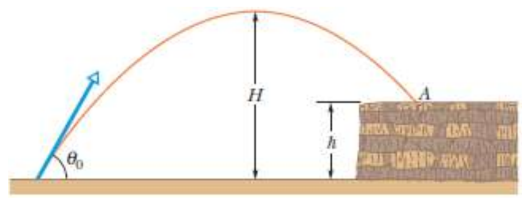
Chapter 4



1- In the figure, a stone is projected at a cliff of height h with an initial speed of 40.0 m/s directed at angle θ_0 = 55.0° above the horizontal. The stone strikes at A, 5.00 s after launching. Find the speed, in m/s, of the stone just before impact at A.

Answer:

The horizontal motion is steady, so $v_x = v_{0x} = v_0 \cos \theta_0$, but the vertical component of velocity varies according to Eq. 4-23. Thus, the speed at impact is

θ_0=50°; v_o = 40 m/s; g = 9.8 m/s^2; t = 5.0 s;

we know, $$v_{Ax} = v_{ox} = v_o \cos\theta_o; \quad v_{Ay} = v_{oy} - gt = v_o \sin\theta_o - gt.$$

$$v = \sqrt{(v_o \cos\theta_o)^2 + (v_o \sin\theta_o - gt)^2} \Rightarrow v = 28.11 \text{ m/s}$$

2- A ball is thrown horizontally from a height of 20 m and hits the ground with a speed that is three times its initial speed. What is the initial speed, in m/s?

Answer:

The initial velocity has magnitude v_0 and because it is horizontal, it is equal to v_x the horizontal component of velocity at impact. Thus, the speed at impact is

$$\sqrt{v_0^2 + v_y^2} = 3v_0$$

where and we have used the equation $v_y^2 = v_{0y}^2 + 2gh = 0 + 2gh \Rightarrow \quad v_y = \sqrt{2gh}$ replaced with h = 20 m. Squaring both sides of the first equality and substituting from the second, we find

$$v_0^2 + 2gh = (3v_0)^2$$

which leads to $gh = 4v_0^2$ and therefore to $v_0 = \sqrt{(9.8 \text{ m/s}^2)(20 \text{ m})}/2 = 7.0$ m/s.

3- The velocity $\vec{v}$ of a particle moving in the xy-plane is given by $\vec{v} = (6.0t - 4.0t^2)\,\hat{\mathrm{i}} + 8.0\,\hat{\mathrm{j}}$, with $\vec{v}$ in m/s and t (>0) in s. When is the acceleration zero?

Answer:

The acceleration as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left((6.0t - 4.0t^2)\hat{\mathrm{i}} + 8.0\,\hat{\mathrm{j}}\right) = (6.0 - 8.0t)\,\hat{\mathrm{i}} \text{ m/s}^2$$

$\vec{a} = (6.0 - 8.0t)\hat{\mathrm{i}} = 0$ ➔ ***t* = 0.75 s.** (note $\vec{a}$ has only x component)

4- A ball is thrown with a velocity $\vec{v}_0 = \left(3\,\hat{\mathrm{i}} + 5\,\hat{\mathrm{j}}\right)$ m/s from the ground. Its velocity just before it strikes the ground is:

Answer: $\vec{v}_0 = \left(3\,\hat{\mathrm{i}} - 5\,\hat{\mathrm{j}}\right)$ m/s

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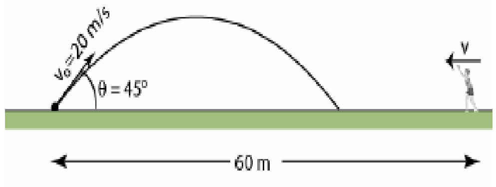
5. A ball is kicked from the ground with an initial speed of 20 m/s at an angle of 45°. A player 60 m away starts running to catch the ball at that instant (see the Figure). What must be his average speed (s) if he has to catch the ball just before it hits the ground? Ans: 6.6 m/s



Answer: Use $R = \frac{v_o^2}{g}\sin(2\theta_o) \Rightarrow R = \frac{v_o^2}{g}\sin(2\times 45^o) = \frac{20^2}{9.8} \approx 41$.

So, the boy's distance that he should travel will be 60-41=19 m. This should be done in the flight time, where the flight time is

$$T = 2\frac{v_o \sin\ \theta_o}{g} = \frac{2\times 20\times \sin(45^o)}{9.8} = 2.89$$

The average speed should be: $v = \frac{19-0}{2.89} = 6.57 \approx 6.6$ m/s

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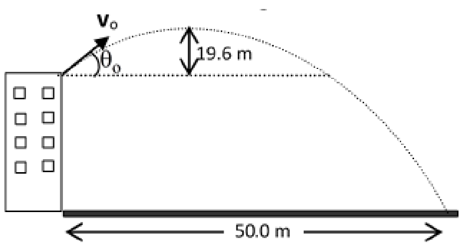
6. A projectile is fired with initial velocity $\mathbf{v}_o$ and angle θ_o = 60º from the top of a building (**Figure**) and is observed to reach a maximum height of 19.6 m. It later hits the ground at a horizontal distance of 50.0 m from the base of the building. Find the time of flight of the projectile. (Neglect air friction). Answer: 4.42 s



Ans. We know the horizontal distance = 50.0 m. If we can calculate the initial horizontal speed, then we can calculate the time of flight. First calculate the initial vertical velocity, with knowing the maximum height. i.e.

$$v_y^2 = v_{0y}^2 - 2g\left(y-y_0\right) \Rightarrow v_{0y} = v_o \sin 60^\circ = 19.6 \text{ m/s}$$

The final answer will be used to calculate the initial horizontal speed:

$$v_{0y} = 19.6 \text{ m/s} \Rightarrow v_{0x} = v_o \sin 60^\circ = 11.3 \text{ m/s} t = \frac{x}{v_{0x}} = 4.4 \text{ s}$$

Then the time will be:

$$t = \frac{x}{v_{0x}} = 4.4 \text{ s}$$

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7. A projectile's launch speed is 4 times its speed at maximum height. Find the launch angle from the horizontal. Answer: 75.5º

Ans: At launch point, the launch speed is v_o. At maximum height $v_y = 0$, $v_x = v_{ox} = v_o \cos\theta_o$.

Use the required condition: $v_o = 4v_x = v_o \cos\theta_o \Rightarrow \theta_o = \cos^{-1}(1/4) = 75.5^\circ$

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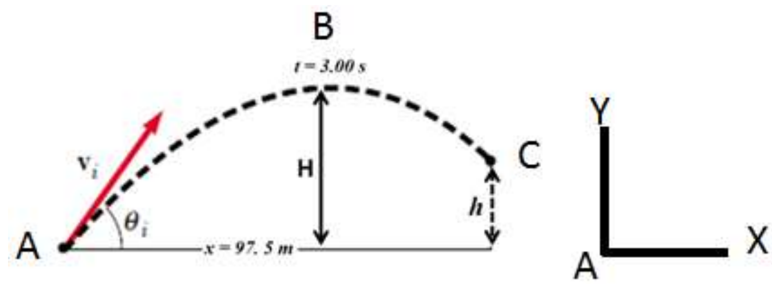
8. A baseball is hit at ground level as shown in **Figure 1**. The ball is observed to reach its maximum height above ground level 3.00 s after being hit. And 2.50 s after reaching this maximum height, the ball is observed to barely clear a fence of height *h* that is at a horizontal distance of 97.5 m from the point where it was hit. What is the height *h* of the fence? (Neglect air resistance)



Answer: Take the point A as the origin and the upward direction is positive.

Initial velocities at A will be v_{Ay} and $.v_{Ax}$

At the maximum height, we have $t=3$ and $v_{By}=0$. So, to calculate v_{Ay}, use the kinematic equation:

$$v_{By} = v_{Ay} - gt \quad \Rightarrow \quad v_{Ay} = gt$$

Now, to calculate H, we can use the equation:

$$d = v_{Ay} t - \frac{1}{2} g t^2 \Rightarrow H = gt^2 - \frac{1}{2} g t^2 = \frac{1}{2} g t^2 = \frac{1}{2}(9.8)3^2 = 44.1 \text{ m}$$

Take the motion from B to C and use the equation:

$$h - H = v_{By} t - \frac{1}{2} g t'^2 = -\frac{1}{2} g t'^2 = -\frac{1}{2}(9.8)(2.5)^2 = -30.6 \text{ m}$$

Then

$$h = (44.1 - 30.6) \text{ m} = 13.5 \text{ m}$$

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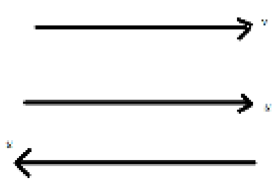
9. A boat takes 3 hours to travel 30 km along the river flow, then 5 hours to return to its starting point. How fast, in km/h, is the river flowing?



Answer:

$velocity\ of\ the\ Boat \rightarrow u;\quad velocity\ of\ the\ river\ \rightarrow v$

Along the flow, the time will be

$$t_1 = \frac{d}{u+v} \Rightarrow u + v = \frac{d}{t_1} \qquad (1)$$

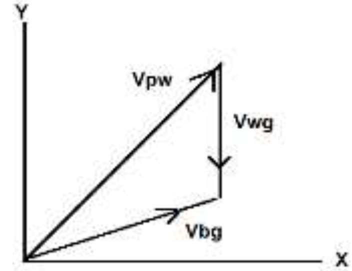
Against the flow, the time will be

$$t_2 = \frac{d}{u-v} \Rightarrow u - v = \frac{d}{t_2} \qquad (2)$$

Subtracting (1) and (2)

$$v = \frac{d}{2}\left(\frac{1}{t_1} - \frac{1}{t_2}\right) = \frac{30}{2}\left(\frac{1}{3} - \frac{1}{5}\right) = 2 \text{ km/h}$$

Q8: Wind blows with a speed of 60.0 km/h from the north towards south. A plane flies at 45.0° north of east at a speed of 200 km/h relative to the wind. The resultant speed of the plane relative to the ground is: **Ans:** 163 km/h



Answer:

w ⟶ wind; p ⟶ plane ; g ⟶ ground

$$\vec{v}_{wg} = -60.0\ \hat{j}\ (km/h)$$

$$\vec{v}_{pw} = (200 \times \cos 45)\hat{\imath} + (200 \times \sin 45)\hat{\jmath} = 141\ \hat{\imath} + 141\hat{\jmath}\ (km/h)$$

$$\vec{v}_{pg} = \vec{v}_{pw} + \vec{v}_{wg} = 141\ \hat{\imath} + 81\ \hat{\jmath}\ (km/h)$$

$$\therefore v_{pg} = [(141)^2 + (81)^2]^{\frac{1}{2}} = 163\ km/h$$

Q3. A jet fighter has a speed of 290.0 km/h and is diving at an angle of $\theta = 30^o$ below the horizontal when the pilot releases a missile (**Figure 1**). The horizontal distance between the release point and the point where the missile strikes the ground is d = 700 m. How high was the release point?

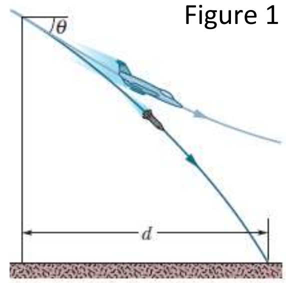


Figure 1

A) 897 m
B) 768 m
C) 954 m
D) 776 m
E) 966 m

Ans:

$$v_0 = 290\frac{km}{h}\ ;\ \theta = -30^o$$

$$d = 700\ m = v_0 \times cos(\theta) \times t \rightarrow t = \frac{700}{v_0 \times cos(\theta)}$$

$$0 - y_0 = v_0 \times sin(\theta) \times t - \frac{1}{2}gt^2 \rightarrow y_0 = 897\ m$$

5- Which of the following is NOT an example of accelerated motion?

A) Horizontal component of projectile motion
B) Circular motion at constant speed
C) A swinging pendulum
D) Earth's motion about sun
E) Vertical component of projectile motion

Ans: A
Difficulty: Easy
Section: 4-3
Learning Objective 4.3.1

6- The velocity of a projectile equals its initial velocity added to:

A) a constant horizontal velocity
B) a constant vertical velocity
C) a constantly increasing horizontal velocity
D) a constantly increasing downward velocity
E) a constant velocity directed at the target

Ans: D
Difficulty: Medium
Section: 4-4
Learning Objective 4.4.0

7- Two bodies are falling with negligible air resistance, side by side, above a horizontal plane. If one of the bodies is given an additional horizontal acceleration during its descent, it:

A) strikes the plane at the same time as the other body.
B) strikes the plane earlier than the other body.
C) has the vertical component of its velocity altered.
D) has the vertical component of its acceleration altered.
E) follows a straight line path along the resultant acceleration vector.

Ans: A
Difficulty: Easy
Section: 4-4
Learning Objective 4.4.0