cos 60^\circ - 2 \times 9.8}{6} = \underline{2.43 \text{ m/s}^2}$$



Q19. Two boxes, one of mass m = 5.00 kg and the other with an unknown mass M are connected with a string passing over a massless frictionless pulley and are placed on frictionless planes as shown in the figure. What must be the mass M, if it goes down the plane with an acceleration of a = 2.45 m/s^2?

Answer: The equations of motion for the masses are:

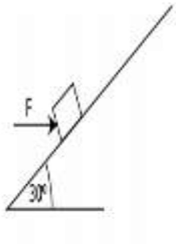
$$Ma = Mg\sin\phi - T; \quad ma = T - mg\sin\theta;$$

$$\Rightarrow \quad (m+M)a = Mg\sin\phi - mg\sin\theta$$

$$\Rightarrow M = \underline{19.14 \text{ kg}}$$

<u>T061</u>

Q13. A 4.0 kg block is pushed upward a 30° inclined frictionless plane with a constant horizontal force F (shown in the figure). If the block moves with a constant speed find the magnitude of the force F.



Answer: Note that; the acceleration is zero.

The equation of motion for the masse along the plane is:

$$Ma = 0 = F\cos\phi - Mg\sin\phi;$$

$$\Rightarrow F = mg\,\frac{\sin\phi}{\cos\phi} = 4\times 9.8\times\frac{0.5\times 0.866}{0.866} = \underline{23.0\text{ N}};$$

<u>T052</u>

Q#16. A 2.3-N weight is suspended by a string from a ceiling and held at an angle θ from the vertical by 4.0-N horizontal force F as shown in Fig 6. The tension in the string is:

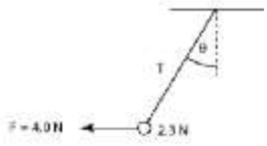


Figure 6

Answer: Note that; the acceleration is zero.

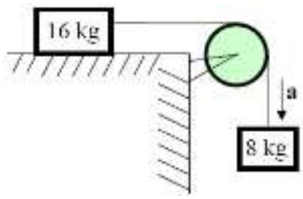
On y-axis: $T\cos\phi = Mg$; On x-axis $T\sin\phi = F$;

dividing both equations, one finds

$$\Rightarrow F = mg\tan\phi \Rightarrow \phi = \tan^{-1}\frac{4}{2.3} = 60^\circ;$$

Then
$$T = \frac{2.3}{\cos 60^\circ} = \underline{4.6\text{ N}}$$

Q1. A 16-kg block and an 8-kg block is connected by a string as shown in the Figure. If the pulley is massless and the surface is frictionless, the magnitude of the acceleration of the 8-kg block is:



a. g/3 b. 4g/3 d. g e. g/2

Answer: The equations of motion for the two masses are:

For 8 kg: $8a = 8g - T$ (1)

For 16 kg: $16a = T$ (2)

Adding the above two equations, we have

$$24a = 8g \quad \Rightarrow \quad a = \frac{g}{3}$$

Q4. A constant force, F, acts on a 19-kg particle. The particle, initially at rest, moves a distance of 22 m in 3.8 s. Find the magnitude of the force F.

a. 58 N b. 86 N c. 50 N d. 41 N e. 12 N

Answer: Start with the equation:

$$F = ma,$$

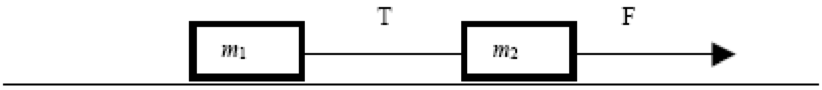
And the kinematic equation:

$$d = v_o t + \tfrac{1}{2}at^2 \Rightarrow 22 = 0 + \tfrac{1}{2}a(3.8)^2 \Rightarrow a = \frac{2\times 22}{(3.8)^2}$$

Then we have

$$F = 19\frac{2\times 22}{(3.8)^2} = 57.9 \text{ m/s}^2$$

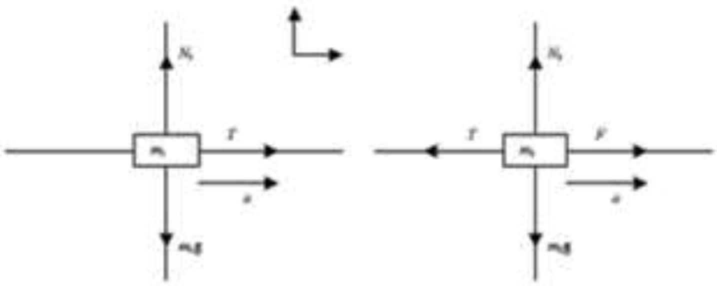
Q5. Consider a system consisting of two blocks (masses = m_1 & m_2) attached by a light cord on a smooth, flat table pulled to the right by a force F. Find the tension in the connecting cord and the acceleration of the system. Consider frictionless table. Take the values F = 46 N, m_1 = 22 kg and m_2 = 37 kg to calculate the acceleration and the tension in the coed.



a. 17 N b. 29 N c. 46 N d. 31 N

Answer:

The FBD's for this system is



FBD₁:
$$\sum F_x = T = m_1 a \quad (1)$$
$$\sum F_y = N_1 - m_1 g = 0$$
$$\therefore N_1 = m_1 g$$

FBD₂:
$$\sum F_x = F - T = m_2 a \quad (2)$$
$$\sum F_y = N_2 - m_2 g = 0$$
$$\therefore N_2 = m_2 g$$

Now, if we combined equations (1) and (2) , we have

$$a = \frac{F}{(m_1 + m_2)} \qquad (3)$$

Note that this is in the form $a = \dfrac{F}{\sum_i m_i}$

Plugging Eq. (3) into Eq. (1) yields

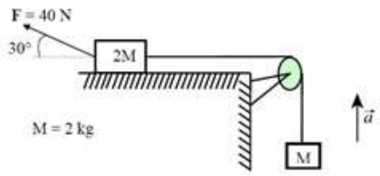
$$T = m_1\left(\frac{F}{m_1 + m_2}\right) \qquad (4)$$

Note that this is in the form $F = ma$

- Eq. (3) for the combined masses gives: $a = 46/59 \text{ m/s}^2$
- Eq (4) yields:

$$T = 22\left(\frac{46}{59}\right) \Rightarrow T = 17.2 \text{ N}$$

Q11. In the Figure, F=40 N and M=2 kg. What is the magnitude of the acceleration of the suspended object M ? (All surfaces are frictionless)
a. 2.5 m/s^2 b. 2.8 m/s^2 c. 3.3 m/s^2 d. 5.4 m/s^2 e. 13 m/s^2

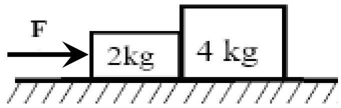


Q11:
Answer: The equations of motion for the two masses are:
For 2M: $2Ma = F\cos 30^o - T$ (1)
For M: $Ma = T - Mg$ (2)
Adding the above two equations, we have

$$\Rightarrow \quad 3Ma = F\cos 30^o - Mg = 40\sqrt{3} - 2\times 9.8$$

$$\therefore a = \frac{40\sqrt{3} - 2\times 9.8}{6} = 2.5 \text{ m/s}^2$$

Q12. The horizontal surface on which the objects (see the Figure) slide is frictionless. If the magnitude of the force of the small block on the large block is 5.2 N, determine F.
a. 7.8 N b. 2.6 N c. 5.2 N

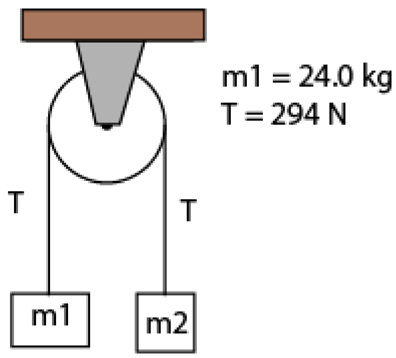


Answer:
For the combined masses: $(2+4)a = F$ (1)
For the 2 kg**,** $2a = F - P$ (1)
Solve (1) and (2) we have: $F = \frac{3P}{2} = \frac{3\times 5.2}{2} = 7.8 \text{ N}$

Q14. A 700-kg elevator accelerates downward at 3.8 m/s^2. The tension force of the cable on the elevator is:
a. 4.2 kN, up b. 2.1 kN, down c. 2.1 kN, up
Answer:

$$\Rightarrow \quad 700a = 700g - T \Rightarrow T = 700(9.8 - 3.8) = 4200 \text{ N, Up}$$

T-072 Q13.
Two blocks of mass m_1= 24.0 kg and m_2, respectively, are connected by a light string that passes over a massless pulley as shown in Fig. 2. If the tension in the string is T = 294 N. Find the value of m_2. (Ignore friction)
A) 40.0 kg



T-072 Q14.
Two horizontal forces of equal magnitudes are acting on a box sliding on a smooth horizontal table. The direction of one force is the north direction; the other is in the west direction. What is the direction of the acceleration of the box?
A) 45° west of north

T-072 Q16.
A 5.0 kg block is lowered with a downward acceleration of 2.8m/s^2 by means of a rope. The force of the block on the rope is: A) 35 N, down

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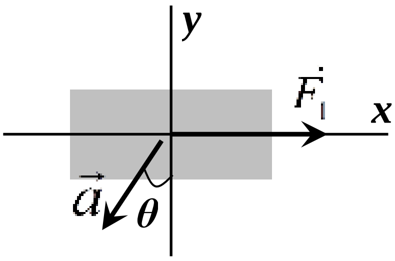
Selected problems from the Textbook

5. There are two forces on the 2.00 kg box in the overhead view of the Figure, but only one is shown. For F_1 = 20.0 N, a = 12.0 m/s^2, and θ = 30.0º , find the second force (a) in unit-vector notation and as (b) a magnitude and (c) an angle relative to the positive direction of the x axis.



Solution:

We denote the two forces $\vec{F}_1$ and $\vec{F}_2$. According to Newton's second law, $\vec{F}_1 + \vec{F}_2 = m\vec{a}$, so $\vec{F}_2 = m\vec{a} - \vec{F}_1$.

(a) In unit vector notation $\vec{F}_1 = (20.0\ \text{N})\hat{\text{i}}$ and

$$\vec{a} = -(12.0\sin 30.0°)\hat{\text{i}} - (12.0\cos 30.0°)\hat{\text{j}} = -(6.00)\hat{\text{i}} - (10.4)\hat{\text{j}}\ \text{m/s}^2.$$

Therefore,

$$\vec{F}_2 = (2.00\text{kg})(-6.00\,\text{m/s}^2)\hat{\text{i}} + (2.00\ \text{kg})(-10.4\ \text{m/s}^2)\hat{\text{j}} - (20.0\ \text{N})\hat{\text{i}}$$
$$= (-32.0)\hat{\text{i}} - (20.8)\hat{\text{j}}\ \text{N}.$$

(b) The magnitude of $\vec{F}_2$ is

$$|\vec{F}_2| = \sqrt{F_{2x}^2 + F_{2y}^2} = \sqrt{(-32.0\ \text{N})^2 + (-20.8\ \text{N})^2} = 38.2\ \text{N}.$$

(c) The angle that $\vec{F}_2$ makes with the positive x axis is found from

$\tan\theta = (F_{2y}/F_{2x})$ = [(–20.8 N)/(–32.0 N)] = 0.656.

Consequently, the angle is either 33.0° or 33.0° + 180° = 213°. Since both the x and y components are negative, the correct result is 213°. An alternative answer is

$$213° - 360° = -147°.$$

Q. In Figure 1, a constant horizontal force $\vec{F}_a$ is applied to block A, which pushes against block B with a 20.0 N force directed horizontally to the right. In Figure 2, the same force $\vec{F}_a$ is applied to block **B**; now block A pushes on block B with a 10.0 N force directed horizontally to the left. The blocks have a combined mass of 12.0 kg. What are the magnitudes of (a) their acceleration in Figure 1 and (b) force $\vec{F}_a$?

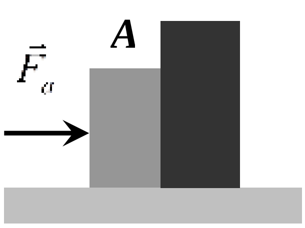


Figure 1

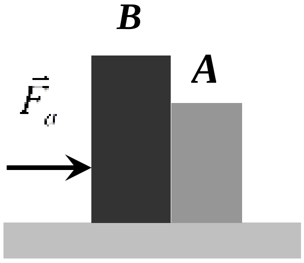


Figure 2

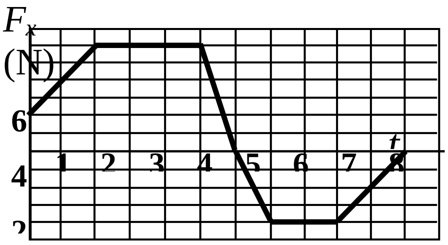
Solution:

Both situations involve the same applied force and the same total mass, so the accelerations must be the same in both figures.

(a) The (direct) force causing B to have this acceleration in the first figure is twice as big as the (direct) force causing A to have that acceleration. Therefore, B has the twice the mass of *A*. Since their total is given as 12.0 kg then *B* has a mass of m_B = 8.00 kg and *A* has mass m_A = 4.00 kg. Considering the first figure, (20.0 N)/(8.00 kg) = 2.50 m/s^2. Of course, the same result comes from considering the second figure ((10.0 N)/(4.00 kg) = 2.50 m/s^2).

(b) F_a = (12.0 kg)(2.50 m/s^2) = 30.0 N

Q. The Figure gives, as a function of time *t*, the force component F_x that acts on a 3.00 kg ice block that can move only along the *x* axis. At *t* = 0, the block is moving in the positive direction of the axis, with a speed of 3.0 m/s. What are its (a) speed and (b) direction of travel at *t* = 9 s?

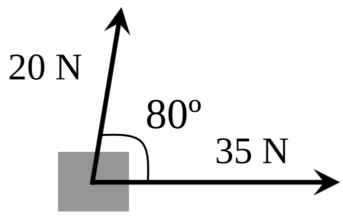


Solution:

(a) The acceleration (which equals *F/m* in this problem) is the derivative of the velocity. Thus, the velocity is the integral of *F/m*, so we find the "area" in the graph of F_x as a function of *t* (½×4×2 +2×2 +6×3 +½×6×1 –½×4×1 –4×2 –½×4×2 = 15 N.s) and divide by the mass (3 kg) to obtain $v - v_o$ = 15/3 = 5. Since v_o = 3.0 m/s, then $v = 8.0\,\text{m/s}$.

(b) Our positive answer in part (a) implies $\vec{v}$ points in the +*x* direction.

Q. The only two forces acting on a body have magnitudes of 20.0 N and 35.0 N and directions that differ by 80.0°. The resulting acceleration has a magnitude of 20.0 m/s^2. What is the mass of the body?



Solution:

We are only concerned with horizontal forces in this problem (gravity plays no direct role since it is mentioned that there are only the two given forces). Without loss of generality, we take one of the forces along the +*x* direction and the other at 80° (measured counterclockwise from the *x* axis).

$$\vec{F}_{net} = 35.0\hat{i} + 20\cos(80)\hat{i} + 20\sin(80)\hat{j} = (38.5\hat{i} + 19.7\hat{j})\ \text{N}$$

$$\|\vec{F}_{net}\| = F_{net} = \sqrt{(38.5)^2 + (19.7)^2} = 43.2\ \text{N}$$

$$m = F_{net}/a = 43.2/20.0 = 2.16\ \text{kg}$$

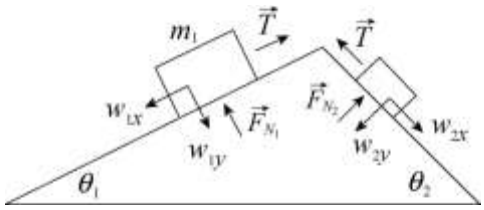
Q. Two boxes on two frictionless inclined planes connected with a cord as shown in the Figure. The left box has m_1 = 3.0 kg and the right box has a mass m_2 = 2.0 kg. Find the tension in the cord if the left plane is inclined at an angle θ_1 = 30° and the right plane is inclined at an angle θ_2 = 60°.

Solution:

The +*x* axis is "uphill" for m_1 = 3.0 kg and "downhill" for m_2 = 2.0 kg (so they both accelerate with the same sign). The *x* components of the two masses along the *x* axis are given by $w_{1x} = m_1 g \sin\theta_1$ and $w_{2x} = m_2 g \sin\theta_2$, respectively.

Applying Newton's second law, we obtain

$$T - m_1 g \sin\theta_1 = m_1 a$$
$$m_2 g \sin\theta_2 - T = m_2 a$$



Adding the two equations allows us to solve for the acceleration:

$$a = \left(\frac{m_2 \sin\theta_2 - m_1 \sin\theta_1}{m_2 + m_1} \right) g$$

With $\theta_1 = 30°$ and $\theta_2 = 60°$, we have a = 0.45 m/s^2. This value is plugged back into either of the two equations to yield the tension T = 16 N.

Also, we can find the tension directly by multiplying first equation by m_2 and second equation by m_1 then subtract them to get the tension after some rearrangements.