

Chapter 4

Q1. A soccer ball is kicked from the ground and follows a parabolic path before landing on the ground. Which one of the following statements is **true**? (Neglect air resistance)

- A) The horizontal component of the velocity of the ball is the same throughout its flight
- B) The acceleration of the ball decreases as the ball moves upward
- C) The velocity of the ball is zero when the ball is at its maximum height
- D) The acceleration of the ball is zero when the ball is at its maximum height
- E) The vertical component of the velocity of the ball is zero just before hitting the ground

Q2. A particle starts from the origin at $t = 0$ with a velocity of $(6.0 \text{ m/s}) \hat{i}$ and moves in the xy plane with a constant acceleration of $(-2.0 \text{ m/s}^2) \hat{i} + (4.0 \text{ m/s}^2) \hat{j}$. At the instant the particle reaches its maximum positive x -coordinate, what is its corresponding y -coordinate?

- A) 18 m
- B) 36 m
- C) 11 m
- D) 27 m
- E) 15 m

Ans:

At maximum positive x -coordinate, $v_{fx} = 0$ but $v_{fx} = v_{ix} + a_x t$

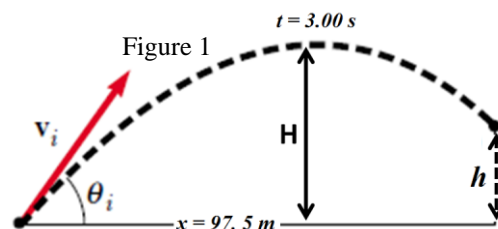
Then $v_{fx} = 0 = 6 - 2.0 t \Rightarrow at t$

$= 3 \text{ sec; and corresponding } y - \text{coordinate is:}$

$$y = v_{iy} + \frac{1}{2} a_y t^2 = 0 + \frac{1}{2} \times 4 \times (3)^2 = 18.0 \text{ m}$$

Q3. A baseball is hit at ground level as shown in **Figure 1**. The ball is observed to reach its maximum height above ground level 3.00 s after being hit. And 2.50 s after reaching this maximum height, the ball is observed to barely clear a fence of height h that is at a horizontal distance of 97.5 m from the point where it was hit. What is the height h of the fence? (Neglect air resistance)

- A) 13.5 m
- B) 30.6 m
- C) 2.80 m
- D) 44.1 m
- E) 4.90 m



Ans:

At $t = 3.00 \text{ s}$; $v_{iy} = 0$

For downward motion:

$$\text{Height } H = v_{iy} t - \frac{1}{2} |g| t^2 = -\frac{1}{2} |g| t^2$$

$$H = -\frac{1}{2} \times 9.8 \times (3)^2 = -44.1 \text{ m}$$

$$H - h = v_{iy} t - \frac{1}{2} |g| t^2 = -\frac{1}{2} |g| t^2$$

$$H - h = -\frac{1}{2} \times 9.8 \times (2.5)^2 = -30.63 \text{ m}$$

$$h = H + 30.63 = -44.1 + 30.63$$

$$h = -13.47$$

$$|h| = 13.47 = 13.5 \text{ m}$$

Q4.

A star with a diameter of 40.0 km rotates about its central axis making two revolutions per second. What is the speed (km/s), of an object on the star's equator?

- A) 251
- B) 628
- C) 400
- D) 100
- E) 450

Ans:

$$T = 0.5 \text{ sec}$$

$$v = \frac{2\pi R}{T} = \frac{2\pi \times 20 \times 10^3}{0.5} = 251327.4 \text{ m/s} = 2.51 \times 10^2 \text{ km/s}$$

Q5.

A boat is traveling upstream towards the east at 10 km/h with respect to the water of a river. The water is flowing at 5.0 km/h with respect to the ground. A man on the boat walks from front to rear at 3.0 km/h with respect to the boat. What are the magnitude and direction of the man's velocity with respect to the ground?

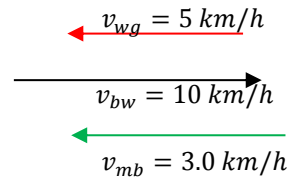
- A) 2.0 km/h, towards the east
- B) 2.0 km/h, towards the west
- C) 8.0 km/h, towards the east
- D) 12 km/h, towards the east
- E) 18 km/h, towards the west

Ans:

$$v_{mg} = v_{mb} + v_{bw} + v_{wg}$$

$$= -3 + 10 - 5$$

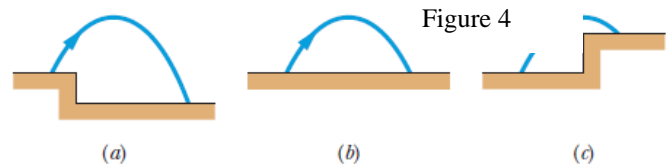
$$v_{mg} = +2 \text{ km/h}$$



Q6.

Figure 4 shows three situations in which identical projectiles are launched (at the same level) at identical initial speeds and angles. The projectiles do not land on the same surface, however. Rank the situations according to the final speeds of the projectiles just before they land, greatest first.

- A) a; b; c
- B) a; c; b
- C) b; a; c
- D) b; c; a
- E) c; a; b



Q7.

A particle's position vector is initially $\vec{r} = 10.0\hat{i} - 12.0\hat{j} + 4.0\hat{k}$, and 10 s later it is $\vec{r} = -4.0\hat{i} + 16.0\hat{j} - 4.0\hat{k}$, all in meters. In unit vector notation, what is its \vec{v}_{avg} during the 10 s?

- A) $-1.4\hat{i} + 2.8\hat{j} - 0.8\hat{k}$
- B) $+1.4\hat{i} + 2.8\hat{j} - 0.8\hat{k}$

- C) $-1.4\hat{i} - 2.8\hat{j} - 0.8\hat{k}$
- D) $-1.4\hat{i} + 2.8\hat{j} + 0.8\hat{k}$
- E) $+1.4\hat{i} - 2.8\hat{j} + 0.8\hat{k}$

Ans:

$$\begin{aligned} \vec{r}_1 &= 10\hat{i} - 12\hat{j} + 4\hat{k} \\ \vec{r}_2 &= -4\hat{i} + 12\hat{j} - 4\hat{k} \\ \Delta\vec{r} &= \vec{r}_2 - \vec{r}_1 = -14\hat{i} + 28\hat{j} - 8\hat{k} \\ \Delta t &= 10 \text{ s}, \vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = -1.4\hat{i} + 2.8\hat{j} - 0.8\hat{k} \end{aligned}$$

Q8.

A small ball rolls horizontally off the edge of a tabletop that is 2.40 m high. It strikes the floor at point 3.00 m horizontally from the table edge. How long is the ball in the air?

- A) 0.700 s
- B) 0.500 s
- C) 0.600 s
- D) 0.400 s
- E) 0.800 s

Ans:

$$y = h - \frac{1}{2}g \times t^2 \rightarrow y = 0 \text{ at } t = \sqrt{\frac{2 \times h}{g}} = \sqrt{\frac{2 \times 2.40 \text{ m}}{9.8 \frac{\text{m}}{\text{s}^2}}} = 0.700 \text{ s}$$

Q9.

A student runs as fast as he can along a moving sidewalk in 2.50 s. He then turns around and runs as fast as he can back along the moving sidewalk to his starting point, taking 10 s. What is the ratio of the student's running speed to the sidewalk's speed?

- A) 1.67
- B) 1.25
- C) 1.27
- D) 1.17
- E) 1.34

Ans:

$$\begin{aligned} t_1 &= \frac{d}{v + V} = 2.5 \text{ s} \\ t_2 &= \frac{d}{v - V} = 10 \text{ s} \\ \frac{t_2}{t_1} &= 4 = \frac{v + V}{v - V} \rightarrow \frac{v}{V} = \frac{5}{3} = 1.67 \end{aligned}$$

Q10.

A ship sails due north at 4.50 m/s relative to the ground while a boat heads northwest with a speed of 5.20 m/s relative to the ground. Find the speed of the ship relative to the boat.

- A) 3.77 m/s
- B) 2.39 m/s
- C) 7.95 m/s
- D) 1.25 m/s
- E) 6.11 m/s

Ans:

$$\vec{v}_{so} = \vec{v}_{bo} + \vec{v}_{bs}$$

$$4.5 \hat{j} = -3.68 \hat{i} + 3.68 \hat{j} + \vec{v}_{bs}$$

$$\vec{v}_{bs} = 3.68 \hat{i} + 0.823 \hat{j}$$

$$v_{bs} = \sqrt{(3.68)^2 + (0.823)^2} = 3.77 \text{ m/s}$$

Q11.

A student throws a red ball from the balcony of a tall building with an initial horizontal speed of 10 m/s. At the same time, a second student drops a blue ball from the same balcony. Neglecting air resistance, which statement is true?

- A) The two balls reach the ground at the same instant.
- B) The blue ball reaches the ground first.
- C) The red ball reaches the ground first.
- D) Both balls hit the ground with the same speed.
- E) The blue ball hits the ground with larger speed.

Ans:

Vertical motions are affected by the same constant acceleration; So The two balls reach the ground at the same instant

Q12.

A projectile is thrown from the ground into the air with an initial speed v_0 . Its velocity, 1.50 s after it was thrown, is 42.3 m/s making an angle 30.4° above the horizontal. Determine the initial velocity v_0 of the projectile.

- A) 51.3 m/s at 44.7° above the horizontal
- B) 43.1 m/s at 34.2° above the horizontal
- C) 21.6 m/s at 49.2° above the horizontal
- D) 32.5 m/s at 23.5° above the horizontal
- E) 12.2 m/s at 54.5° above the horizontal

Ans:

$$v_y(t = 1.5 \text{ sec}) = v \sin \theta = 42.3 \sin(30.4) = 21.4 \text{ m/s}$$

$$v_{0x} = v \cos \theta = 42.3 \cos(30.4) = 36.48 \text{ m/s}$$

$$v_{0y} = v_y + gt = 21.41 + 9.8 \times 1.5 = 36.11 \text{ m/s}$$

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{36.11^2 + 36.48^2} = (51.3 \text{ m})/\text{s}$$

$$\theta = \tan^{-1} \left(\frac{v_{0y}}{v_{0x}} \right) = \tan^{-1} \left(\frac{36.11}{36.48} \right) = 44.7^\circ$$

Q13.

A stone is tied to the end of a string and is rotated in a horizontal circle at 400 revolutions per minute. If the magnitude of its acceleration is $1.5 \times 10^3 \text{ m/s}^2$, what is the radius of the circle?

- A) 0.85 m
- B) 0.35 m
- C) 0.64 m
- D) 0.71 m
- E) 0.53 m

Ans:

$$a_r = 1.5 \times 10^3 = \frac{v^2}{r} \Rightarrow v = 38.7 \sqrt{r}$$
$$T = \frac{2\pi r}{v} = \frac{1}{f} = \frac{r}{\frac{400}{60}} = \frac{2\pi r}{v}$$
$$0.15 \text{ s} = T = \frac{2\pi r}{38.7 \sqrt{r}} \Rightarrow \mathbf{r = 0.85 \text{ m}}$$

Q14.

A ball is thrown straight upward and returns to the thrower's hand (at the same initial level) after 3.00 s. A second ball thrown from the same height at an angle of 37.0° with the horizontal reaches the same maximum height as the first ball. With what speed was the second ball thrown?

- A) 24.4 m/s
- B) 14.7 m/s
- C) 29.1 m/s
- D) 49.3 m/s
- E) 35.2 m/s

Ans:

For 1st ball $\Rightarrow v_{yf} = 0v_{oy} - gt$
 $\Rightarrow v_{oy} = (9.8)(1.5) = 14.7 \text{ m/s}$
maximum height $\Rightarrow v_{fy}^2 = v_{oy}^2 - 2g(\Delta y)$
 $0 = (14.7)^2 - (2)(9.8)(\Delta y)$
 $\Delta y = 11.0 \text{ m}$
2nd ball
 $v_{fy}^2 = 0 = v_{oy}^2 \sin^2(37) - (2)(9.8)(11)$
 $v_{oy} = \mathbf{24.4 \text{ m/s}}$

Q15.

A particle starts from the origin of an xy plane. Its acceleration is given by $\vec{a} = (2.0\hat{i} + 4.0\hat{j}) \text{ m/s}^2$. At time $t = 0$, the velocity is $-4.0\hat{i} \text{ m/s}$. What is the particle's velocity if the y -component of its displacement is $+18 \text{ m}$?

- A) $(2.0\hat{i} + 12\hat{j}) \text{ m/s}$
- B) $(4.0\hat{i} - 6.0\hat{j}) \text{ m/s}$
- C) $(2.0\hat{i} + 2.0\hat{j}) \text{ m/s}$
- D) $(3.0\hat{i} + 12\hat{j}) \text{ m/s}$
- E) $(4.0\hat{i} - 4.0\hat{j}) \text{ m/s}$

Ans:

$$\Delta y = v_{oy}t + \frac{1}{2}a_y t^2$$

$$18 = 0 + \frac{1}{2}(4)t^2$$

$$\Rightarrow t = 3 \text{ s}$$

$$v_x = v_{ox} + a_{xt} = -4 + (2)(3) = \mathbf{2 \text{ m/s}}$$

$$v_y = v_{oy} + a_{yt} = 0 + (4)(3) = \mathbf{12 \text{ m/s}}$$

Q3.

A projectile's launch speed is 4 times its speed at maximum height. Find the launch angle from the horizontal.

A) 75.5°

B) 70.6°

C) 45.3°

D) 32.0°

E) 49.2°

Ans:

At maximum height $v_y = 0 \Rightarrow v = v_x = v_{0x}$

$$v_0 = 4 v_{0x} \Rightarrow v_0 = 4v_0 \cos\theta_0$$

$$\Rightarrow \theta = \mathbf{75.5^\circ}$$

Q4.

A particle moves at constant speed in a circular path. The instantaneous velocity and instantaneous acceleration vectors are both:

A) Perpendicular to each other

B) Perpendicular to the circular path

C) tangent to the circular path

D) Opposite to each other

E) Parallel to each other

Ans:

In uniform circular motion, the acceleration is perpendicular to velocity at each instant.

Q13.

A 0.150 kg ball, attached to the end of a string, is revolving uniformly in a horizontal circle of radius 0.600 m. The ball makes 10.0 revolutions in 5.00 seconds. Calculate the centripetal acceleration of the ball?

A) 94.8 m/s²

B) 25.7 m/s²

C) 12.6 m/s²

D) 9.81 m/s²

E) zero

Ans:

$$a = \frac{v^2}{R}; v = \frac{2\pi R}{T}; T = \frac{5 \text{ sec}}{10 \text{ rev}} = \frac{1}{2} \text{ sec}$$

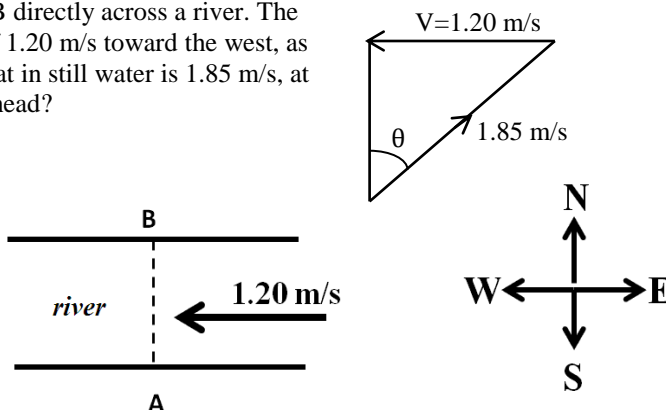
$$v = \frac{2\pi \times 0.6}{0.5} = 7.54 \text{ m/s}$$

$$a = \frac{(7.54)^2}{0.6} = 94.8 \text{ m/s}^2$$

Q14.

A boat is to travel from point A to point B directly across a river. The water in the river flows with a velocity of 1.20 m/s toward the west, as shown in **Figure 3**. If the speed of the boat in still water is 1.85 m/s, at what angle from the north must the boat head?

- Figure # 3
- A) 40.4° east of north
 - B) 30.2° west of north
 - C) 10.5° east of north
 - D) 90.0° west of north
 - E) 55.0° west of north



Ans:

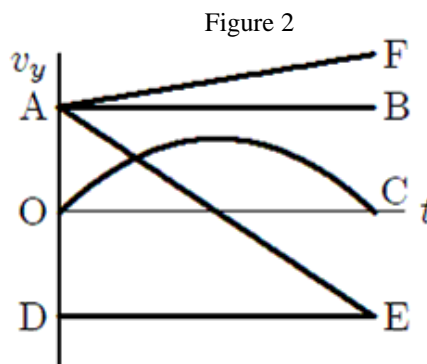
$$\theta = \sin^{-1}\left(\frac{1.2}{1.85}\right)$$

$$= 40.44^\circ \text{ of north}$$

Q15.

Which one of the curves shown in **Figure 2** best represents the vertical component of the velocity v_y versus time t for a projectile fired at an angle of 45° above the horizontal?

- A) AE
- B) AB
- C) OC
- D) DE
- E) AF



Ans:

$$v_y = v_{iy} - gt$$

$$\frac{\partial y}{\partial t} = -g, \text{ line with } - \text{ve slope}$$

Q16.

A stone is tied to a string and rotated in a circle of radius 4 m at a constant speed. If the magnitude of its acceleration is 16 m/s^2 , what is the period of the motion?

- A) $\pi \text{ s}$
- B) $2\pi \text{ s}$
- C) $3\pi \text{ s}$
- D) $\pi/2 \text{ s}$
- E) $4\pi \text{ s}$

Ans:

$$V = \sqrt{R \cdot a} = \sqrt{(4.0 \text{ m} \times 16 \text{ m/s}^2)} = 8.0 \text{ m/s}$$

$$T = 2\pi R/v = (2\pi \times 4.0 \text{ m}) / (8.0 \text{ m/s}) = \pi \text{ s}$$

Q17.

The minimum speed of a projectile during the whole flight is 5.0 m/s. It takes 4.0 s to reach its horizontal range. What is the horizontal range of the projectile?

- A) 20 m
- B) 30 m
- C) 40 m
- D) 50 m
- E) 10 m

Ans:

The minimum speed of the projectile motion is at the highest point, when $V_y = 0$. So minimum speed = V_x

$$\text{Horizontal range} = V_x \times t = 5 \text{ m/s} \times 4 \text{ s} = 20 \text{ m}$$

Q18.

A projectile is launched from ground level with an initial velocity:

$$\vec{V}_0 = (20 \hat{i} + 12 \hat{j}) \text{ m/s.}$$

How far, from the launch point, will it travel horizontally as it hits the ground? Ignore air resistance.

- A) 49 m
- B) 51 m
- C) 69 m
- D) 25 m
- E) 92 m

Ans:

$$v_{0x} = 20; v_{0y} = 12; g = 9.8; v_y = 0$$

$t \equiv$ time for maximum height

$$-v_y = v_{0y} - gt = 0$$

$$-t = \frac{v_{0y}}{g} = \frac{12}{9.8} = 1.22449$$

$$x = v_0 \times 2t$$

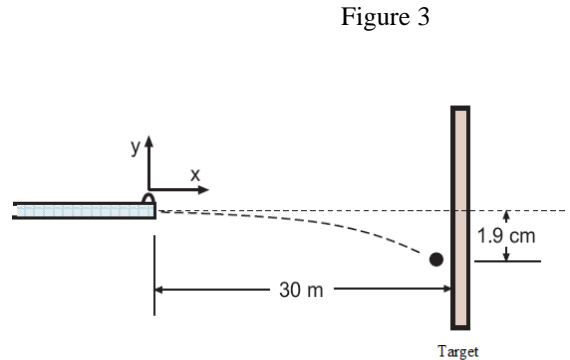
$$20 \times 2 \times 1.22449 = 49 \text{ m}$$

Q19.

A rifle is aimed horizontally at a target 30 m away. The bullet hits the target 1.9 cm below the aiming point, as shown in **Figure 3**. At what time will the bullet hit the target after being fired?

- A) $6.2 \times 10^{-2} \text{ s}$
- B) $2.0 \times 10^{-2} \text{ s}$
- C) $3.9 \times 10^{-3} \text{ s}$
- D) $2.9 \times 10^{-3} \text{ s}$
- E) $4.2 \times 10^{-2} \text{ s}$

Ans:



$$\Delta y = -\frac{1}{2}gt^2 \Rightarrow -1.9 \times 10^{-2} = -\frac{1}{2}(9.8)t^2 \Rightarrow t^2 = \frac{2(1.9 \times 10^{-2})}{9.8} = 3.9 \times 10^{-3} \text{ s}^2$$
$$t = 6.2 \times 10^{-2} \text{ s}$$

Q15.

A particle is moving in uniform circular motion with speed V , period T and radius R . What is the magnitude of the average acceleration of the particle over one period?

- A) 0
- B) $(3V)/T$
- C) $(2V)/T$
- D) V/T
- E) $V/(2T)$

Ans: For one period, the particle return back to the initial position with the same velocity.

Q4.

A 1.0 kg stone is tied to a 0.50 m string and rotated at a constant speed of 4.0 m/s in a vertical circle. Find the magnitude of the tension in the string when the stone is at the bottom of the circle? [Ignore air resistance]

- A) 42 N
- B) 24 N
- C) 64 N
- D) 33 N
- E) 98 N

Ans:

$$T - mg = \frac{mv^2}{r}$$

$$T = mg + \frac{mv^2}{r} = (1)(9.8) + \frac{(1)(4)^2}{0.5} = 42 \text{ N}$$

Q2.

A projectile is fired from ground level at an angle of 45° above the horizontal. Which of curves in **Figure 8** represents the vertical component of the velocity as a function of time?

- A) **AE**
- B) OC
- C) DE
- D) AB
- E) AF

Ans:

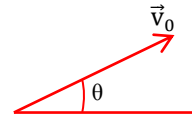
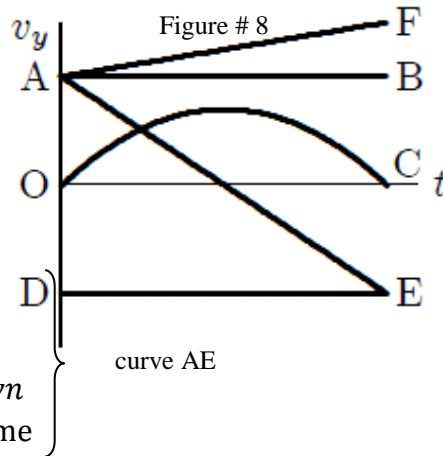
$$v_y = v_{0y} - \frac{1}{2}gt^2$$

$$\therefore v_y > 0 \rightarrow \text{going up}$$

$$v_y = 0 \rightarrow \text{top}$$

$$v_y < 0 \rightarrow \text{going down}$$

Also, v_y is linear vs time



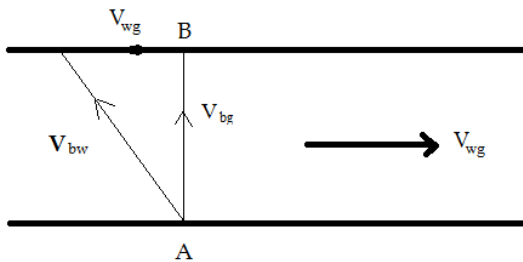
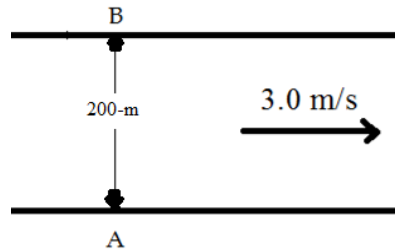
Q14.

Figure 4 shows a 200-m wide river which has a uniform flow speed of 3.0 m/s toward the east. A boat with a speed of 8.0 m/s relative to the water leaves the south bank at point A and crosses the river to point B directly north of its departure point. How long does it take the boat to cross the river?

- A) **27 s**
- B) 23 s
- C) 25 s
- D) 29 s
- E) 17 s

Ans:

Figure 4



$$V_{bg} = \text{SQRT}(V_{bw}^2 - V_{wg}^2) = \text{SQRT}(8.0^2 - 3.0^2) = 7.4 \text{ m/s} \rightarrow t = 200/7.4 = 27 \text{ s}$$

Q: Wind blows with a speed of 60.0 km/h from the north towards south. A plane flies at 45.0° north of east at a speed of 200 km/h relative to the wind. The resultant speed of the plane relative to the ground is

- A) 163 km/h
- B) 176 km/h
- C) 100 km/h
- D) 143 km/h
- E) 120 km/h

Ans:

w → wind; p → plane ; g → ground

$$\vec{v}_{wg} = -6.00 \hat{j} \text{ (k/h)}$$

$$\vec{v}_{pw} = (200 \times \cos 45^\circ) \hat{i} + (200 \times \sin 45^\circ) \hat{j} = 141 \hat{i} + 141 \hat{j} \text{ (km/h)}$$

$$\vec{v}_{pg} = \vec{v}_{pw} + \vec{v}_{wg} = 141 \hat{i} + 81 \hat{j} \text{ (km/h)}$$

$$\therefore v_{pg} = [(141)^2 + (81)^2]^{\frac{1}{2}} = 163 \text{ km/h}$$

Q3.

An object is moving on a horizontal circular path of radius 1.5 meters at a constant speed. The time required for one revolution is 3.2 s. The acceleration of the object is:

- A) 5.8 m/s²
- B) 2.6 m/s²
- C) 7.7 m/s²
- D) 1.4 m/s²
- E) zero

Ans:

$$a = \frac{v^2}{R} = \frac{1}{R} \left(\frac{2\pi R}{T} \right)^2$$

Q4.

A particle moves at constant speed in a circular path. The instantaneous velocity and instantaneous acceleration vectors are both:

- A) Perpendicular to each other
- B) Perpendicular to the circular path
- C) tangent to the circular path
- D) Opposite to each other
- E) Parallel to each other

Ans:

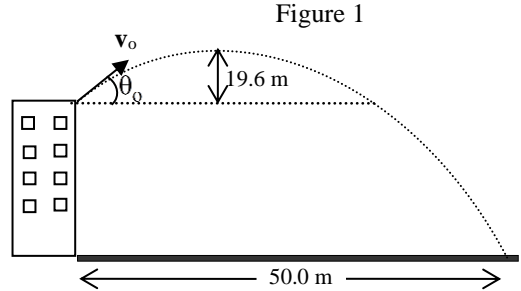
In uniform circular motion, the acceleration is perpendicular to velocity at each instant.

Q4.

A projectile is fired with initial velocity v_0 and angle $\theta_0 = 60^\circ$ from the top of a building (**Figure 1**) and is observed to reach a maximum height of 19.6 m. It later hits the ground at a horizontal distance of 50.0 m from the base of the building. Find the time of flight of the projectile. (Neglect air friction)

- A) 4.42 s
- B) 5.00 s
- C) 9.80 s
- D) 3.32 s
- E) 2.50 s

Ans:



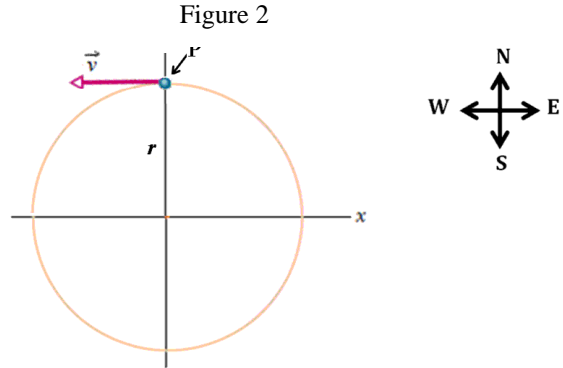
$$v_y^2 = v_{0y}^2 - 2g(y - y_0) \Rightarrow v_{0y} = 19.6 \text{ m/s} \Rightarrow v_{0x} = 11.3 \text{ m/s} \Rightarrow t = \frac{x}{v_{0x}} = 4.4 \text{ s}$$

Q3.

A particle P moves in counterclockwise nonuniform circular motion around a circle of radius r as shown in **Figure 2**. At a certain instant the velocity \vec{v} of the particle is 24 m/s west, and the acceleration of the particle has components of 2.4 m/s² east and 1.8 m/s² south. What is the radius of the circle?

- A) 0.32 km
- B) 0.19 km
- C) 0.54 km
- D) 0.14 km
- E) 0.27 km

Ans:



$$a_r = \frac{v^2}{r}; \text{ then } r = \frac{v^2}{a_r}$$

$$r = \frac{v^2}{a_r} = \frac{(24)^2}{1.8} = 320 \text{ m} = 0.32 \text{ km}$$

Q4.

A 50 kg boy and a 10 kg box are on a frictionless ice of a frozen pond. They are 15 m apart and connected by a rope of negligible mass. The boy exerts a horizontal 5.0 N force on the rope to pull the box. How far from the boy's initial position do they meet?

- A) 2.5 m
- B) 3.0 m
- C) 5.6 m
- D) 0.50 m
- E) 4.3 m

Ans:

$$a_{boy} = \frac{5}{50} = 0.1 \text{ m/s}^2; a_{sled} = \frac{5}{10} = 0.5 \text{ m/s}^2$$

$$t = \frac{d}{\frac{1}{2}a_{boy}} = \frac{15 - d}{\frac{1}{2}a_{sled}}; \text{ then } d \times a_{sled} = (15 - d) a_{boy}$$

$$d = \frac{15 \times a_{boy}}{a_{sled} + a_{boy}} = \frac{15 \times 0.1}{0.1 + 0.5} = 2.5 \text{ m}$$

Q3.

A projectile's launch speed is 4 times its speed at maximum height. Find the launch angle from the horizontal.

- A) 75.5°
- B) 70.6°
- C) 45.3°
- D) 32.0°
- E) 49.2°

Ans:

At maximum height $v_y = 0 \Rightarrow v = v_x = v_{0x}$
 $v_0 = 4 v_{0x} \Rightarrow v_0 = 4v_0 \cos\theta_0$
 $\Rightarrow \theta = 75.5^\circ$

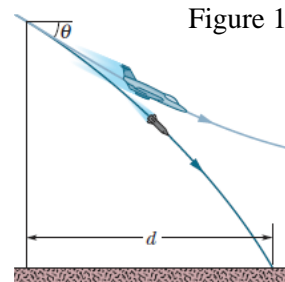
Q3.

A jet fighter has a speed of 290.0 km/h and is diving at an angle of $\theta = 30^\circ$ below the horizontal when the pilot releases a missile (**Figure 1**). The horizontal distance between the release point and the point where the missile strikes the ground is $d = 700$ m. How high was the release point?

- A) 897 m
- B) 768 m
- C) 954 m
- D) 776 m
- E) 966 m

Ans:

$v_0 = 290 \frac{\text{km}}{\text{h}} ; \theta = -30^\circ$
 $d = 700 \text{ m} = v_0 \times \cos(\theta) \times t \rightarrow t = \frac{700}{v_0 \times \cos(\theta)}$
 $0 - y_0 = v_0 \times \sin(\theta) \times t - \frac{1}{2}gt^2 \rightarrow y_0 = 897 \text{ m}$



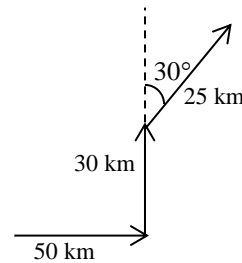
Q3.

A car is driven east for a distance of 50 km, then north for 30 km, and then in a direction 30° east of north for 25 km. Calculate the magnitude of the car's total displacement from its starting point.

- A) 81 km
- B) 55 km
- C) 97 km
- D) 35 km
- E) 11 km

Ans:

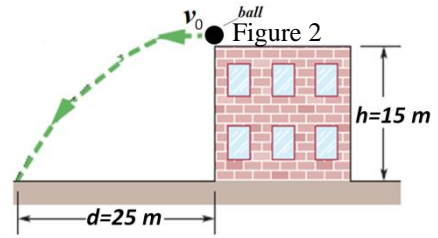
Total displacement $\vec{D} = 50\vec{i} + 30\vec{j} + 25 \cos 30^\circ \vec{j} + 25 \sin 30^\circ \vec{i}$
 $\vec{D} = (50 + 25 \sin 30^\circ)\vec{i} + (30 + 25 \cos 30^\circ)\vec{j}$
 $|\vec{D}| = \sqrt{(62.5)^2 + (51.65)^2} = 81.08 \text{ m} \approx 81 \text{ km}$



Q4.

A ball is thrown horizontally leftward from the left edge of a roof, at height $h = 15.0$ m above the ground. The ball hits the ground at a distance $d = 25.0$ m from the building, as shown in **Figure 2**. Find the speed of the ball when it hits the ground. (Ignore air resistance).

- A) 22.3 m/s
- B) 17.2 m/s
- C) 14.3 m/s
- D) 32.8 m/s
- E) 42.9 m/s



Ans:

$$|v_{fy}| = \sqrt{v_{iy}^2 - 2gy} = \sqrt{-2gh} = \sqrt{2 \times 9.8 \times 15}$$

$$= 17.14\text{ m/s}; v_{fy} = -17.14\text{ m/s}$$

$$t = \frac{v_{fy} - v_{iy}}{-g} = \frac{v_{fy}}{-g} = \frac{-17.14}{-9.8} = 1.75\text{ s}$$

$$v_{fx} = \frac{d}{t} = \frac{25}{1.75} = 14.3\text{ m/s}$$

$$|v_f| = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(14.3)^2 + (-17.14)^2} = 22.3\text{ m/s}$$

Q4.

A boat takes 3 hours to travel 30 km along the river flow, then 5 hours to return to its starting point. How fast, in km/h, is the river flowing?

- A) 2
- B) 8
- C) 6
- D) 4
- E) 9

Ans:

Boat $\rightarrow u$; river $\rightarrow v$

$$t_1 = \frac{d}{u + v}$$

$$u + v = \frac{d}{t_1} \rightarrow (1)$$

$$u - v = \frac{d}{t_2} \rightarrow -u + v = -\frac{d}{t_2} \rightarrow (2)$$

Adding (1) and (2)

$$v = \frac{d}{2} \left(\frac{1}{t_1} - \frac{1}{t_2} \right)$$

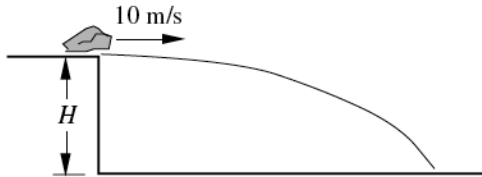
$$= \frac{30}{2} \left(\frac{1}{3} - \frac{1}{5} \right) = 2\text{ km/h}$$

More Projectile Motion Problems

PROBLEM 1

A rock is kicked horizontally at a speed of 10 m/s from the edge of a cliff of height $H = 45$ m.

- a) How far from the base of the cliff does the rock land?
- b) What is the velocity of the rock at the instant it lands?



PROBLEM 2

A soccer ball is kicked at ground level at an angle of 53° to the horizontal with an initial velocity of 15 m/s.

- a) What is the maximum height of the ball?
- b) What is the total time the ball is in the air?
- c) How far away does the ball land?

PROBLEM 3

A projectile is shot from the edge of a cliff 125 m above the ground level with an initial speed of 100 m/s at an angle of 37.0° with the horizontal.

- a) Determine the time taken by the projectile to hit the ground.
- b) Determine the range of the projectile as measured from the base of the cliff.
- c) Determine the total velocity vector the instant the projectile strikes the ground.

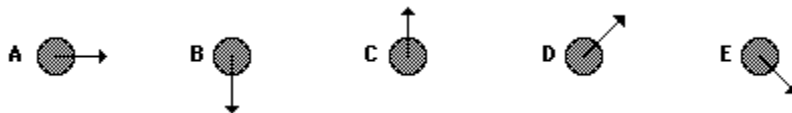
PROBLEM 4

A projectile is launched at a velocity of 125 m/s at an angle of 20° . It strikes a building at a height of 58.6 m above the ground.

- a) How far away is the building?
- b) Why are there two possible answers?

Projectile Motion Practice Questions

1. Consider these diagrams. Which diagram, if any, would show:

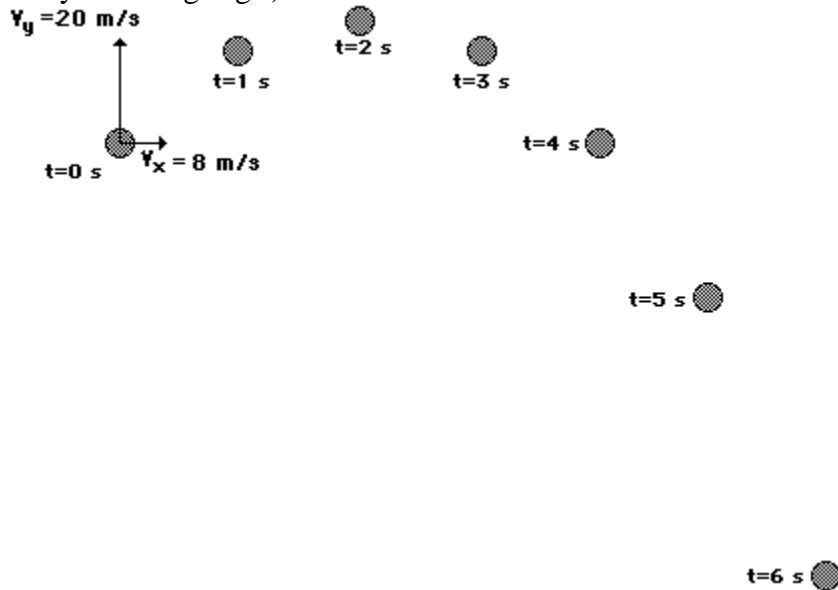


- a) The initial horizontal velocity?
- b) The initial vertical velocity?
- c) The horizontal acceleration?
- d) The vertical acceleration?

Circle the best answer for questions 2 and 3.

2. Suppose a tank is able to point its barrel up at an angle of 90° . If the tank is in motion and fires a shell straight up, where will the shell land if the tank maintains a constant motion. (Don't consider any air resistance)

- a) in front of the tank
 - b) behind the tank
 - c) on top of the tank
3. Suppose an airplane drops a package out of its cargo hold while it is travelling through the sky with a constant horizontal speed. Where will the package land once it hits the ground?
- a) directly below the plane
 - b) below the plane and ahead of it
 - c) below the plane and behind it
4. A cannonball is launched horizontally from the top of an 80.0 m high cliff. How much time will it take for the ball to reach the ground and at what height will the ball be after each second of travel?
5. Fill in the table below indicating the value of the horizontal and vertical components of velocity and acceleration for a projectile. (Don't worry about sig. figs.)
6. The diagram below shows the path for a projectile launched upwards and at an angle from the top of a tall building. The initial horizontal and vertical components of velocity are shown at $t = 0$ s. The path of the projectile is shown in time intervals of one second. For each time interval, state the horizontal and vertical components of velocity. (Don't worry about sig. figs.)



7. A cannonball is launched horizontally from the top of a 45 m high cliff.

a) How long is the cannonball in the air? (Don't worry about sig. figs.)

b) At what height is the cannonball after each second?

8. Determine the horizontal (v_{ix}) and vertical components (v_{iy}) for the following situations:

a) A water balloon is launched with a speed of 40.0 m/s at an angle of 60.0° to the horizontal.

b) A motorcycle stunt person traveling 70.0 m/s jumps off a ramp at an angle of 35.0° to the horizontal.

c) A springboard diver jumps with a velocity of 10.0 m/s at an angle of 80.0° to the horizontal.

Solutions

1. a) A

b) B or C

c) None, as there is no horizontal acceleration for projectiles.

d) None, since none of the diagrams on their own properly shows any acceleration. A series of diagrams showing either a speeding up or slowing down would be necessary.

2. C

3. A

4. $y = 80.0 \text{ m}$
 $g = -10 \text{ m/s}^2$

$$y = 0.5gt^2$$

$$-80.0 \text{ m} = 0.5(-10)t^2$$

$$-80.0 = -5t^2$$

$$-80.0/-5 = t^2$$

$$16.0 = t^2$$

$$t = 4.00 \text{ s}$$

So it will take 4.00 seconds to fall 80.0 m.

We can use the same equation to find the height as well after each second of motion.

$$y = \frac{1}{2}gt^2$$

So for the first second:

$$y = \frac{1}{2}(-10)(1.00)^2$$

$$y = -5.00 \text{ m}$$

So now to find the height (which was initially at 80 m) we will take 80 m and subtract the distance that the ball has dropped. In other words:

$$80.0 \text{ m} - 5.00 \text{ m} = \text{a height of } 75.0 \text{ m}$$

Then continue to plug in each second and compute:

$$\text{So at } t = 2.00 \text{ s, } y = 20.0 \text{ m (down)... } 80.0 \text{ m} - 20.0 \text{ m} = 60.0 \text{ m}$$

$$\text{At } t = 3.00 \text{ s, } y = 45.0 \text{ m (down)... } 80.0 \text{ m} - 45.0 \text{ m} = 35.0 \text{ m}$$

$$\text{At } t = 4.00 \text{ s, } y = 80.0 \text{ m (down)... } 80.0 \text{ m} - 80.0 \text{ m} = 0.00 \text{ m}$$

5. The v_x values remain 10 m/s for the entire 6 seconds; the a_x values will be 0 m/s² for the entire 6 seconds.

The v_y values will be changing by 10 m/s each second, so $v_y = 20$ m/s at the first second, 10 m/s at the second second, 0 m/s at the third second, -10 m/s at the fourth second, -20 m/s at the fifth second, -30 m/s at the sixth second.

The a_y values will be -10 m/s²

6. The v_x values will remain 8 m/s for the entire 6 seconds.

The v_y values will be changing by -10 m/s each second.

So $v_y = 10$ m/s at the first second, $v_y = 0$ m/s at the second second, $v_y = -10$ m/s at the third second, $v_y = -20$ m/s at the fourth second, $v_y = -30$ m/s at the fifth second, and $v_y = -40$ m/s at the sixth second.

7. a) $y = 0.5gt^2$
 $-45 = 0.5(-10)t^2$
 $-45 = -5t^2$
 $-45/-5 = -5t^2/-5$
 $9.0 = t^2$
 $t = 3.0$ s

b) First second: $45 - 5 = 40$ m high
Second second: $45 - 20 = 25$ m high
Third second: $45 - 45 = 0$ m (hits ground)

8. a) $\cos 60.0 = v_x/40.0$ m/s, $v_x = (40.0 \text{ m/s})(\cos 60.0)$, $v_x = 20.0$ m/s.
 $\sin 60 = v_y/40.0$ m/s, $v_y = (40.0 \text{ m/s})(\sin 60.0)$, $v_y = 34.6$ m/s

b) $\cos 35.0 = v_x/70.0$ m/s, $v_x = (70.0 \text{ m/s})(\cos 35.0)$, $v_x = 57.3$ m/s.
 $\sin 35.0 = v_y/70.0$ m/s, $v_y = (70.0 \text{ m/s})(\sin 35.0)$, $v_y = 40.2$ m/s.

c) $\cos 80.0 = v_x/10.0$ m/s, $v_x = (10.0 \text{ m/s})(\cos 80.0)$, $v_x = 1.74$ m/s.
 $\sin 80.0 = v_y/10.0$ m/s, $v_y = (10.0 \text{ m/s})(\sin 80.0)$, $v_y = 9.85$ m/s