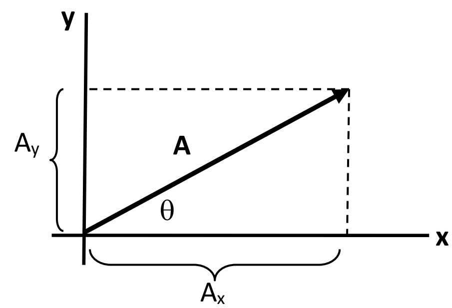
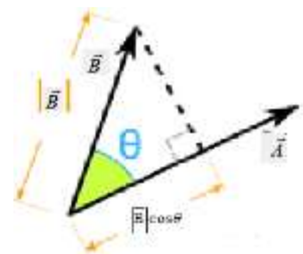
Chapter 3
Summary

Consider vector $\mathbf{A}$, with a direction of θ above the horizontal.



$A_x = |\mathbf{A}|\cos\theta \equiv$ The projection of $\mathbf{A}$ along x-axis ,

$\equiv$ The component of $\mathbf{A}$ along x-axis,



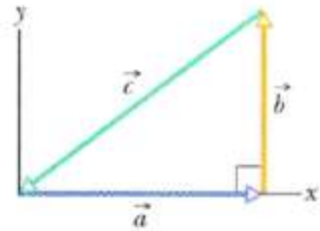
$A_y = |\mathbf{A}|\sin\theta \equiv$ The projection of $\mathbf{A}$ along y-axis

$\equiv$ The component of $\mathbf{A}$ along y-axis.

Define the two vectors $\vec{A} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)$, $\vec{B} = \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$ and the angle between them is θ. Then we can construct the following table:

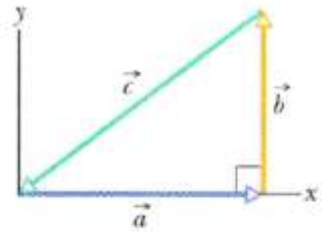
	Dot-product	Cross-product
Definition	$\vec{A}\cdot\vec{B} = \lvert\vec{A}\rvert\lvert\vec{B}\rvert\cos\theta$	$\vec{A}\times\vec{B} = \lvert\vec{A}\rvert\times\lvert\vec{B}\rvert\sin\theta\ \hat{n}$
Unit vector	$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \cdots = 1,$ $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i} = \cdots = 0$	$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \hat{0},$ $\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}$
Expansion	$\vec{A}\cdot\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$ $= A_xB_x + A_yB_y + A_zB_z$	$\vec{A}\times\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$ $= \left(A_yB_z - A_zB_y\right)\hat{i} - \left(A_xB_z - A_zB_x\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$ $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

,



Q1: For the vectors in the figure, with $a = 4$, $b = 3$, and $c = 5$, what are
(a) the magnitude and (b) the direction of $\vec{a} \times \vec{b}$,
(c) the magnitude and (d) the direction of $\vec{a} \times \vec{c}$, and
(e) the magnitude and (f) the direction of $\vec{b} \times \vec{c}$?
(The z axis is not shown. $+z$ direction is out of the page)
What about the average speed and velocity? Use t = 5.0 s for the total time.

Answer: Examining the figure, we see that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.
(a) $|\vec{a} \times \vec{b}| = (3.0)(4.0) = 12$ since the angle between them is 90º.
(b) Using the Right Hand Rule, the vector $\vec{a}\times\vec{b}$ points in the $\hat{\mathrm{i}}\times\hat{\mathrm{j}}=\hat{\mathrm{k}}$, or the $+z$ direction.
(c) $|\vec{a} \times \vec{c}| = |\vec{a} \times (-\vec{a} - \vec{b})| = |-(\vec{a} \times \vec{b})| = 12.$
(d) The vector $-\vec{a}\times\vec{b}$ points in the $-\hat{\mathrm{i}}\times\hat{\mathrm{j}}=-\hat{\mathrm{k}}$, or the $-z$ direction.
(e) $|\vec{b} \times \vec{c}| = |\vec{b} \times (-\vec{a} - \vec{b})| = |-(\vec{b} \times \vec{a})| = |(\vec{a} \times \vec{b})| = 12.$
(f) The vector points in the $+z$ direction, as in part (a).



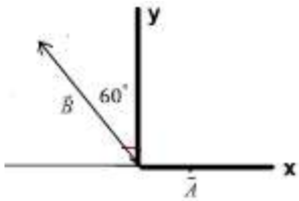
Q2: For the vectors shown in the Figure, $a = 4$, $b = 3$, and $c = 5$, calculate
(a) $\vec{a} \,.\, \vec{b}$, (b) $\vec{a} \,.\, \vec{c}$, and (c) $\vec{b} \,.\, \vec{c}$.

Answer: From the figure, it is clear that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.
(a) $\vec{a} \cdot \vec{b} = 0$ since the angle between them is 90°.
(b) $\vec{a} \cdot \vec{c} = \vec{a} \cdot (-\vec{a} - \vec{b}) = -|\vec{a}|^2 = -16$.
Another solution:
$\vec{a} \cdot \vec{c} = |\vec{a}||\vec{c}|\cos\Theta = 4\text{x}5\text{x}(4/5) = -16$.

(c) Similarly, $\vec{b} \cdot \vec{c} = -9.0$.

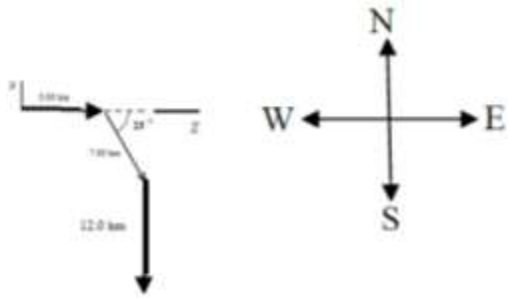
Q3. Find the sum of the following two vectors: $\vec{\mathrm{A}}$: 8.66 in +x-direction, $\vec{\mathrm{B}}$: 10.0 , at 60° from +y-axis measured counterclockwise.

Answer:
It is valuable to plot the vector in Cartesian coordinate.



$$\vec{\mathrm{A}}+\vec{\mathrm{B}}=8.66\,\hat{\mathrm{i}}+10.0\,\mathrm{Cos}\left(60^{\circ}\right)\hat{\mathrm{j}}-10.0\,\mathrm{Sin}\left(60^{\circ}\right)\hat{\mathrm{i}}\ =5.00\,\hat{\mathrm{j}}.$$

Q6: A man walks 3.00 km due East, then 7.00 km 25° South of East, and then 12.0 km due South. What is the final location, in km, of the man from the starting point?



Answer: use the figure to calculate the components of the resultant $\vec{A}$

$$A_x = 3 + 7\cos(25^\circ) \approx 9.34,$$

$$A_y = -12 - 7\sin(25^\circ) \approx -15$$

Q10: A certain vector ($\vec{A}$) in the xy plane has an x component of $A_x = 4.0$ m and a y component of $A_y = 10$ m. It is then rotated in the xy plane so its x component is doubled. Its new y component is about:

Answer:

The rotation will not change the magnitude of $\vec{A}$. If we have the vector $\vec{A'}$ after rotation, then

$$\left|\vec{A}\right|^2 = \left|\vec{A'}\right|^2 \Rightarrow 4^2 + 100^2 = 8^2 + y'^2$$

$$y' = \underline{7.2\text{ m}}$$

Q10. Vector $\vec{A}$ = 1.00 $\hat{i}$ +3.00 $\hat{j}$, vector $\vec{B}$ = 4.00 $\hat{i}$ −1.00 $\hat{j}$ and the vector $\vec{C}$ = 2.00 $\hat{k}$. Find the angle (in degrees) between vector $\vec{A}$ and vector $\vec{B}\times\vec{C}$.

A) 176

Ans:

$$\vec{D} = \vec{B} \times \vec{C} \; then \; angle \; between \; \vec{A} \; and \; \vec{D} = \theta = \cos^{-1}\left(\frac{A_x D_x + A_y D_y}{|A||B|}\right)$$

$$\vec{D} = \vec{B} \times \vec{C} = (4\vec{\imath} - 1.0\vec{\jmath}) \times 2\vec{k} = -8\vec{\jmath} - 2\vec{\imath}$$

$$|D| = \sqrt{68} = 8.25; \; |A| = \sqrt{10} = 3.16$$

$$\theta = \cos^{-1}\left(\frac{-2 - 24}{8.25 \times 3.16}\right) = 175.8^\circ \cong 176^\circ$$

Another solution:

Check the expression: $\vec{A}\times\vec{D} = \left|\vec{A}\right|\left|\vec{D}\right|\sin\theta\,\hat{k}$,

$$\vec{A}\times\vec{D} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 0 \\ -2 & -8 & 0 \end{vmatrix} = (-8+6)\hat{k} = -2\,\hat{k}$$, to find $\theta = -4^\circ + 180 = 176^\circ$